INVESTIGATION OF
SUPERSYMMETRIC PARTICLES
PRODUCTION IN HADRONS COLLISIONS

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Abstract

We present the SUSY QCD corrections to the production of a light Higgs boson ($M_h$) with a pair of supersymmetric scalar partners of the top quark ($\tilde{t}_1$) at CERN proton-proton Large Hadron Collider (LHC). Detailed results of the lowest order (LO) calculations of production are presented. The partonic processes responsible for the production involve both quark-antiquark ($q\bar{q}$) and gluon-gluon (gg) initial states, the dominant mechanism is the gluon-fusion. The lowest order cross-sections depend strongly on the renormalization/factorization scale. Compared to the lowest order, SUSY QCD corrections are significant and stabilize the cross sections.
Acknowledgements

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I specially dedicate this work and any progress in my life to my father soul and my mother.
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CHAPTER 1

Introduction

One of the major objectives of future high-energy experiments is to search for scalar Higgs particles and investigate the symmetry breaking mechanism of the electroweak interactions. In the standard model (SM) [1], one doublet of complex scalar fields is introduced to spontaneously break the symmetry, leading to a single neutral Higgs boson $h^0$. But there exists the problem of the quadratically divergent contributions to the corrections to the Higgs boson mass. This is the so-called naturalness problem. One of the hopeful methods, which can solve this problem, is the supersymmetric (SUSY) extension to the SM. In these extension models, the quadratic divergences of the Higgs boson mass can be cancelled by loop diagrams involving the supersymmetric partners of the SM particles exactly. The most attractive and simplest supersymmetric extension of the SM is the minimal supersymmetric standard model (MSSM)[2, 3]. In this model, there are two Higgs doublets $H_1$ and $H_2$ to give masses to up- and down-type fermions. The Higgs sector consists of three neutral Higgs bosons, one $CP$-odd particle ($A^0$), two $CP$-even particles ($h^0$ and $H^0$), and a pair of charged Higgs bosons ($H^\pm$).

However, these Higgs bosons haven’t been directly explored experimentally until now. The published experimental lower mass bounds for the Higgs bosons presented by LEP experiments are: $M_{h^0} > 114.4$ GeV (at 95% CL) for the SM Higgs boson, and for the MSSM bosons $M_{h^0} > 91.0$ GeV and $M_{A^0} > 91.9$ GeV (at 95% CL, $0.5 < \tan \beta < 2.4$ excluded). The SM fits to precision electroweak data [4] indirectly set a limitation of the light Higgs boson, $M_{h^0} < 200$ GeV, while there should has a scalar Higgs boson lighter than about 130 GeV in MSSM [5, 6].

If Supersymmetry (SUSY) exists, it will be discovered at the next generation of hadronic machines’, has been a recurring motto so far. Indeed sooner (at the Tevatron, $\sqrt{s} = 2$ TeV) or later (at the Large Hadron Collider (LHC), $\sqrt{s} = 14$ TeV), depending on the mass scale of the Higgs bosons and of the Superpartners of ordinary matter, several Supersymmetric
‘signatures’ should clearly be viable. Typical SUSY events at hadron colliders will involve either the production and decay of heavy spartons, squarks and gluinos, whose foreseen mass range is expected to be around the TeV scale [7], or of Higgs bosons [8], primarily of the lightest one, for which the SUSY theory imposes a stringent mass bound of the order of the electroweak (EW) scale. At the Large Hadron Collider (LHC), the most promising channel [9] for detecting the $h$ boson is the rare decay into two photons, with the Higgs particle dominantly produced via the gluon fusion mechanism [10]; the two LHC collaborations expect to detect the narrow $\gamma\gamma$ peak for $M_h \lesssim 130$ GeV, with an integrated luminosity $\int L dt \sim 300$ fb$^{-1}$ [9].

Supersymmetry predicts the existence of scalar partners $\tilde{t}_L, \tilde{t}_R$ to each SM chiral fermion, which mix to produce two mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$, and spin–1/2 partners to the gauge bosons and to the scalar Higgs bosons. In the MSSM, the genuine SUSY particles, neutralino/charginos and sfermions could have masses not too far from the electroweak symmetry breaking scale. In particular the lightest neutralino, which is expected to be the lightest SUSY particle (LSP), could have a mass in the range of $\sim 100$ GeV. Another particle which could also be light is one of the spin–zero partners of the top quark, the lightest stop $\tilde{t}_1$. Indeed, because of the large $m_t$ value, the two current stop eigenstates could strongly mix [11], leading to a mass eigenstate $\tilde{t}_1$ much lighter than the other squarks [which are constrained to be rather heavy [12, 13] by the negative searches at the Tevatron] and even lighter than the top quark itself. Similar features can also occur in the sbottom sector. These particles could therefore be also easily accessible at the next generation of hadron colliders.

If the mixing between third generation squarks is large, stops/sbottoms can not only be rather light but at the same time their couplings to Higgs bosons can be strongly enhanced. In particular, the (normalized) $h\tilde{t}_1\tilde{t}_1$ coupling can be the largest electroweak coupling in the MSSM. This might have a rather large impact on the phenomenology of the MSSM Higgs bosons as was stressed in Ref. [14]. The measurement of this important coupling would open a window to probe directly some of the soft–SUSY breaking terms of the potential. To measure Higgs–squarks couplings directly, one needs to consider the three–body associated production of Higgs bosons with scalar quark pairs which has been studied recently in [15, 16, 17]. The cross sections for the production of squarks ($\tilde{q}$) in association with Higgs boson in hadron collisions have been calculated at the Born level already quite some time ago [18, 19, 20].

In this thesis we study the production of neutral Higgs particles in association with stop pair ($pp \rightarrow \tilde{t}_1\tilde{t}_1^c, h$) in the minimal SUGRA at LHC. Theoretical predictions are improved by calculations of the next-to-leading order (NLO) SUSY-QCD corrections with the final-state squarks restricted to $\tilde{t}_1$. This NLO calculation is motivated by two requirements: First, to
stabilize the theoretical predictions for the cross sections with respect to the renormalization and factorization scales, which introduce spurious parameters into fixed-order calculations. And second, to improve the accuracy of the theoretical predictions for the cross sections. The structure of this Thesis is as follows: in chapter 2 we give the basic notation of the MSSM used throughout this Thesis; in chapter 3 we explore the renormalization of the MSSM. In chapter 4, we present analytical expressions of the matrix elements for the partonic and hadronic LO cross sections. Chapter 5 is dedicated to the classification of the NLO SUSY-QCD contributions into virtual and real corrections with the treatment of soft and collinear singularities. Numerical results and discussions are represented in chapter 6 and finally Chapter 7 is devoted to the general conclusions.
CHAPTER 2

The Minimal Supersymmetric Standard Model
(MSSM)

2.1 MSSM: the field content and Lagrangian

Supersymmetry (SUSY) can be introduced in many manners, maybe the most straightforward one is adding to the space-time coordinates \((t, \vec{x})\) another set of coordinates \(\theta_\alpha\) \((\alpha = 1, \ldots, n\) the space-time dimension\) that are Grassmann variables, i.e. they anti-commute. Now the general “rotations” in this space are a superset of the Poincaré transformations of space-time. It is clear that being \(\theta_\alpha\) Grassmann variables the generators of the rotations that involve these coordinates will behave in a special way, and indeed they do. These generators (usually called \(Q_\alpha\)) anti-commute with themselves, so they do not form an Algebra, but a Super-Algebra, and the SUSY transformations do not form a Lie Group. However it turns out that it is the only external transformation that can be added to the Poincaré Group, and leave the Scattering \((S)\) matrix untransformed. One can add as many “supersymmetries” (i.e. sets of \(\theta\) variables) as the dimension of the space-time, thus if we introduce a single set of \(\theta\) it is said that we have a \(N = 1\) supersymmetry, and so on. The structure of the full set of coordinates \((t, \vec{x}, \theta_\alpha)\) is called Superspace. For a \(N = 1\) SUSY, it was found that to any chiral fermion there should be a scalar particle with exactly the same properties. This fact is on the basis of the absence of quadratic divergences in boson mass renormalization, since for any loop diagram involving a scalar particle there should be a fermionic loop diagram, which will cancel quadratic divergences between each other, though logarithmic divergences remain.

As no scalar particles have been found at the electroweak scale we may infer that, if SUSY exists, it is broken. We can allow SUSY to be broken maintaining the property that no quadratic divergences are allowed: this is the so called Soft-SUSY-Breaking mechanism [21].
We can achieve this by only introducing a small set of SUSY-Breaking terms in the Lagrangian, to wit: masses for the components of lowest spin of a supermultiplet; and triple scalar interactions. However, other terms like explicit fermion masses for the matter fields would violate the Soft-SUSY-Breaking condition.

The simplest—and most popular—supersymmetric model is the straightforward supersymmetrization of the Standard Model (SM), where one introduces a minimal set of supersymmetric partner fields to the Standard Model [SM]. These fields contain scalar partners—sleptons and squarks—of all chiral eigenstates of the Dirac fermions, incorporated into chiral supermultiplets. Absorbing Standard Model gauge fields into vector supermultiplets leaves Majorana fermion partners of the neutral U(1), SU(2), SU(3) gauge fields and Dirac fermion partners of the charged SU(2) gauge fields, called gauginos. The SU(3) ghost fields are defined by re-writing the Fadeev-Popov determinant; therefore they do not receive supersymmetric partners, which enter by requiring the original Lagrangean being invariant under a global supersymmetry transformation. This is known as the \textit{Minimal Supersymmetric Standard Model} (MSSM) [22, 23, 24]. The MSSM is build up as follows:

- In addition to the gauge boson fields, spin-$\frac{1}{2}$ “gaugino” fields are introduced. The partners of $B_\mu$ and $W^i_\mu$ are denoted $\tilde{B}$ and $\tilde{W}^i$. In analogy to the photon, the $Z$ and the $W^\pm$ bosons, one can form a photino $\tilde{\gamma}$, a $Z$–ino $\tilde{Z}$, and $\tilde{W}^\pm$–inos from the $\tilde{B}$ and $\tilde{W}^i$ fields. The superpartners of the gluons are the gluinos $\tilde{g}$.

- Quarks and leptons get spin-0 partners called squarks and sleptons. As there has to be a superpartner for each degree of freedom, two bosonic fields are needed per SM fermion. They are named “left” and “right” states $\tilde{q}_L$, $\tilde{q}_R$ and $\tilde{\ell}_L$, $\tilde{\ell}_R$.

- Moreover, one needs two complex Higgs doublets with hypercharges $\pm 1$ in order to give masses to up– and down–type quarks and leptons, and to cancel anomalies. The Higgs fields are also assigned spin-$\frac{1}{2}$ partners, the so–called higgsinos.

The field content of the Minimal Supersymmetric Standard Model is shown in Table 2.1 (the tilde over the symbol of a particle denotes a superpartner of a usual particle).

The labels L or R for squarks or sleptons do not mean that they are left or right handed. Being spin zero particles they have no handedness. This is used to mark that they are superpartners of left or right handed quarks and leptons. The presence of the additional Higgs boson is a generic property of the supersymmetric theory. In the MSSM there are two doublets of scalar fields with quantum numbers $(1, 2, -1)$ and $(1, 2, 1)$.
Table 2.1: **Particle Content of the MSSM**

<table>
<thead>
<tr>
<th>Superfield</th>
<th>Bosons</th>
<th>Fermions</th>
<th>$SU_c(3)\times SU_L(2)\times U_Y(1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gauge</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G^a$</td>
<td>gluon</td>
<td>$g^a$</td>
<td>gluino $\tilde{g}^a$</td>
</tr>
<tr>
<td>$V^k$</td>
<td>Weak</td>
<td>$W^k$ ($W^\pm, Z$)</td>
<td>wino, zino $\tilde{w}^k (\tilde{w}^\pm, \tilde{z})$</td>
</tr>
<tr>
<td>$V'$</td>
<td>Hypercharge</td>
<td>$B (\gamma)$</td>
<td>bino $\tilde{b}(\gamma)$</td>
</tr>
<tr>
<td><strong>Matter</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_i$</td>
<td>sleptons</td>
<td>$\tilde{L}_i = (\tilde{\nu}, \tilde{e})_L$</td>
<td>leptons $L_i = (\nu, e)_L$</td>
</tr>
<tr>
<td>$E_i$</td>
<td></td>
<td>$\tilde{E}_i = \tilde{e}_R$</td>
<td>$E_i = e_R$</td>
</tr>
<tr>
<td>$Q_i$</td>
<td></td>
<td>$\tilde{Q}_i = (\tilde{u}, \tilde{d})_L$</td>
<td>quarks $Q_i = (u, d)_L$</td>
</tr>
<tr>
<td>$U_i$</td>
<td></td>
<td>$\tilde{U}_i = \tilde{u}_R$</td>
<td>$U_i = u_R^c$</td>
</tr>
<tr>
<td>$D_i$</td>
<td></td>
<td>$\tilde{D}_i = \tilde{d}_R$</td>
<td>$D_i = d_R$</td>
</tr>
<tr>
<td><strong>Higgs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H_1$</td>
<td>Higgses</td>
<td>$H_1$</td>
<td>higgsinos $\tilde{H}_1$</td>
</tr>
<tr>
<td>$H_2$</td>
<td></td>
<td>$H_2$</td>
<td>$\tilde{H}_2$</td>
</tr>
</tbody>
</table>

Table 2.1 does not contain gravitational fields. Supersymmetry (SUSY) in its local version includes gravity; the resulting theory is know as *supergravity*. The model then also includes the graviton (spin–2) and its fermionic partner the gravitino (spin–$\frac{3}{2}$).

The Lagrangian of the MSSM consists of two parts; the first part is SUSY generalization of the Standard Model, while the second one represents the SUSY breaking. The supersymmetric part of the Lagrangian consists of the gauge invariant kinetic terms corresponding to the $SU(3)\times SU(2)_L\times U(1)_Y$ gauge groups depending on 3 gauge couplings as in the Standard Model and of the superpotential. Usually the superpotential is chosen in the form repeating that of the Yukawa interaction in the SM. Let us now turn to the Lagrangian of the MSSM. Clearly, gauge interactions are determined by the gauge group, which is the same as in the Standard Model: $SU(3)_C\times SU(2)_L\times U(1)_Y$. For a concise derivation of these interactions, see e.g. [23, 25, 26]. Masses and couplings of the matter fields are determined by the superpotential $W$. The choice of the gauge group constrains $W$ but does not fix it completely. Holding to the principle of minimality, that means introducing only those terms that are
necessary to build a consistent model, we get:

\[
\mathcal{W} = \sum_{i,j=1}^{3} \left[ (h_E)_{ij} H_1 L_i E_j^c + (h_D)_{ij} H_1 Q_i D_j^c + (h_U)_{ij} Q_i H_2 U_j^c \right] + \mu H_1 H_2 \tag{2.1}
\]

where \(i\) and \(j\) are generation indices. Contractions over SU(2) and SU(3) indices are understood. In particular,

\[
H_1 H_2 \equiv \epsilon^{\alpha\beta} H_{1\alpha} H_{2\beta} = H_0^1 H_0^2 - H_1^0 - H_2^0 \tag{2.2}
\]

with \(\epsilon^{\alpha\beta}\) a totally anti–symmetric tensor used to contract over the SU(2) weak isospin indices \(\alpha, \beta = 1, 2\). Likewise, \(H_1^a Q_D^c \equiv \epsilon^{\alpha\beta} H_{1\alpha} Q_a^\beta D_c^a\) where \(a = 1, 2, 3\) is a colour index, etc.

The 3×3 matrices \(h_D\), \(h_U\), and \(h_E\) are dimensionless Yukawa couplings giving rise to quark and lepton masses. Moreover, \(h_D\) and \(h_U\) account for the mixing between the quark current eigenstates as described by the CKM matrix [27]. Notice that the same superpotential is obtained by requiring that baryon and lepton numbers be conserved (which is automatically fullfilled in the SM but not in the MSSM).

The Lagrangian derived from (2.1) is

\[
\mathcal{L}_{SUSY} = -\left[ \sum_{j,k} \frac{\partial^2 \mathcal{W}}{\partial \phi_j \partial \phi_k} \psi_j \psi_k + h.c. \right] - \sum_{j} \left| \frac{\partial \mathcal{W}}{\partial \phi_j} \right|^2 \tag{2.3}
\]

where \(\phi_i\) are scalar and \(\psi_i\) fermion fields; \(\mathcal{W}\) only depends on the scalar fields. The first term in (2.3) describes masses and Yukawa interactions of fermions, while the second term describes scalar mass terms and scalar interactions.

The interactions obtained in this way respect a symmetry called R–parity under which the “ordinary” fields (matter fermions, Higgs and gauge bosons) are even while their superpartners (sfermions, higgsinos and gauginos) are odd.

\[
R = (-1)^{3(B-L)+2S} \tag{2.4}
\]

where \(B\) — is the baryon number, \(L\) — is the lepton number, and \(S\) — is the spin of the particle. As a consequence, all interactions involve an even number of SUSY particles (“sparticles”). This means that sparticles can only be produced in pairs, and any sparticle decay must lead to an odd number of sparticles. Hence in the MSSM the lightest supersymmetric particle (LSP) is stable. This leads to another important feature of the MSSM: Since the LSPs cannot decay some of them must have survived from the Big
Bang era. Searches for so-called “exotic isotopes” have led to very stringent bounds which exclude any strongly or electromagnetically interacting stable particles with masses below a few TeV. The LSP of the MSSM must therefore be electrically and colour neutral [28]. In turn, a neutral stable LSP is a good candidate for dark matter. For collider experiments this means that any decay chain of a sparticle will end in an arbitrary number of SM particles plus at least one LSP which escapes the detector, carrying away some energy and momentum. In the context of the MSSM the typical SUSY signature is thus distinguished by missing (transverse) energy/momentum. Depending on different variants of the model, the LSP can be the lightest neutralino or the gravitino; the sneutrino has already been ruled out [29]. We want to stress that R–parity is a symmetry that is somehow built in by hand due to our assumption of strict minimality. In principle, the following R–parity violating terms are allowed:

\[ W_R = \lambda L L E^c + \lambda' L Q D^c + \lambda'' D^c D^c U^c + \mu' H_1 L. \] (2.5)

Here also generation indices have been suppressed. Within the MSSM breaking R–parity therefore means breaking baryon and/or lepton number.

The scalar potential \( V \) is obtained from \( F \)– and \( D \)–terms:

\[ V = F_i^* F_i + \frac{1}{2} \left[ D_a D^a + D^i D^i + (D')^2 \right] \] (2.6)

with

\[ F_i = \frac{\partial W(\phi_i)}{\partial \phi_i}, \] (2.7)

and

\[ D^a = g_3 \phi^* T_a \phi, \quad D^i = \frac{1}{2} g_2 \phi^* \sigma^i \phi, \quad D' = \frac{1}{2} g_1 Y_\phi \phi^*_i \phi_i. \] (2.8)

\( T^a = \lambda^a/2 \) \((a = 1, \ldots 8)\) with \( \lambda^a \) the Gell-Mann–Low matrices. \( \sigma^i \) \((i = 1, \ldots 3)\) are the Pauli matrices and \( Y_\phi = 2(Q - I_3) \) is the hypercharge.

The Lagrangian as given in (2.3) conserves supersymmetry. However, in a realistic model SUSY must be broken. Otherwise the masses of ordinary particles and their superpartners would be equal, which is not the case as we know from experiment. As the genuine mechanism of (dynamical) SUSY breaking is not yet understood, we parametrize it by inserting SUSY breaking terms by hand into the Lagrangian. The terms that break SUSY softly, i.e. do not induce quadratic divergencies, are [30]
• gaugino mass terms $-\frac{1}{2}M_a\bar{\lambda}_a\lambda_a$, where $a$ is the group index;

• scalar mass terms $-M_{\phi_i}^2|\phi_i|^2$;

• trilinear scalar interactions $A_{ijk}\phi_i\phi_j\phi_k$; and

• bilinear terms $-B_{ij}\phi_i\phi_j + h.c.$.

They lead to the following explicit form of the soft SUSY breaking Lagrangian:

$$-\mathcal{L}_{\text{soft}} = \frac{1}{2}M_1\bar{B}B + \frac{1}{2}M_2\bar{W}W + \frac{1}{2}M_3\bar{g}g + m^2_H_1|H_1|^2 + m^2_H_2|H_2|^2$$

$$+ M_Q^2|\bar{q}_L|^2 + M_U^2|\bar{u}_R|^2 + M_D^2|\bar{d}_R|^2 + M_L^2|\bar{\ell}_L|^2 + M_E^2|\bar{\ell}_R|^2$$

$$+ \left( h_{AE}H_1\bar{\ell}_L\bar{c}_R + h_{AD}A_DH_1\bar{q}_L\bar{d}_R + h_{AU}A_UH_2\bar{q}_L\bar{u}_R + B\mu H_1H_2 + h.c. \right).$$

(2.9)

$M_1$, $M_2$, and $M_3$ are the U(1), SU(2), and SU(3) gaugino masses, respectively. $m^2_{H_1}$, $m^2_{H_2}$, and $B\mu$ are mass terms for the Higgs fields. The scalar mass terms $M_Q^2$, $M_U^2$, $M_D^2$, $M_L^2$, and $M_E^2$ are in general hermitean $3 \times 3$ matrices in generation space, while $h_{AE}A_E$, $h_{AD}A_D$, and $h_{AU}A_U$ are general $3 \times 3$ matrices. Allowing all the parameters in (2.9) to be complex, we end up with 124 masses, phases and mixing angles as free parameters of the model. Notice that $\mathcal{L}_{\text{soft}}$ also respects R–parity. Indeed, a R–parity violating term in Eq. (2.9) would lead to an unstable vacuum unless the same term also appears in the superpotential Eq. (2.1).

### 2.2 Renormalization Group Equations

The physical quantities at the electroweak scale are related to their values at some high energy scale by renormalization group (RG) equations. We will not perform a RG analysis here but just add some quantitative arguments:

One of the very appealing features of the MSSM is that it allows for gauge coupling unification at $M_X \sim 10^{16}$ GeV [31]. The MSSM is thus compatible with a Grand Unified Theory (GUT). As in GUT models the gauginos all live in the same representation of the unified gauge group, gaugino masses are also unified at scales $Q \geq M_X$. The 1–loop RG equations [32] for the gauge couplings and the gaugino masses are

$$\frac{d}{dt} g_a = \frac{b_a}{16\pi^2} g_a^3,$$

$$\frac{d}{dt} M_a = \frac{b_a}{8\pi^2} g_a^2 M_a$$

(2.10)
with $t = \ln(Q/M_X)$ and $b_a = 33/5, 1, -3$ for $a = 1, 2, 3$, respectively. One therefore has
\[
\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2}
\] (2.11)
at any RG scale, up to small 2–loop effects.

We next consider the evolution of scalar masses. For simplicity we here assume that the soft masses of squarks and sleptons are flavour–diagonal e.g., $M_Q^2 = \text{diag}(M_Q^2, M_{\tilde{Q}}^2, M_{\tilde{Q}}^2)$. Moreover, we neglect Yukawa and trilinear couplings of the first and second generation, i.e. $h_U = \text{diag}(0, 0, h_t)$, $A_U = \text{diag}(0, 0, A_t)$, etc.. The 1–loop RG equations for the masses of squarks and sleptons of the first two generations are then given by
\[
16\pi^2 \frac{d}{dt} M_\phi^2 = -\sum_{a=1}^{3} 8g_a^2 C_\phi^a |M_a|^2
\] (2.12)
with the sum running over the gauge groups. The $C_\phi^a$ are Casimir operators: $C_1^\phi = \frac{3}{5} \left(\frac{Y_\phi}{2}\right)^2$ for each scalar $\phi$ with hypercharge $Y_\phi$ ($Y_\tilde{Q} = \frac{1}{3}$, $Y_u = -\frac{4}{3}$, $Y_\tilde{d} = \frac{2}{3}$, etc.); $C_2^\phi = 3/4 \ (0)$ for $\phi = \tilde{Q}, \tilde{U}, \tilde{D}$; and $C_3^\phi = 4/3 \ (0)$ for $\phi = \tilde{Q}, \tilde{U}, \tilde{D}$ ($\tilde{L}, \tilde{E}$). The right side of Eq. (2.12) is strictly negative, so $M_\phi^2$ grows when being evolved down to the low scale. Moreover, owing to SU(3) contributions, squark masses grow faster than slepton masses.

The mass–squared parameters of the third generation squarks and sleptons as well as of the Higgs fields also obey (2.12) but get additional contributions from Yukawa and trilinear couplings:
\[
16\pi^2 \frac{d}{dt} M_{Q_3}^2 = X_t + X_b - \frac{32}{3} g_3^2 |M_3|^2 - 6g_2^2 |M_2|^2 - \frac{2}{15} g_1^2 |M_1|^2, \tag{2.13}
\]
\[
16\pi^2 \frac{d}{dt} M_{U_3}^2 = 2X_t - \frac{32}{3} g_3^2 |M_3|^2 - \frac{32}{15} g_1^2 |M_1|^2, \tag{2.14}
\]
\[
16\pi^2 \frac{d}{dt} M_{D_3}^2 = 2X_b - \frac{32}{3} g_3^2 |M_3|^2 - \frac{8}{15} g_1^2 |M_1|^2, \tag{2.15}
\]
\[
16\pi^2 \frac{d}{dt} M_{L_3}^2 = X_\tau - 6g_2^2 |M_2|^2 - \frac{3}{5} g_1^2 |M_1|^2, \tag{2.16}
\]
\[
16\pi^2 \frac{d}{dt} M_{E_3}^2 = 2X_\tau - \frac{24}{5} g_1^2 |M_1|^2, \tag{2.17}
\]
and

\[ 16 \pi^2 \frac{d}{dt} m_{H_1}^2 = 3X_b + X_\tau - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2, \]  
\[ 16 \pi^2 \frac{d}{dt} m_{H_2}^2 = 3X_t - 6g_2^2 |M_2|^2 - \frac{6}{5} g_1^2 |M_1|^2, \]

with

\[ X_t = 2|h_t|^2 (m_{H_2}^2 + M_{Q_3}^2 + M_{U_3}^2) + 2|A_t|^2, \]  
\[ X_b = 2|h_b|^2 (m_{H_1}^2 + M_{Q_3}^2 + M_{D_3}^2) + 2|A_b|^2, \]  
\[ X_\tau = 2|h_\tau|^2 (m_{H_1}^2 + M_{L_3}^2 + M_{E_3}^2) + 2|A_\tau|^2. \]

\( X_t, X_b, \) and \( X_\tau \) are always positive, so they decrease the scalar masses as one runs downwards to the low scale. Therefore, the soft breaking parameters of the third generation are in general smaller than those of the first and second generation [unless one starts with very different values at the high scale].

The RG equations for \( m_{H_1}^2 \) and \( m_{H_2}^2 \) are still a special case. Compared to those for the soft squark masses they do not get the large contributions proportional to \( |M_3|^2 \). Moreover, \( X_t \) and \( X_b \) enter with larger coefficients. This can cause \( m_{H_1}^2 \) or \( m_{H_2}^2 \) to become negative near the electroweak scale [33]. Since this leads to a breakdown of the electroweak symmetry solely by quantum corrections this is called radiative electroweak symmetry breaking [34].

Here we note that owing to the large top quark mass one can expect an especially large effect from \( X_t \), favoring \( m_{H_2}^2 < m_{H_1}^2 \) (unless \( h_t \sim h_b \)).

The gaugino masses do not only enter Eqs. (2.12) to (2.19) but also the RG equations for the \( A \) parameters. Non–zero gaugino masses at \( M_X \) are therefore sufficient to create all the other soft SUSY breaking terms. On the other hand, if the gaugino masses vanished at \( M_X \) they would only be generated through 2–loop and higher order effects and hence be very small.

Another (technical) remark seems appropriate at this point: For computing radiative corrections in supersymmetry, it is important to choose regularization and renormalization schemes that preserve supersymmetry (and gauge symmetry, of course). Dimensional regularization (DREG), for instance, explicitly violates SUSY because the continuation of the number of spacetime dimensions to \( D = 4 - \epsilon \) introduces a mismatch between the number of gauge boson and gaugino degrees of freedom. A solution is to perform the momentum integrals in \( D = 4 - \epsilon \) dimensions while taking the vector index \( \mu \) of the gauge boson fields.
over all four dimensions [35, 36, 37]. This is known as \textit{dimensional reduction} (DRED) and nicely respects both gauge symmetry and SUSY, at least up to 2–loop order. We will come back to this when discussing supersymmetric QCD corrections in Chapter 2.

\subsection{2.3 Models of SUSY Breaking}

Above we have introduced explicit SUSY breaking terms because we are ignorant of the fundamental mechanism that breaks supersymmetry. If SUSY is broken spontaneously there exists a Goldstone fermion called the goldstino. In global supersymmetry the goldstino is massless. In local supersymmetry (supergravity) the goldstino is “eaten” by the gravitino ($\tilde{g}_{3/2}$) which in this way acquires a mass $m_{3/2}$ [38]. This is called the \textit{super–Higgs mechanism} and is completely analogous to the ordinary Higgs mechanism in gauge theories.

Present models of spontaneously–broken low–energy supersymmetry assume that SUSY is broken in a “hidden” or “secluded” sector which is completely neutral with respect to the SM gauge group. The information of SUSY breaking is then mediated to the “visible” sector, which contains the MSSM, by some mechanism. There are no renormalizable tree–level interactions between the hidden and visible sectors. Two scenarios have been studied in detail: gravity–mediated and gauge–mediated SUSY breaking.

In \textit{gravity–mediated SUSY breaking}, SUSY breaking is transmitted to the MSSM via gravitational interactions [39]. The breakdown of the symmetry occurs at $O(10^{10})$ GeV or higher, and the gravitino gets a mass of the order of the electroweak scale. The simplest realization of such a framework is the \textit{minimal supergravity model} (mSUGRA) [22, 40]. In this approach, one assumes a universal gaugino mass $M_{1/2}$, a universal scalar mass $M_0$, and a universal cubic coupling $A_0$ at $M_X$. In addition, one just needs to specify $\tan \beta$ and the sign of $\mu_0$. RGEs are then used to derive the MSSM parameters at the electroweak scale. Since mSUGRA involves only five parameters (in addition to the 18 SM parameters) it is highly predictive and thus used for most experimental searches. However, one should keep in mind that it also highly restrictive.

\textit{Gauge–mediated SUSY breaking} (GMSB) [41] models involve a “secluded” sector where SUSY is broken and a “messenger” sector consisting of particles with SU(3)×SU(2)×U(1) quantum numbers. The messengers directly couple to the particles of the secluded sector. This generates a SUSY breaking spectrum in the messenger sector. Finally, SUSY breaking is mediated to the MSSM via virtual exchange of the messengers. A basic feature of such models is that SUSY is broken at much lower scales than in the gravity–mediated case, typically at $O(10^4–10^5)$ GeV. Moreover, the gravitino gets a mass in the eV to keV range,
and is therefore the LSP. This can be crucial for SUSY signatures at collider experiments because the next–to–lightest SUSY particle (NLSP) will eventually decay into its SM partner plus a gravitino. A long–lived $\tilde{\chi}_1^0$–NLSP that decays outside the detector leads to the usual SUSY signature of large missing energy plus leptons and/or jets. If, in contrast, the decay $\tilde{\chi}_1^0 \rightarrow \gamma \tilde{g}_{3/2}$ occurs inside the detector SUSY events would in addition contain photons. The NLSP may, however, also be a charged particle e.g., $\tilde{\tau}_L^\pm$. This would lead either to a long–lived charged particle or to SUSY signatures characterized by $\tau$–leptons.

Since gauge interactions are flavour–blind one has universal boundary conditions in GMSB as in mSUGRA. The low–energy spectrum is determined by the mass of the messengers. Minimal GMSB is thus even more restrictive than mSUGRA. In the most general case, however, both supergravity and gauge–mediated effects may contribute to the breaking of supersymmetry.

A more detailed discussion of SUSY breaking is beyond the scope of this thesis. For a thorough introduction to supergravity, see e.g. [26]; for the phenomenology of mSUGRA, see e.g. [42, 43]. A review of gauge–mediated SUSY breaking is given in [44].

2.4 Electroweak Symmetry Breaking
(The Higgs sector of the MSSM)

The scalar potential for the Higgs fields — including all soft–breaking terms — is

$$
V_{\text{Higgs}} = (|\mu|^2 + m_{H_1}^2) |H_1|^2 + (|\mu|^2 + m_{H_2}^2) |H_2|^2 + (B \mu H_1 H_2 + \text{h.c.})
+ \frac{1}{8} (g_1^2 + g_2^2) (|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g_2^1 g_2^0 |H_1^+ H_2^0 + H_2^0 H_1^-|^2.
$$

(2.23)

Here the terms proportional to $|\mu|^2$ come from $F$–terms, the quartic interactions come from $D$–terms, and the terms proportional to $m_{H_1}^2$, $m_{H_2}^2$ and $B \mu$ come from soft SUSY breaking, see Eq. (2.9).

First, we use SU(2)$_L$ gauge transformations to rotate away any VEV of one of the charged Higgs fields e.g., $\langle H_1^- \rangle = 0$. Then $\partial V_{\text{Higgs}} / \partial H_1^- = 0$ requires $\langle H_2^+ \rangle = 0$ as well. This is good because we are now sure that electric charge is conserved in the Higgs sector. We shall thus ignore the charged components in (2.23) when minimizing the potential.

Next, we choose $B \mu$, the only term in (2.23) that depends on complex phases, to be real and positive. This can be done through a redefinition of the phases of $H_1$ and $H_2$. Then $\langle H_1^0 \rangle$ and $\langle H_2^0 \rangle$ are also real. This means that CP is not spontaneously broken by the Higgs scalar potential. The Higgs mass eigenstates are thus also eigenstates of CP.
The scalar potential has a local minimum other than its origin if
\[(|\mu|^2 + m_{H_1}^2)(|\mu|^2 + m_{H_2}^2) < (B\mu)^2.\] (2.24)

However, this is not enough; $V_{\text{Higgs}}$ must also be bounded from below. This is the case if
\[(|\mu|^2 + m_{H_1}^2) + (|\mu|^2 + m_{H_2}^2) \geq 2B\mu.\] (2.25)

These two conditions can only be satisfied simultaneously if $m_{H_1} \neq m_{H_2}$ — implying that in the MSSM electroweak symmetry breaking is not possible without first breaking SUSY!

We are finally ready to minimize the Higgs potential by solving $\partial V_{\text{Higgs}}/\partial H_0^1 = \partial V_{\text{Higgs}}/\partial H_0^2 = 0$. The VEVs at the minimum of the potential, $v_1 \equiv \langle H_1^0 \rangle$ and $v_2 \equiv \langle H_2^0 \rangle$, are related by
\[v^2 \equiv v_1^2 + v_2^2 = 2m_Z^2/(g_1^2 + g_2^2) \approx (174 \text{ GeV})^2,\] (2.26)
so that only the ratio of the two remains a free parameter. Defining
\[\tan \beta \equiv v_2/v_1\] (2.27)
the minimalization conditions are given by
\[|\mu|^2 + m_{H_1}^2 = -B\mu \tan \beta - \frac{1}{2} m_Z^2 \cos 2\beta,\] (2.28)
\[|\mu|^2 + m_{H_2}^2 = -B\mu \cot \beta + \frac{1}{2} m_Z^2 \cos 2\beta.\] (2.29)

The two complex scalar Higgs doublets consist of eight degrees of freedom, three of which are eaten by the longitudinal modes of the $Z$ and $W$ bosons. The remaining five physical degrees of freedom form a neutral CP–odd, two neutral CP–even and two charged Higgs bosons denoted by $A^0$, $h^0$, $H^0$, and $H^\pm$, respectively. They are given by
\[
\begin{bmatrix}
G^0 \\
A^0
\end{bmatrix} = \sqrt{2} \begin{bmatrix}
\sin \beta & \cos \beta \\
-\cos \beta & \sin \beta
\end{bmatrix} \begin{bmatrix}
\text{Im}(H_2^0) \\
\text{Im}(H_1^0)
\end{bmatrix},
\] (2.30)
\[
\begin{bmatrix}
G^+ \\
H^+
\end{bmatrix} = \begin{bmatrix}
\sin \beta & \cos \beta \\
-\cos \beta & \sin \beta
\end{bmatrix} \begin{bmatrix}
H_2^+ \\
H_1^-
\end{bmatrix}
\] (2.31)
with $G^0$ and $G^+$ the would–be Nambu–Goldstone bosons, and
\[
\left( \begin{array}{c}
H^0 \\
H^0
\end{array} \right) = \sqrt{2} \left( \begin{array}{cc}
\cos \alpha & \sin \alpha \\
-\sin \alpha & \cos \alpha
\end{array} \right) \left( \begin{array}{c}
\text{Re}(H^0_2) - v_2 \\
\text{Re}(H^0_1) - v_1
\end{array} \right)
\] (2.32)

with the Higgs mixing angle \( \alpha \). [\text{Re}(H) \) denotes the real and \text{Im}(H) \) the imaginary part of \( H \).] The mass eigenvalues at tree level are

\[
m^2_A = 2B\mu/\sin 2\beta, \\
m^2_{H^\pm} = m^2_A + m^2_W, \\
m^2_{h^0,H^0} = \frac{1}{2} \left( m^2_A + m^2_Z \mp \sqrt{(m^2_A + m^2_Z)^2 - 4m^2_A m^2_Z \cos 2\beta} \right).
\] (2.35)

We can therefore take \( m_A \) and \( \tan \beta \) as the free parameters of the MSSM Higgs sector. From (2.35) it follows that

\[
m_{h^0} < \min(m_Z, m_A) |\cos 2\beta|
\] (2.36)

at tree level. However, the Higgs masses and mixing angle are subject to large radiative corrections. In this thesis we use the formulae of Ref. [45] for the \( (h^0) \). Assuming that the masses of the sparticles in the loops do not exceed 1 TeV one obtains an upper bound for the lightest Higgs mass of

\[
m_{h^0} \lesssim 130 \text{ GeV}.
\] (2.37)

On the other hand, \( A^0, H^0, \) and \( H^\pm \) can be almost arbitrarily heavy. If \( m_A \gg m_Z \) they become nearly degenerate and decouple from the low–energy regime. In this case \( h^0 \) is very difficult to distinguish from a Standard Model Higgs boson. Up–type quarks get masses proportional to \( v_2 = v \sin \beta \); the masses of down–type quarks and electron–type leptons are proportional to \( v_1 = v \cos \beta \). At tree level the quark and lepton masses are therefore related to the respective Yukawa couplings by

\[
h_u = \frac{g m_u}{\sqrt{2} m_W \sin \beta}, \quad h_d = \frac{g m_d}{\sqrt{2} m_W \cos \beta}, \quad h_e = \frac{g m_e}{\sqrt{2} m_W \cos \beta}.
\] (2.38)

This is why we neglected the Yukawa couplings of the first and second generation, but not those of \( t, b, \) and \( \tau \). Obviously, \( h_t \) is significant due to the large top quark mass. Moreover, \( h_t : h_b : h_\tau = m_t : m_b \tan \beta : m_\tau \tan \beta \) from (2.38), so also the bottom and tau Yukawa couplings are important if \( \tan \beta \) is large. In fact, certain models predict the unification of top and bottom (or top, bottom, and tau) Yukawa couplings for \( \tan \beta \sim m_t/m_b \). One can, moreover, require that none of the Yukawa couplings become nonpertubatively large. This
gives rough bounds of $1.2 \lesssim \tan \beta \lesssim 65$.

### 2.5 MSSM parametrization

#### 2.5.1 MSSM parameters

If SUSY were an exact symmetry then only one parameter should be added to the SM, but we have to deal with a plethora of Soft-SUSY-Breaking parameters, namely

- masses for Left- and Right-chiral sfermions,
- a mass for the Higgs sector,
- gaugino masses,
- triple scalar couplings for squarks and Higgs.

This set of parameters is often simplified to allow a comprehensive study. Most of these simplifications are based on some universality assumption at the unification scale. In minimal supergravity (mSUGRA) all the parameters of the MSSM are computed from a restricted set of parameters at the Unification scale, to wit: $\tan \beta$; a common scalar mass $m_0$; a common fermion mass for gauginos $m_{1/2}$; a common trilinear coupling for all sfermions $A_0$; and the higgsino mass parameter $\mu$. Then one computes the running of each one of these parameters down to the EW scale, using the Renormalization Group Equations (RGE), and the full spectrum of the MSSM is found.

We will not restrict ourselves to such a simplified model. As stated in the introduction we treat the MSSM as an effective Lagrangian, to be embedded in a more general framework that we don’t know about. This means that essentially all the parameters quoted above are free. However for the kind of studies we have performed there is an implicit asymmetry of the different particle generations. We are mostly interested in the phenomenology of the third generation, thus we will treat top and bottom supermultiplet as distinguished from the rest. This approach is well justified by the great difference of the Yukawa couplings of top and bottom with respect to the rest of fermions. We are mainly interested on effects on the Higgs sector, so the smallness of the Yukawa couplings of the first two generations will result on small effects in our final result. We include them, though, in the numerical analysis and the numerical dependence is tested. On the other hand, if we suppose that there is unification at some large scale, at which all sfermions have the same mass, and then evolve these masses to the EW scale, then the RGE have great differences[46]. Slepton RGE are
dominated by EW gauge interactions, 1st and 2nd generation squarks RGE are dominated by QCD, and for the 3rd generation squarks there is an interplay between QCD and Yukawa couplings. Also, as a general rule, the gauge contribution to the RGE equations of left- and right-handed squark masses are similar, so when Yukawa couplings are not important they should be similar at the EW scale.

With the statement above in mind we can simplify the MSSM spectrum by taking an unified parametrization for 1st and 2nd generation squarks (same for sleptons). We will use: a common mass for $\tilde{u}_L$ and $\tilde{q}_R$ ($m_{\tilde{u}}$); an unified trilinear coupling $A_u$ for 1st and 2nd generation; a common mass for all $\tilde{\nu}_L$ and $\tilde{l}_R$ ($m_{\tilde{\nu}}$); and a common trilinear coupling $A_\tau$.

For the third generation we will use different trilinear couplings $A_t$ and $A_b$, as these can play an important role in the kind of processes we are studying. Stop masses can present a large gap (due to its Yukawa couplings), being the right-handed stop the lightest one. We will use a common mass for both chiral sbottoms, which we parametrize with the lightest sbottom mass ($m_{\tilde{b}_1}$), and the lightest stop quark mass ($m_{\tilde{t}_1}$), as the rest of mass inputs in this sector. This parametrization is useful in processes where squarks only appear as internal particles in the loops (such as the ones studied in, as one-loop corrections to these parameters would appear as two-loop effects in the process subject of study. However in we deal with squarks as the main subject of the process and in this case a more physical set of inputs must be used. We have chosen to use the physical sbottom masses ($m_{\tilde{b}_1}$, $m_{\tilde{b}_2}$) and the sbottom and stop mixing angles ($\theta_{\tilde{b}}$, $\theta_{\tilde{t}}$) to be our main inputs. Again one-loop effects on other parameters (such as $A_b$) would show up as two-loop corrections to the observables we are interested in.

For the same reasons EW gaugino sector is also supposed to have small effects in our studies. Gluino mass ($m_{\tilde{g}}$), on the other hand, is let free.

For the Higgs sector two choices are available, we can use the pseudoscalar mass $M_{A^0}$, or the charged Higgs mass $m_{H^\pm}$. Both choices are on equal footing. As the charged Higgs particle is a main element for most of our studies we shall use its mass as input parameter in most of our work. However in it is more useful to use $M_{A^0}$.

### 2.5.2 Constraints

The MSSM reproduces the behaviour of the SM up to energy scales probed so far [47]. Obviously this is not for every point of the full parameter space!

There exists direct limits on sparticle masses based on direct searches at the high energy colliders (LEP II, SLC, Tevatron). Although hadron colliders can achieve larger center of
mass energies than \( e^+e^- \) ones, its samples contain large backgrounds that make the analysis more difficult. This drawback can be avoided if the ratio signal-to-background is improved, in fact they can be used for precision measurements of “known” observables (see e.g. [48]). \( e^+e^- \) colliders samples are more clean, and they allow to put absolute limits on particle masses in a model independent way.

The most stringent bound to the MSSM parameter space is the LEP II bound to the mass of charged particles beyond the SM. At present [49, 50, 51] this limit is roughly

\[
M_{\text{charged}} \gtrsim 90 \text{ GeV} .
\]  

(2.39)

Specific searches for Supersymmetric particles are being performed at LEP II, negative neutralino searches rise up a limit on neutralino masses of[50]

\[
M_{\chi_1^0} \gtrsim 30 \text{ GeV} ,
\]  

(2.40)

it turns out that after translating this limit to the \( \mu - \mu \) parameters it is less restrictive than the one obtained for the charginos from (2.39).

Actual Higgs searches at LEP II imply that, for the MSSM neutral Higgs sector[52]

\[
m_h > 72.2 \text{ GeV} , \ M_{A^0} > 76.1 \text{ GeV} .
\]  

(2.41)

Notice that without the MSSM relations there is no model independent bound on \( M_{A^0} \) from LEP [53]. Actual fits to the MSSM parameter space project a preferred value for the charged Higgs mass of \( m_{H^\pm} \simeq 120 \text{ GeV} \) [54].

Hadron colliders bounds are not so restrictive as those from \( e^+e^- \) machines. Most bounds on squark and gluino masses are obtained by supposing squark mass unification in simple models, such as mSUGRA. At present the limits on squarks (1st and 2nd generation) and gluino masses are[55]

\[
m_{\tilde{g}} > 176 \text{ GeV} , \ m_{\tilde{g}} > 173 \text{ GeV} .
\]  

(2.42)

From the top quark events at the Tevatron a limit on the branching ratio \( BR(t \to H^+ b) \) can be extracted, and thus a limit on the \( \tan \beta - m_{H^\pm} \) relation.

Finally indirect limits on sparticle masses are obtained from the EW precision data. We apply these limits through all our computations by computing the contribution of sparticles to these observables and requiring that they satisfy the bounds from EW measurements. We require new contributions to the \( \rho \) parameter to be smaller than present experimental error
on it, namely

\[ \delta \rho_{\text{new}} < 0.003 . \]  

(2.43)

We notice that as \( \delta \rho_{\text{new}} \) is also the main contribution from sparticle contributions to \( \Delta r \) [56], new contributions to this parameter are also below experimental constrains. Also the corrections in the \( \alpha \)- and \( G_F \)-on-shell renormalization schemes will not differ significantly.

There exist also theoretical constrains to the parameters of the MSSM. As a matter of fact the MSSM has a definite prediction: there should exist a light neutral scalar Higgs boson \( h^0 \). Tree-level analysis put this bound to the \( Z \) mass, however the existence of large radiative corrections to the Higgs bosons mass relations grow this limit up to \( \sim 130 \) GeV. Recently the two-loop radiative corrections to Higgs mass relations in the MSSM have been performed[57, 58, 59], and the present upper limit on \( m_h \) is

\[ m_h \leq 130 - 135 \text{ GeV} . \]  

(2.44)

It is very important to know as precise as possible this limit, as by means of a possible Run III of the Tevatron collider (TEV33, at the same energy, but higher luminosity) either a \( h^0 \) should be found, or on the contrary a lower limit to its mass in the ballpark of 130 GeV will be put. Thus it is of extreme importance to have both, a very precise prediction for the bound (2.44), and a very precise analysis of the Tevatron data. Of course if the MSSM is extended in some way this limit can be evaded, though not to values larger of \( \sim 200 \) GeV [60, 61]. Whatever the spectrum of the MSSM is, it should comply with the benefits that SUSY introduces into the SM which apply the following condition is fulfilled:

\[ M_{\text{SUSY}} \lesssim \mathcal{O}(1 \text{ TeV}) . \]  

(2.45)

If supersymmetric particles had masses heavier than the TeV scale then problems with GUT’s appear. This statement does not mean that SUSY would not exist, but that then the SM would not gain practical benefit from the inclusion of SUSY.

A similar upper bound is obtained when making cosmological analyses, in these type of analyses one supposes the neutralino to be part of the cold dark matter of the universe, and requires its annihilation rate to be sufficiently small to account for the maximum of cold dark matter allowed for cosmological models, while at the same time sufficiently large so that its presence does not becomes overwhelming. Astronomical observations also restrict the parameters of SUSY models, usually in the lower range of the mass parameters (see e.g. ([62],[63],[64])).
For the various RGE analysis to hold the couplings of the MSSM should be perturbative all the way from the unification scale to the EW scale. This implies, among other restrictions, that top and bottom Yukawa couplings should be below certain limits. In terms of $\tan \beta$ this amounts it to be confined in the approximate interval

$$0.5 \lesssim \tan \beta \lesssim 70 .$$

(2.46)

### 2.5.3 Unconstrained MSSM versus specific models for soft SUSY breaking

In the unconstrained MSSM no specific assumptions are made about the underlying SUSY-breaking mechanism, and a parametrization of all possible soft SUSY-breaking terms is used that do not alter the relation between the dimensionless couplings (which ensures that the absence of quadratic divergences is maintained). This parametrization has the advantage of being very general, but the disadvantage of introducing more than 100 new parameters in addition to the SM. While in principle these parameters (masses, mixing angles, complex phases) could be chosen independently of each other, experimental constraints from flavour-changing neutral currents, electric dipole moments, etc. seem to favour a certain degree of universality among the soft SUSY-breaking parameters.

Within a specific SUSY-breaking scenario, the soft SUSY-breaking terms can be predicted from a small set of input parameters [65]. The most prominent scenarios in the literature are minimal Supergravity (mSUGRA) [66, 67], minimal Gauge Mediated SUSY Breaking (mGMSB) [68] and minimal Anomaly Mediated SUSY Breaking (mAMSB) [69, 70]. The mSUGRA and mGMSB scenarios have four parameters and a sign, while the mAMSB scenario can be specified in terms of three parameters and a sign.
CHAPTER 3

Regularisation and Renormalization

Squark Renormalization

3.1 Regularisation

In higher-order perturbation theory the relations between the formal parameters and measurable quantities are different from the tree-level relations in general. Moreover, the procedure is obscured by the appearance of ultraviolet (UV) and infrared (IR) divergences in the loop integrations. This can be seen, e.g. in the one loop vertex correction

The corresponding integral is

\[
I = \int \frac{d^4k}{(2\pi)^4} \frac{f(k^2)}{k^2 + (k - P_2)^2 + (k + P_1)^2} \tag{3.1}
\]

Two types of divergences can appear in such an integral:

(i) UV divergences, which are associated with singularities occurring at large loop momenta: \( k \to \infty \to I \to \infty \),

(ii) IR divergences, which are generated, if one of the propagators in the loop vanishes:
\[ k \rightarrow 0, -p_1, +p_2 \text{ (soft) or } \cos \theta \rightarrow 1 \text{ (collinear)} \]

\[ \implies I \rightarrow \infty \text{ for } P_1^2 = P_2^2 = 0 \]

For a mathematically consistent treatment one has to regularize the theory, e.g. by dimensional regularization (DREG), where the regularization is performed by analytically continuing the space-time dimension from 4 to \( D \) \([71, 72]\).

### 3.1.1 Dimensional Regularization and Dimensional Reduction

The most popular in gauge theories is the so-called dimensional regularization. In this case, one modifies the integration measure.

The technique of dimensional regularization consists of analytical continuation from an integer to a noninteger number of dimensions. Basically one goes from some \( D \) to \( D - 2\epsilon \), where \( \epsilon \rightarrow 0 \). In particular, we will be interested in going from 4 to \( 4 - 2\epsilon \) dimensions. In this case, all the ultraviolet and infrared singularities manifest themselves as pole terms in \( \epsilon \).

To perform this continuation to non-integer number of dimensions, one has to define all the objects such as the metric, the measure of integration, the \( \gamma \) matrices, the propagators, etc. Though this continuation is not unique, one can define a self-consistent set of rules, which allows one to perform the calculations.

**The metric:** \( g_{\mu\nu}^4 \rightarrow g_{\mu\nu}^{4-2\epsilon} \). Though it is rather tricky to define the metric in non-integer dimensions, one usually needs only one relation, namely \( g^{\mu\nu} g_{\mu\nu} = \delta^\mu_\mu = D = 4 - 2\epsilon \).

**The measure:** \( d^4q \rightarrow (\mu^2)^\epsilon d^{4-2\epsilon}q \), where \( \mu \) is a parameter of dimensional regularization with dimension of a mass. The integration with this measure is defined by an analytical continuation from the integer dimensions.

**The \( \gamma \) matrices:** The usual anticommutation relation holds \( \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \); however, some relations involving the dimension are modified:

\[
\gamma^\mu \gamma_\mu = D = 4 - 2\epsilon; \quad Tr \gamma^\mu \gamma^\nu = g^{\mu\nu} Tr 1 = g^{\mu\nu} \left\{ \begin{array}{l} 2^{D/2} \frac{2^{D/2}}{4} \end{array} \right. 
\]

Usually \( Tr 1 = 4 \) is taken. Then the \( \gamma \)-algebra is straightforward:

\[
Tr \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma = Tr 1 [g^{\mu\nu} g^{\rho\sigma} + g^{\mu\sigma} g^{\nu\rho} - g^{\nu\rho} g^{\mu\sigma}],
\]

\[
\gamma^\mu \gamma^\nu \gamma^\mu = -\gamma^\mu \gamma^\mu \gamma^\nu + 2g^{\mu\nu} \gamma^\mu = -(4 - 2\epsilon)\gamma^\nu + 2\gamma^\nu = -(2 - 2\epsilon)\gamma^\nu, \text{ etc.}
\]

What is not well-defined is the \( \gamma^5 \) since \( \gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3 \) and cannot be continued to an
arbitrary dimension. This creates a problem in dimensional regularization since there is no consistent way of definition of $\gamma^5$.

Transfer to dimension $4 - 2\varepsilon$ and introduce the Feynman parametrization. This gives

$$I = \mu^{2\varepsilon} \int \frac{d^{4-2\varepsilon}}{(2\pi)^{4-2\varepsilon}} \frac{f(k^2)}{k^2 + (k - P_2)^2 + (k + P_1)^2}.$$ (3.2)

In supersymmetric theories, however, a $D$-dimensional treatment of vector fields leads to a mismatch between the fermionic and bosonic degrees of freedom, which gives rise to a breaking of the supersymmetric relations. This led to the development of dimensional reduction (DRED) [73]. In this scheme only the momenta are treated as $D$-dimensional, while the fields and the Dirac algebra are kept 4-dimensional. It leads to ambiguities related to the treatment of $\gamma_5$ [74], and therefore cannot be consistently applied at all orders (for a review, see Ref. [75]). Hence, renormalization and the structure of counterterms have to be adapted by exploiting the basic symmetries expressed in terms of the supersymmetric BRS transformations [76]. An additional complication in the conventional approach assuming an invariant regularization scheme, however, arises from the modification of the symmetry transformations themselves by higher-order terms.

### 3.1.2 Renormalization

So far we have isolated the divergences, but they are still there. How do we get rid of them? The crucial insight is that the parameters of the Lagrangean, the “bare” parameters, are not observable. Rather, the sum of bare parameters and loop-induced corrections are physical. Hence, divergencies of bare parameters can cancel against divergent loop corrections, leaving physical observables finite. One may replace the bare parameters by renormalized ones by multiplicative renormalization for each bare parameter $a_0$,

$$a_0 = Z_a a = a + \delta a$$ (3.3)

with renormalization constants $Z_a$ different from 1 by a higher-order term. The renormalized parameters $a$ are finite and fixed by a set of renormalization conditions. The decomposition (3.3) is to a large extent arbitrary. Only the divergent parts are determined directly by the structure of the divergences of the loop amplitudes. The finite parts depend on the choice of the explicit renormalization conditions. These conditions determine the physical meaning of the renormalized parameters.

In the following we discuss the renormalization of several sectors.
i. **On-Shell scheme (OS):** The renormalized masses are chosen at the poles of the propagators, the renormalization constant of the field are adjusted such that all the external self-energies vanishes and that the residue of the renormalized fermion propagator is one. The fermion self-energy contains scalar, vectorial and axial vectorial contributions:

\[
\Sigma_f(p) = \gamma^\nu \Sigma^V_f(p^2) + \gamma^5 \Sigma^A_f(p^2) + m_f \Sigma^S_f(p^2).
\]

The fermionic renormalization constant are defined as:

\[
\delta m^{\text{OS}}_f = m_f \left( \Sigma^V_f(m^2_f) + \Sigma^S_f(m^2_f) \right),
\]

\[
\delta Z^{\text{OS}}_V = \Sigma^V_f(m^2_f) + 2m_f \frac{\partial}{\partial p^2} \left[ \Sigma^V_f(p^2) + \Sigma^S_f(p^2) \right] \bigg|_{p^2=m^2_f},
\]

The gluon vacuum polarization is given by:

\[
\delta Z^{\text{OS}}_A = \Sigma^A_f(m^2_f),
\]

The left and right handed states contain vectorial and axialvectorial contributions which have to be renormalized separately:

\[
\delta Z^{\text{OS}}_{f,R} = \delta Z^{\text{OS}}_V + \delta Z^{\text{OS}}_A \quad \text{and} \quad \delta Z^{\text{OS}}_{f,L} = \delta Z^{\text{OS}}_V - \delta Z^{\text{OS}}_A.
\]

The gluon vacuum polarization is given by:

\[
\Pi^{\mu\nu}(q) = (g^{\mu\nu} - q^{\mu}q^{\nu}/q^2)\Pi(q) = (g^{\mu\nu} - q^{\mu}q^{\nu}/q^2)\delta Z_G + \Pi^{\mu\nu,R}(q),
\]

leading for the gluon renormalization constant to:

\[
\delta Z^{\text{OS}}_G = \Pi(q)|_{q^2=0}.
\]

ii. **Modified minimal subtraction scheme (MS):** in one-loop correction the UV poles always occur in the combination:

\[
\Delta^{\text{OS}}_{UV} = \frac{\Gamma(1+\epsilon)}{\epsilon}(4\pi)^\epsilon = \frac{1}{\epsilon} - \gamma_E + \log(4\pi) + O(\epsilon),
\]

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with the Gamma function Γ(\(x\)) and the Euler constant \(\gamma_E\). In the \textit{minimal subtraction scheme} (MS), we subtract just the singular parts. One can make the subtraction differently, for instance, subtract also the finite parts. It is useful to subtract the Euler constant and \(\log 4\pi\) which accompany the pole terms. This subtraction scheme is called the \textit{modified minimal subtraction scheme} or the \(\overline{MS}\)-scheme. It is equivalent to the redefinition of the parameter \(\mu^2\). The renormalization constants are defined as:

\[
\delta m_f^{\overline{MS}} = m_f \left( \Sigma_f^V(p^2) + \Sigma_f^S(p^2) \right) |_{\text{div}},
\]

\[
\delta Z_V^{\overline{MS}} = \frac{\partial}{\partial p} \left[ m_f \Sigma_f^S(p^2) + \phi \Sigma_f^V(p^2) \right] |_{\text{div}},
\]

\[
\delta Z_A^{\overline{MS}} = \Sigma_f^A(p^2) |_{\text{div}},
\]

\[
\delta Z_G^{\overline{MS}} = \Pi(p^2) |_{\text{div}}.
\]

In supersymmetric theories, however, a complication occurs. In \(n \neq 4\) dimensions the \(\overline{MS}\) scheme introduces a mismatch between the number of gluon \((n - 2)\) and gluino \((2)\) degrees of freedom. Since this \(\mathcal{O}(\varepsilon)\) mismatch will result in finite non-zero contributions, the \(\overline{MS}\) scheme violates supersymmetry explicitly in higher orders. In particular, the Yukawa coupling \(\tilde{g}_s\), which by supersymmetry should coincide with the gauge coupling \(g_s\), deviates from it by a finite amount at the one-loop level. Requiring the physical amplitudes to preserve this supersymmetric relation, a shift between the bare Yukawa coupling and the bare gauge coupling must be introduced in the \(\overline{MS}\) scheme:

\[
\tilde{g}_s = g_s \left[ 1 + \frac{\alpha_s}{4\pi} \left( \frac{2}{3}N - \frac{1}{2}C_F \right) \right] = g_s \left[ 1 + \frac{\alpha_s}{3\pi} \right],
\]

which effectively subtracts the contributions of the false, non-supersymmetric degrees of freedom [also called \(\varepsilon\) scalars].

The need for introducing a finite shift is best demonstrated for the effective [one-loop corrected] Yukawa coupling, which must be equal to the effective gauge coupling in an exact supersymmetric world with massless gluons/gluinos and equal-mass quarks/squarks. For the sake of simplicity we define the effective couplings \(\Gamma^{\text{eff}}(Q^2)\) and \(\tilde{\Gamma}^{\text{eff}}(Q^2)\) in the limit of on-shell quarks/squarks and almost on-shell gluons/gluinos, with virtuality \(Q^2 \ll m_{\tilde{q}}^2 = m_q^2\); in this limit the couplings do not contain gauge-dependent terms. In the \(\overline{MS}\) scheme we
find, after charge renormalization:

\[
\overline{\text{MS}} : \Gamma_{\text{eff}}(Q^2) = g_s \left\{ 1 + \frac{\alpha_s}{4\pi} N \left[ -\frac{1}{\varepsilon} - \log \left( \frac{\mu^2}{m^2_{\tilde{q}}} \right) - \frac{1}{2} \log \left( \frac{Q^2}{m^2_{\tilde{q}}} \right) + \frac{7}{6} \right] \right\}
\]

\[
\hat{\Gamma}_{\text{eff}}(Q^2) = \hat{g}_s \left\{ 1 + \frac{\alpha_s}{4\pi} \left( N \left[ -\frac{1}{\varepsilon} - \log \left( \frac{\mu^2}{m^2_{\tilde{q}}} \right) - \frac{1}{2} \log \left( \frac{Q^2}{m^2_{\tilde{q}}} \right) + 1 \right] + \frac{C_F}{2} \right) \right\}.
\]

The singular term \(1/\varepsilon\) represents the combination \(1/\varepsilon - \gamma_E + \log(4\pi)\). The remaining \(1/\varepsilon\) poles are IR and collinear singularities. Inspecting \(\Gamma_{\text{eff}}\) and \(\hat{\Gamma}_{\text{eff}}\), it is easy to prove that the difference between the two effective couplings coincides with the shift in Eq. (3.4). Taking into account this finite shift of the bare couplings in the \(\overline{\text{MS}}\) scheme, both effective couplings become identical at the one-loop level. In this way supersymmetry is preserved and the \(\overline{\text{MS}}\) renormalization becomes a consistent scheme.

An alternative renormalization scheme is the modified Dimensional Reduction (\(\overline{\text{DR}}\)) scheme in which the fields are treated in four dimensions, but the phase space and loop momenta in \(n\) dimensions. In this scheme no mismatch between bosonic and fermionic degrees of freedom is apparently introduced and supersymmetry is preserved \(ab\ \text{initio}\). At the level of the effective couplings \(\Gamma_{\text{eff}}\) and \(\hat{\Gamma}_{\text{eff}}\), this is reflected in the equalities

\[
\overline{\text{DR}} : \quad \Gamma_{\text{eff}}(Q^2) = g_s \left\{ 1 + \frac{\alpha_s}{4\pi} N \left[ -\frac{1}{\varepsilon} - \log \left( \frac{\mu^2}{m^2_{\tilde{q}}} \right) - \frac{1}{2} \log \left( \frac{Q^2}{m^2_{\tilde{q}}} \right) + 1 \right] \right\}
\]

\[
= \hat{\Gamma}_{\text{eff}}(Q^2).
\]

As a result, both couplings are identical order by order. \(\text{It should be noted that the transition from the effective gauge coupling in } \overline{\text{MS}} \text{ to the gauge coupling in } \overline{\text{DR}} \text{ involves a well-known finite renormalization } \alpha_s N/(24\pi) = \alpha_s/(8\pi) \) \([77]\).

In the following we use the \(\overline{\text{MS}}\) renormalization scheme, supplemented by the finite shift of the Yukawa coupling. In this way supersymmetry is preserved on the one hand, while on the other hand the definition of the strong gauge coupling corresponds to the usual Standard-Model measurements.

Below we list the various renormalizations needed for the production of squarks and gluinos. In order to preserve the form of the Ward identity, non-zero particle masses have to be renormalized in an on-shell scheme. We have opted for a real mass renormalization, involving the subtraction of the real part of the on-shell self-energies at the real-valued pole
masses. In the case of squarks and gluinos, this is equivalent to replacing the bare masses in the lowest-order propagators by

\[
(m^2_{\tilde{q}})^{\text{bare}} \rightarrow m^2_{\tilde{q}} \left\{ 1 + \frac{\alpha_s}{4\pi} \left[ \left( -\frac{1}{\varepsilon} - \log \left( \frac{\mu^2}{m^2_{\tilde{q}}} \right) \right) \left( \frac{m^2_{\tilde{q}}}{m^2_{\tilde{g}}} \right)^2 - 2 - 6 \frac{m^2_{\tilde{q}}}{m^2_{\tilde{g}}} + \left( -2 + 4 \frac{m^2_{\tilde{q}}}{m^2_{\tilde{g}}} - 2 \frac{m^4_{\tilde{q}}}{m^4_{\tilde{g}}} \right) \log \left| 1 - \frac{m^2_{\tilde{q}}}{m^2_{\tilde{g}}} \right| \right] \right\} \quad (3.8)
\]

\[
(m^2_{\tilde{g}})^{\text{bare}} \rightarrow m_{\tilde{g}} \left\{ 1 + \frac{\alpha_s}{4\pi} \left[ \left( -\frac{1}{\varepsilon} - \log \left( \frac{\mu^2}{m^2_{\tilde{g}}} \right) \right) \left( 3N - n_f - 1 \right) - 4N + \frac{m^2_{\tilde{g}}}{m^2_{\tilde{g}}} - \frac{m^2_{\tilde{r}}}{m^2_{\tilde{g}}} + n_f \left( 2 - \frac{m^2_{\tilde{g}}}{m^2_{\tilde{g}}} \right) \right] \right\}_{B_0} \quad (3.9)
\]

with

\[
B_0 = \text{Re} \left[ 2 - \log \left( \frac{m^2_{\tilde{q}}}{m^2_{\tilde{g}}} \right) + x_1 \log(1 - 1/x_1) + x_2 \log(1 - 1/x_2) \right] \quad (3.10)
\]

\[
x_{1,2} = \frac{1}{2m^2_{\tilde{g}}} \left[ m^2_{\tilde{g}} + m^2_{\tilde{r}} - m^2_{\tilde{q}} \pm \sqrt{(m^2_{\tilde{g}} - m^2_{\tilde{r}} - m^2_{\tilde{q}})^2 - 4m^2_{\tilde{r}}m^2_{\tilde{q}}} \right] \quad (3.11)
\]

The parameters \(m_{\tilde{g}}\) and \(m_{\tilde{q}}\) are the pole masses.

As discussed above, the couplings are renormalized in the \(\overline{MS}\) scheme, including the finite shift of the bare Yukawa coupling given by Eq. (3.4). This leads to the following replacements of the bare couplings in the LO expressions:

\[
(g_s)^{\text{bare}} \rightarrow g_s(\mu^2_R) \left\{ 1 + \frac{\alpha_s(\mu^2_R)}{4\pi} \left[ \left( -\frac{1}{\varepsilon} + \log \left( \frac{\mu^2_R}{\mu^2} \right) \right) \frac{\beta_0}{2} - \frac{N}{3} \log \left( \frac{m^2_{\tilde{g}}}{\mu^2} \right) - \frac{n_f + 1}{6} \log \left( \frac{m^2_{\tilde{g}}}{Q^2_R} \right) - \frac{1}{3} \log \left( \frac{m^2_{\tilde{g}}}{\mu^2} \right) \right] \right\} \quad (3.12)
\]

\[
(\hat{g}_s)^{\text{bare}} \rightarrow g_s(\mu^2_R) \left\{ 1 + \frac{\alpha_s(\mu^2_R)}{4\pi} \left[ \left( -\frac{1}{\varepsilon} + \log \left( \frac{\mu^2_R}{\mu^2} \right) \right) \frac{\beta_0}{2} - \frac{N}{3} \log \left( \frac{m^2_{\tilde{g}}}{\mu^2} \right) - \frac{n_f + 1}{6} \log \left( \frac{m^2_{\tilde{g}}}{\mu^2} \right) - \frac{1}{3} \log \left( \frac{m^2_{\tilde{g}}}{\mu^2} \right) + \frac{2N}{3} - C_F \right] \right\} \quad (3.13)
\]
The first coefficient $\beta_0$ of the SUSY-QCD $\beta$ function can be decomposed into a sum of contributions from light and heavy particles:

$$\beta_0 = \left[ \frac{11}{3} N - \frac{2}{3} n_f \right] + \left[ - \frac{2}{3} N - \frac{2}{3} - \frac{1}{3} (n_f + 1) \right] = \beta_0^L + \beta_0^H. \quad (3.14)$$

In addition to the poles, also some logarithms are subtracted in order to decouple the heavy particles [top quark, squarks, gluinos] from the running of $\alpha_s(Q_R^2)$. In this decoupling scheme the $Q_R^2$ evolution of the strong coupling is determined completely by the light-particle spectrum [gluons and $n_f = 5$ massless quarks]:

$$\frac{\partial g_s^2(\mu_R^2)}{\partial \log(\mu_R^2)} = g_s(\mu_R^2)\beta(g_s) = -\alpha_s(\mu_R^2)\beta_0^L. \quad (3.15)$$

The methods described above to renormalize the UV divergences lead to cross-sections that are UV-finite. Nevertheless, there are still divergences left. The IR divergences will cancel against the contribution from soft-gluon radiation. The collinear singularities, finally, will be removed by applying mass factorization. These steps will be discussed in detail in the next subsections.

### 3.2 The scalar quark sector of the MSSM

Renormalization in the squark sector is needed in the present calculation at the one-loop level, i.e. at $O(\alpha_s)$. As above, we work in the on-shell scheme. In the following the formulas are written for one flavor. The squark mass term of the MSSM Lagrangian is given by

$$\mathcal{L}_{m_f} = -\frac{1}{2} \left( \bar{\tilde{f}}_L^i, \bar{\tilde{f}}_R^i \right) Z \left( \begin{array}{c} \bar{\tilde{f}}_L^i \\ \bar{\tilde{f}}_R^i \end{array} \right), \quad (3.16)$$

where

$$Z = \begin{pmatrix} M_Q^2 + M_Z^2 \cos 2\beta \ (I_3^f - Q_f s_W^2) + m_f^2 & m_f(A_f - \mu \{\cot \beta; \tan \beta\}) \\ m_f(A_f - \mu \{\cot \beta; \tan \beta\}) & M_{Q'}^2 + M_Z^2 \cos 2\beta \ Q_f s_W^2 + m_f^2 \end{pmatrix}, \quad (3.17)$$
and \{\cot \beta; \tan \beta\} corresponds to \{u; d\}-type squarks. The soft SUSY breaking term \(M_{\tilde{Q}'}\) is given by:

\[
M_{\tilde{Q}'} = \begin{cases} 
M_{\tilde{D}} & \text{for right handed } u\text{-type squarks} \\
M_{\tilde{D}} & \text{for right handed } d\text{-type squarks}
\end{cases}.
\] (3.18)

In order to diagonalize the mass matrix and to determine the physical mass eigenstates the following rotation has to be performed:

\[
\begin{pmatrix}
\tilde{f}_1 \\
\tilde{f}_2
\end{pmatrix} = \begin{pmatrix}
\cos \theta \tilde{f} & \sin \theta \tilde{f} \\
-\sin \theta \tilde{f} & \cos \theta \tilde{f}
\end{pmatrix} \begin{pmatrix}
\tilde{f}_L \\
\tilde{f}_R
\end{pmatrix}.
\] (3.19)

The mixing angle \(\theta \tilde{f}\) is given for \(\tan \beta > 1\) by:

\[
\cos \theta \tilde{f} = \sqrt{\frac{(m_{\tilde{f} R}^2 - m_{\tilde{f} L}^2)^2}{m_{\tilde{f} R}^2 (A_f - \mu \{\cot \beta; \tan \beta\})^2 + (m_{\tilde{f} R}^2 - m_{\tilde{f} L}^2)^2}}
\] (3.20)

\[
\sin \theta \tilde{f} = \pm \text{sgn} \left[ A_f - \mu \{\cot \beta; \tan \beta\} \right] \times \sqrt{\frac{m_{\tilde{f} R}^2 (A_f - \mu \{\cot \beta; \tan \beta\})^2}{m_{\tilde{f} R}^2 (A_f - \mu \{\cot \beta; \tan \beta\})^2 + (m_{\tilde{f} R}^2 - m_{\tilde{f} L}^2)^2}}.
\] (3.21)

The negative sign in (3.21) corresponds to \(u\)-type squarks, the positive sign to \(d\)-type ones. \(m_{\tilde{f} R}^2 = M_{\tilde{Q}'}^2 + M_{\tilde{Z}'}^2 \cos 2\beta Q_f s_w^2 + m_f^2\) denotes the lower right entry in the squark mass matrix (3.17). The masses are given by the eigenvalues of the mass matrix:

\[
m_{\tilde{f}_{1,2}}^2 = \frac{1}{2} \left[ M_{\tilde{Q}'}^2 + M_{\tilde{Q}'}^2 \right] + \frac{1}{2} M_{\tilde{Z}'}^2 \cos 2\beta I_3^f + m_f^2
\]

\[
\pm \frac{c_f}{2} \sqrt{\left[ M_{\tilde{Q}'}^2 - M_{\tilde{Q}'}^2 \cos 2\beta (I_3^f - 2Q_f s_w^2) \right]^2 + 4m_f^2 (A_u - \mu \cot \beta)^2}
\] (3.22)

\[
c_f = \text{sgn} \left[ M_{\tilde{Q}'}^2 - M_{\tilde{Q}'}^2 \cos 2\beta (I_3^f - 2Q_f s_w^2) \right]
\] (3.23)

for \(u\)-type and \(d\)-type squarks, respectively. We make the choice

\[
M_{\tilde{Q}} = M_{\tilde{Q}'} =: m_{\tilde{q}}.
\] (3.24)
Since the non-diagonal entry of the mass matrix eq. (3.17) is proportional to the fermion mass, mixing becomes particularly important for $\tilde{f} = \tilde{t}$, in the case of $\tan \beta \gg 1$ also for $\tilde{f} = \tilde{b}$. For an on-shell renormalization it is convenient to express the squark mass matrix in terms of the physical masses $m_{\tilde{f}_1}, m_{\tilde{f}_2}$ and the mixing angle $\theta_{\tilde{f}}$:

$$Z = \begin{pmatrix} \cos^2 \theta_{\tilde{f}} m_{\tilde{f}_1}^2 + \sin^2 \theta_{\tilde{f}} m_{\tilde{f}_2}^2 & \sin \theta_{\tilde{f}} \cos \theta_{\tilde{f}} (m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2) \\ \sin \theta_{\tilde{f}} \cos \theta_{\tilde{f}} (m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2) & \sin^2 \theta_{\tilde{f}} m_{\tilde{f}_1}^2 + \cos^2 \theta_{\tilde{f}} m_{\tilde{f}_2}^2 \end{pmatrix}. \quad (3.25)$$

$A_{\tilde{f}}$ can be written as follows:

$$A_{\tilde{f}} = \frac{\sin \theta_{\tilde{f}} \cos \theta_{\tilde{f}} (m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2)}{m_{\tilde{f}}} + \mu \{ \cot \beta ; \tan \beta \}. \quad (3.26)$$

### 3.3 Renormalization of squarks

The renormalization of the fields, the masses, and the mixing angle is then performed via

$$\begin{align*}
\tilde{f}_L & \rightarrow \tilde{f}_L (1 + \frac{1}{2} \delta Z_{\tilde{f}_L}) \quad (3.27) \\
\tilde{f}_R & \rightarrow \tilde{f}_R (1 + \frac{1}{2} \delta Z_{\tilde{f}_R}) \quad (3.28) \\
m_{\tilde{f}_i} & \rightarrow m_{\tilde{f}_i} + \delta m_{\tilde{f}_i} \quad (3.29) \\
\theta_{\tilde{f}} & \rightarrow \theta_{\tilde{f}} + \delta \theta_{\tilde{f}}. \quad (3.30)
\end{align*}$$

In the mass eigenstate basis, the field renormalization reads:

$$\begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 + \frac{1}{2} \delta Z_{\tilde{f}_1} & \frac{1}{2} \delta Z_{\tilde{f}_{12}} \\ \frac{1}{2} \delta Z_{\tilde{f}_{21}} & 1 + \frac{1}{2} \delta Z_{\tilde{f}_2} \end{pmatrix} \begin{pmatrix} \tilde{f}_1 \\ \tilde{f}_2 \end{pmatrix}, \quad (3.31)$$

with

$$\begin{align*}
\delta Z_{\tilde{f}_1} &= \begin{pmatrix} \cos^2 \theta_{\tilde{f}} & \sin^2 \theta_{\tilde{f}} \\ \sin^2 \theta_{\tilde{f}} & \cos^2 \theta_{\tilde{f}} \end{pmatrix} \begin{pmatrix} \delta Z_{\tilde{f}_L} \\ \delta Z_{\tilde{f}_R} \end{pmatrix} \quad (3.32) \\
\delta Z_{\tilde{f}_{12}} &= \sin \theta_{\tilde{f}} \cos \theta_{\tilde{f}} (\delta Z_{\tilde{f}_R} - \delta Z_{\tilde{f}_L}) = \delta Z_{\tilde{f}_{21}} \quad (3.33) \\
&= -\frac{\sin \theta_{\tilde{f}} \cos \theta_{\tilde{f}}}{\cos^2 \theta_{\tilde{f}} - \sin^2 \theta_{\tilde{f}}} (\delta Z_{\tilde{f}_2} - \delta Z_{\tilde{f}_1}).
\end{align*}$$
The renormalized diagonal and non-diagonal self-energies in this basis have the following structure:

\begin{align*}
\hat{\Sigma}_{\tilde{f}_1}(k^2) &= \Sigma_{\tilde{f}_1}(k^2) - \delta m_{\tilde{f}_1}^2 + (k^2 - m_{\tilde{f}_1}^2)\delta Z_{\tilde{f}_1}, \\
\hat{\Sigma}_{\tilde{f}_2}(k^2) &= \Sigma_{\tilde{f}_2}(k^2) - \delta m_{\tilde{f}_2}^2 + (k^2 - m_{\tilde{f}_2}^2)\delta Z_{\tilde{f}_2}, \\
\hat{\Sigma}_{\tilde{f}_1\tilde{f}_2}(k^2) &= \Sigma_{\tilde{f}_1\tilde{f}_2}(k^2) - (m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2)\delta \theta_f + (k^2 - \frac{1}{2}(m_{\tilde{f}_1}^2 + m_{\tilde{f}_2}^2))\delta Z_{\tilde{f}_1\tilde{f}_2}.
\end{align*}

We impose the following on-shell renormalization conditions:

\begin{align*}
\text{Re } \hat{\Sigma}_{\tilde{f}_1}(m_{\tilde{f}_1}^2) &= 0 \quad (3.37) \\
\text{Re } \hat{\Sigma}_{\tilde{f}_1\tilde{f}_1}(m_{\tilde{f}_1}^2) &= 0 \quad (3.38) \\
\text{Re } \hat{\Sigma}_{\tilde{f}_2}(m_{\tilde{f}_2}^2) &= 0 \quad (3.39) \\
\text{Re } \hat{\Sigma}_{\tilde{f}_1\tilde{f}_2}(m_{\tilde{f}_1}^2) &= -\text{Re } \Sigma_{\tilde{f}_1\tilde{f}_1}(m_{\tilde{f}_1}^2) + \text{Re } \Sigma_{\tilde{f}_1\tilde{f}_2}(m_{\tilde{f}_2}^2) \quad (3.40) \\
\text{Re } \hat{\Sigma}_{\tilde{f}_1\tilde{f}_2}(m_{\tilde{f}_1}^2) &= 0, \quad (3.41)
\end{align*}

which determines the renormalization constants to be

\begin{align*}
\delta m_{\tilde{f}_1}^2 &= \text{Re } \Sigma_{\tilde{f}_1}(m_{\tilde{f}_1}^2), \quad (3.42) \\
\delta m_{\tilde{f}_2}^2 &= \text{Re } \Sigma_{\tilde{f}_2}(m_{\tilde{f}_2}^2), \quad (3.43) \\
\delta Z_{\tilde{f}_1} &= -\Sigma_{\tilde{f}_1\tilde{f}_1}(m_{\tilde{f}_1}^2), \quad (3.44) \\
\delta Z_{\tilde{f}_2} &= \delta Z_{\tilde{f}_1} \Rightarrow \delta Z_{\tilde{f}_1\tilde{f}_2} = 0 \quad (3.45) \\
\delta \theta_f &= \frac{1}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2} \Sigma_{\tilde{f}_1\tilde{f}_2}(m_{\tilde{f}_1}^2). \quad (3.46)
\end{align*}

The unsymmetric renormalization condition (3.40) is chosen for convenience since it leads to \(\delta Z_{\tilde{f}_2} = \delta Z_{\tilde{f}_1}\) and accordingly to \(\delta Z_{\tilde{f}_1\tilde{f}_2} = 0\), which simplifies the expression for the counterterm of the mixing angle. In eq. (3.41) we have imposed the condition that the non-diagonal self-energy vanishes at \(q^2 = m_{\tilde{f}_1}^2\). Alternatively one could choose \(q^2 = m_{\tilde{f}_2}^2\), instead; the numerical difference arising from these different choices is irrelevant for the results of the Higgs-boson masses, as we have checked explicitly. Taking into account that neither \(\delta \mu\) nor \(\delta \tan \beta\) are of \(O(\alpha_s)\), one obtains from eq. (3.26):

\[
\delta A_f = \frac{\sin \theta_f \cos \theta_f (m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2)}{m_f} \left[ 1 + 2 \sin^2 \theta_f \delta \theta_f + \frac{\delta m_{\tilde{f}_1}^2 - \delta m_{\tilde{f}_2}^2}{m_{\tilde{f}_1}^2 - m_{\tilde{f}_2}^2} - \frac{\delta m_f}{m_f} \right]. \quad (3.47)
\]
For completeness we also list the expression for the quark mass counterterm in the on-shell scheme,

$$\delta m_f = m_f \left( \Sigma_f^V(m_f^2) + \Sigma_f^S(m_f^2) \right), \quad \text{(3.48)}$$

where the scalar functions in the decomposition of the fermion self-energy $\Sigma_f(p)$ are defined according to

$$\Sigma_f(p) = \psi \Sigma_f^V(p^2) + \psi \gamma_5 \Sigma_f^A(p^2) + m_f \Sigma_f^S(p^2). \quad \text{(3.49)}$$
CHAPTER 4

Associated production of MSSM neutral Higgs bosons with top squark

4.1 Motivation

Associated production of MSSM neutral Higgs bosons with third generation scalar quarks potentially substantial: due to the large Yukawa couplings, the lightest top and bottom squarks can have relatively small masses, and their couplings to Higgs bosons can possibly be large.

The LO predictions for the cross sections are plagued by large uncertainties due to the strong dependence on the renormalization and factorization scales, originating from the parton densities and the strong coupling constant. The NLO SUSY-QCD correction should reduce the scale dependence and thus stabilise the theoretical predictions.

4.2 General Framework

The total cross section for \( pp \to \tilde{t}_1 \tilde{\bar{t}}_1 h \) can be written as:

\[
\sigma_{NLO}(pp \to \tilde{t}_1 \tilde{\bar{t}}_1 h) = \sum_{ij} \int dx_1 dx_2 \mathcal{F}_{i}(x_1, \mu) \mathcal{F}_{j}(x_2, \mu) \hat{\sigma}_{NLO}^{ij}(x_1, x_2, \mu),
\]

where \( \mathcal{F}_{i} \) is the NLO parton distribution function (PDF) for parton \( i \) in a proton, defined at a generic factorization scale \( \mu_f = \mu \), and \( \hat{\sigma}_{NLO}^{ij} \) is the parton-level total cross section for incoming partons \( i \) and \( j \), composed of the two channels \( q\bar{q}, gg \to \tilde{t}_1 \tilde{\bar{t}}_1 h \), and renormalized at an arbitrary scale \( \mu_r \) which we also take to be \( \mu_r = \mu \). Throughout this paper we will always assume the factorization and renormalization scales to be equal, \( \mu_r = \mu_f = \mu \). The partonic center-of-mass energy squared, \( \hat{s} \), is given in terms of the total hadronic center-of-mass energy...
We write the NLO parton-level total cross section $\hat{\sigma}^{ij}_{NLO}(x_1, x_2, \mu)$ as:

$$\hat{\sigma}^{ij}_{NLO}(x_1, x_2, \mu) = \hat{\sigma}^{ij}_{LO}(x_1, x_2, \mu) + \delta\hat{\sigma}^{ij}_{NLO}(x_1, x_2, \mu),$$

where $\hat{\sigma}^{ij}_{LO}(x_1, x_2, \mu)$ is the Born cross section, and $\delta\hat{\sigma}^{ij}_{NLO}(x_1, x_2, \mu)$ consists of the corrections to the Born cross section. The NLO QCD contribution, $\delta\hat{\sigma}^{ij}_{NLO}(x_1, x_2, \mu)$, contains both virtual and real corrections to the lowest-order cross section and can be written as the sum of two terms:

$$\delta\hat{\sigma}^{ij}_{NLO}(x_1, x_2, \mu) = \int d(PS_3) \sum |A^{\text{virt}}(ij \rightarrow \tilde{t}_1\tilde{t}_1 h)|^2 + \int d(PS_4) \sum |A^{\text{real}}(ij \rightarrow \tilde{t}_1\tilde{t}_1 h + g)|^2$$

$$\equiv \hat{\sigma}^{ij}_{\text{virt}}(x_1, x_2, \mu) + \hat{\sigma}^{ij}_{\text{real}}(x_1, x_2, \mu),$$

(4.2)

where $|A^{\text{virt}}(ij \rightarrow \tilde{t}_1\tilde{t}_1 h)|^2$ and $|A^{\text{real}}(ij \rightarrow \tilde{t}_1\tilde{t}_1 h + g)|^2$ are respectively the squared matrix elements for the $ij \rightarrow \tilde{t}_1\tilde{t}_1 h$ and $ij \rightarrow \tilde{t}_1\tilde{t}_1 h + g$ processes, and $\sum$ indicates that they have been averaged over the initial-state degrees of freedom and summed over the final-state ones. Moreover, $d(PS_3)$ and $d(PS_4)$ denote the integration over the corresponding three and four-particle phase spaces respectively. The first term in Eq. (4.2) represents the contribution of the virtual corrections, while the second one is due to the real gluon emission.

### 4.3 The leading order cross sections

The associated Higgs production with stop quarks production mechanism at the parton level contributing to the hadronic process $pp \rightarrow \tilde{t}_1\tilde{t}_1 h$, involves $q\bar{q}$ annihilation and gluongluon fusion channels. The subprocess via the $q\bar{q}$ annihilation is written as

$$q(p_1)\bar{q}(p_2) \rightarrow \tilde{t}_1(p_3)\tilde{t}_1(p_4)h(p_5),$$

(4.3)

The Feynman diagrams of this subprocess are plotted in Fig.4.1. They are s-channel, gluon exchange diagrams with Higgs boson radiation off stop-quark and anti-stop-quark, respectively. The subprocess via gluon-gluon fusion which is written as

$$g(p_1)g(p_2) \rightarrow \tilde{t}_1(p_3)\tilde{t}_1(p_4)h(p_5),$$

(4.4)

at the tree level in the MSSM are described by the Feynman diagrams of Fig.4.2. The momenta of the particles are given in brackets. The momenta obey the on-shell conditions...
\[ p_1^2 = p_2^2 = 0, \quad p_3^2 = p_4^2 = m_{\tilde{t}_1}^2, \quad \text{and} \quad p_5^2 = M_h^2. \]

For later use, the following set of kinematical invariants is defined:

\[ \hat{s} = (p_1 + p_2)^2, \]
\[ s_{ij} = (p_i + p_j)^2, \quad i, j = 3, 4, 5, \]
\[ t_{ij} = (p_i - p_j)^2, \quad i = 1, 2, \quad j = 3, 4, 5. \]

(4.5)

Fig. 4.1: The tree-level Feynman diagrams for the \( q \bar{q} \rightarrow \tilde{t}_1 \tilde{t}_1 h \) subprocess.
According to the different topologies of Feynman diagrams, it is convenient to rearrange the amplitudes in terms of their colour structure. Therefore, for the subprocess $q\bar{q} \rightarrow \tilde{t}_1 \tilde{t}_1 h$, the corresponding LO and NLO amplitudes can be expressed in the form:

$$M_{LO,NLO}^{q\bar{q}} = C_{q\bar{q}} A_{LO,NLO}^{q\bar{q}}$$

(4.6)

where $C^{q\bar{q}}$ is the only color factor involved in the LO amplitude of the subprocess $q\bar{q} \rightarrow \tilde{t}_1 \tilde{t}_1 h$, which can be written as:

$$C^{q\bar{q}} = \lambda^c \otimes \lambda^c,$$

(4.7)

The first $3 \times 3$ $SU(3)$ Gell-Mann matrix $\lambda^c$ arises from $q\bar{q}$ color state, the second Gell-Mann matrix $\lambda^c$ arises from $\tilde{t}_1 \tilde{t}_1$ color state. Similarly, the LO amplitude of the subprocess
\( gg \rightarrow \tilde{t}_1 \tilde{t}_1 h \) can be expressed as:

\[
M_{gO}^{gg} = \left( \frac{2}{3} C_1^{gg} + C_2^{gg} + C_3^{gg} \right) M_1^{gg} + \left( \frac{2}{3} C_{-1}^{gg} - C_2^{gg} + C_3^{gg} \right) M_2^{gg}, \quad (4.8)
\]

with

\[
C_1^{gg} = \delta^{c_1 c_2}, \quad C_2^{gg} = i f^{c_1 c_2 c} \lambda^c, \quad C_2^{gg} = d^{c_1 c_2 c} \lambda^c, \quad (4.9)
\]

\[
M_1^{gg} = M_{\tilde{q} g}^{gg} + \frac{1}{2} M_{\tilde{s} g}^{gg} + M_{\tilde{t} g}^{gg}, \quad (4.10)
\]

\[
M_2^{gg} = M_{\tilde{q} g}^{gg} - \frac{1}{2} M_{\tilde{s} g}^{gg} + M_{\tilde{u} g}^{gg}, \quad (4.11)
\]

where \( c_n (n = 1, 2) \) are the color indexes of incoming gluons, \( f^{abc} \) and \( d^{abc} \) are the \( SU(3) \) antisymmetric and symmetric structure constants respectively, matrixes \( 1 \) and \( \lambda^c \) arise from \( \tilde{t}_1 \tilde{t}_1 \) color state. \( M_{\tilde{q} g}^{gg} = \sum_{i=1}^{2} M_{L_0, i} \) \( M_{\tilde{s} g}^{gg} = \sum_{i=3}^{4} M_{L_0, i} \) \( M_{\tilde{t} g}^{gg} = \sum_{i=5}^{7} M_{L_0, i} \) and \( M_{\tilde{u} g}^{gg} = \sum_{i=8}^{10} M_{L_0, i} \). Where \( M_{\tilde{q} g}^{gg}, M_{\tilde{s} g}^{gg}, M_{\tilde{t} g}^{gg} \) and \( M_{\tilde{u} g}^{gg} \) represent the amplitudes of quartic-channel, s-channel, t-channel and u-channel respectively and the \( M_{L_0, i}, i = 1, \ldots 10 \), are the leading order Feynman amplitudes associated to the graphs in Fig.4.2. Taking the trace of color factors, we get:

\[
Tr(C_i^{gg \dagger} C_j^{gg}) = c_i^{gg} \delta_{jk} \quad \text{with} \quad c_1^{gg} = 24, \quad c_2^{gg} = 48, \quad c_1^{gg} = \frac{80}{3}. \quad (4.12)
\]

The squared amplitude for the LO can be written as:

\[
|M_{gO}^{gg}|^2 = \frac{256}{3} (|M_1^{gg}|^2 + |M_2^{gg}|^2) - \frac{32}{3} \cdot 2 \Re (M_1^{gg} \cdot M_2^{gg}). \quad (4.13)
\]

Then the LO cross section for the subprocesses \( q \bar{q}, gg \rightarrow \tilde{t}_1 \tilde{t}_1 h \) can be obtained by using the following formula:

\[
\hat{\sigma}^{q \bar{q}, gg}_{LO} = \int d\Phi_3 \sum |M_{gO}^{q \bar{q}, gg}|^2, \quad (4.14)
\]

where \( d\Phi_3 \) is the three-particle phase space element. The summation is taken over the spins and colors of initial and final states, and the bar over the summation recalls averaging over the spins and colors of initial partons.
The explicit expression for the amplitudes of subprocess $q\bar{q} \rightarrow \tilde{t}_1\tilde{t}_1 h$ at tree level can be written as:

$$M^{q\bar{q}}_{LO} = g_s^2 C_{\tilde{t}_1\tilde{t}_1 h}^{(SUSY)} \bar{v}_k(p_2) \gamma_\mu u_i(p_1) \frac{ig_{\mu\nu}}{\hat{s}} \frac{(-p_3 + p_4 + p_5)_\nu}{(p_4 + p_5)^2 - m_{\tilde{t}_1}^2} \gamma_\nu T^a_{ik} T^b_{jl} + (p_3 \leftrightarrow p_4) \quad (4.15)$$

where $\hat{s} = (p_1 + p_2)^2$, $T^a$ is the $SU(3)$ color matrix. $g_s$ is the strong coupling constant. $C_{\tilde{t}_1\tilde{t}_1 h}^{(SUSY)}$ is the coupling between Higgs boson and stop quarks in the MSSM, which from Sec.(3.2) is expressed as:

$$C_{h\tilde{t}_1\tilde{t}_1}^{(SUSY)} = \frac{1}{2} \sin(\alpha + \beta) \left[ \frac{\cos^2 \theta_t - \frac{4}{3} s_W^2 \cos 2\theta_t}{\sin \beta M_Z^2} \right] - \frac{\cos \alpha m_t^2}{\sin \beta M_Z^2}$$

$$+ \frac{m_t \sin 2\theta_t}{2M_Z^2} \left[ \frac{\cos \alpha}{\sin \beta A_t} + \frac{\sin \alpha}{\sin \beta \mu} \right]$$

$$\quad (4.16)$$

where,

$$\tan \theta_t = \frac{2 m_t M_{LR}}{M_{LL}^2 - M_{RR}^2 - \sqrt{(M_{LL}^2 - M_{RR}^2)^2 + 4m_t^2 M_{LR}^2}}$$

$$\quad (4.17)$$

and

$$m_{\tilde{t}_1,2}^2 = m_t^2 + \frac{1}{2} \left[ M_{LL}^2 + M_{RR}^2 \pm \sqrt{(M_{LL}^2 - M_{RR}^2)^2 + 4m_t^2 M_{LR}^2} \right]$$

$$\quad (4.18)$$

where

$$M_{LL}^2 = m_{\tilde{t}_L}^2 + (I_3^t - e_t s_W^2) \cos 2\beta M_Z^2$$

$$M_{RR}^2 = m_{\tilde{t}_R}^2 + e_t s_W^2 \cos 2\beta M_Z^2$$

$$M_{LR} = A_t + \mu (\tan \beta)^{-2I_3^t} \quad (4.19)$$

$I_3^t$ and $e_t$ are the weak isospin and electric charge of the stop $\tilde{t}$, and $s_W^2 = 1 - c_W^2 \equiv \sin^2 \theta_W$.

For the amplitude parts $M_{\text{Lo},i}, i = 1, \ldots, 10$, we have the expressions as:
\[M_{\text{Lo},1} = 2 C^{(SUSY)}_{i_{1}i_{1}h} \alpha_{a} e \pi g_{\mu\nu} \frac{1}{(p_{3} - p_{5})^{2} - m_{t_{i}}^{2}} \quad (4.20)\]

\[M_{\text{Lo},2} = 2 C^{(SUSY)}_{i_{1}i_{1}h} \alpha_{a} e \pi g_{\mu\nu} \frac{1}{(p_{4} - p_{5})^{2} - m_{t_{i}}^{2}} \quad (4.21)\]

\[M_{\text{Lo},3} = 2 C^{(SUSY)}_{i_{1}i_{1}h} i \alpha_{a}^{2} e \pi (p_{3} - p_{4} + p_{5}) \theta ((p_{2} - p_{1})\phi g_{\mu\nu} + (p_{1} + p_{3} + p_{4} + p_{5})\nu g_{\mu\phi} + (-p_{2} - p_{4} - p_{3} - p_{5})\mu g_{\nu\phi}) \frac{1}{(p_{1} + p_{2})^{2} (-p_{3} - p_{5})^{2} - m_{t_{i}}^{2}} \quad (4.22)\]

\[M_{\text{Lo},4} = 2 C^{(SUSY)}_{i_{1}i_{1}h} i \alpha_{a}^{2} e \pi (p_{4} - p_{3} + p_{5}) \theta ((p_{2} - p_{1})\phi g_{\mu\nu} + (p_{1} + p_{4} + p_{3} + p_{5})\nu g_{\mu\phi} + (-p_{2} - p_{4} - p_{3} - p_{5})\mu g_{\nu\phi}) \frac{1}{(p_{1} + p_{2})^{2} (-p_{4} - p_{5})^{2} - m_{t_{i}}^{2}} \quad (4.23)\]

\[M_{\text{Lo},5} = 2 C^{(SUSY)}_{i_{1}i_{1}h} \alpha_{a} e \pi (p_{2} - 2p_{4})\nu (-p_{2} - p_{3} + p_{4} + p_{5})\mu \phi^{\nu}(p_{1})\phi^{\nu}(p_{2}) \quad (4.24)\]

\[M_{\text{Lo},6} = 2 C^{(SUSY)}_{i_{1}i_{1}h} \alpha_{a} e \pi (p_{2} - 2p_{3})\nu (-p_{2} - p_{4} + p_{3} + p_{5})\mu \phi^{\nu}(p_{1})\phi^{\nu}(p_{2}) \quad (4.25)\]

\[M_{\text{Lo},7} = 2 C^{(SUSY)}_{i_{1}i_{1}h} \alpha_{a} e \pi (-p_{2} - p_{3} + p_{4} + p_{5})\mu (-p_{2} + 2p_{4} + 2p_{5})\nu \phi^{\mu}(p_{2})\phi^{\nu}(p_{1}) \quad (4.26)\]

\[M_{\text{Lo},8} = 2 C^{(SUSY)}_{i_{1}i_{1}h} \alpha_{a} e \pi (-p_{1} - p_{3} + p_{4} + p_{5})\mu (-p_{1} + 2p_{4} + 2p_{5})\nu \phi^{\mu}(p_{1})\phi^{\nu}(p_{2}) \quad (4.27)\]

\[M_{\text{Lo},9} = 2 C^{(SUSY)}_{i_{1}i_{1}h} \alpha_{a} e \pi (-p_{2} - p_{4} + p_{3} + p_{5})\mu (-p_{2} + 2p_{3} + 2p_{5})\nu \phi^{\mu}(p_{2})\phi^{\nu}(p_{1}) \quad (4.28)\]

where \(\epsilon_{1}^{\nu}\) and \(\epsilon_{2}^{\nu}\) are the polarization four-vectors of the incoming gluons.
In terms of the momenta \( p_1, p_2 \) of the initial gluons or quarks, and the momenta \( p_3, p_4, p_5 \) of the final two squarks and Higgs boson respectively, the amplitude squared \( |\mathcal{M}|^2_{q\bar{q}} \) of the quark initiated subprocess \( q\bar{q} \to \tilde{t}_1\tilde{t}_1 h \) [averaged over color and spin], is given by:

\[
|\mathcal{M}|^2_{q\bar{q}} = \frac{1}{36} \left( 4\sqrt{2} G_F M_Z^4 C_W^2 \lambda^2 g_s^4 \right) \times 2 \times \frac{4}{s^2} \left[ \frac{1}{(s + (2p_1 \cdot (p_1 + p_2)))^2} \right] \\
\left[ 2((2p_4 + (p_1 + p_2)) \cdot p_1)((2p_4 + (p_1 + p_2)) \cdot p_2) \right. \\
\left. - \frac{s}{2}(2p_4 + (p_1 + p_2))^2 \right] + \frac{1}{(s - (2p_3 \cdot (p_1 + p_2)))^2} \\
\left[ 2((2p_3 - (p_1 + p_2)) \cdot p_1)((2p_3 - (p_1 + p_2)) \cdot p_2) \right. \\
\left. - \frac{s}{2}(2p_3 - (p_1 + p_2))^2 \right] \\
\left[ 2((2p_4 + (p_1 + p_2)) \cdot p_1)((2p_4 + (p_1 + p_2)) \cdot p_2) \right. \\
\left. + \frac{s}{2}(2p_4 + (p_1 + p_2))^2 \right] \\
\left[ 2((2p_3 - (p_1 + p_2)) \cdot p_1)((2p_3 - (p_1 + p_2)) \cdot p_2) \right. \\
\left. - \frac{s}{2}(2p_3 - (p_1 + p_2))^2 \right]
\]

(4.29)

In the case of the amplitude squared \( |\mathcal{M}|^2_{gg} \) of the gluon initiated subprocess \( gg \to \tilde{t}_1\tilde{t}_1 h \) [averaged over color and spin], is given by:

\[
|\mathcal{M}|^2_{gg} = \frac{1}{256} \times \left( 4\sqrt{2} G_F M_Z^4 C_W^2 \lambda^2 g_s^4 \right) \times \frac{16}{3} \times \frac{1}{4s^2[(p_3 \cdot p_1)(p_3 \cdot p_2)]^2[(p_4 \cdot p_1)(p_4 \cdot p_2)]^2} \\
\left[ - (p_3 \cdot p_4 s - 2[(p_3 \cdot p_2)(p_4 \cdot p_1) + (p_3 \cdot p_1)(p_4 \cdot p_2)])^2[(p_3 \cdot p_1)(p_3 \cdot p_2)] \right. \\
\left. + (p_3 \cdot p_4)^2 - 2[(p_3 \cdot p_1)(p_3 \cdot p_2)][(p_4 \cdot p_1)(p_4 \cdot p_2)] \right] + \frac{1}{(s + 2p_4 \cdot (p_1 + p_2))} \\
\left[ \frac{(p_3 \cdot p_4 s - 2[(p_3 \cdot p_2)(p_4 \cdot p_1) + (p_3 \cdot p_1)(p_4 \cdot p_2)])}{4s^2[(p_3 \cdot p_1)(p_3 \cdot p_2)][(p_4 \cdot p_1)(p_4 \cdot p_2)]^2} \left[ 2m_{t_1}^2 s(8p_3 \cdot (p_1 - p_2)(p_4 \cdot p_1))^2 \right. \\
\left. - 8p_3 \cdot p_1((p_4 \cdot (p_1 + p_2))^2 - 2[(p_4 \cdot p_1)(p_4 \cdot p_2)]) + p_3 \cdot (p_1 + p_2)[(p_4 \cdot p_1)(p_4 \cdot p_2)]\right. \\
\left. + 46p_3 \cdot (p_1 - p_2)(p_4 \cdot p_1)^2 + 7s[(p_3 \cdot p_2)(p_4 \cdot p_1) + (p_3 \cdot p_1)(p_4 \cdot p_2)] + 46p_3 \cdot p_1((p_4 \cdot (p_1 + p_2))^2 \right. \\
\left. - 2[(p_4 \cdot p_1)(p_4 \cdot p_2)] + 10p_3 \cdot (p_1 + p_2)[(p_4 \cdot p_1)(p_4 \cdot p_2)] \right] \right]
\]

(4.30)
where $g_s$ and $G_F$ are the strong and Fermi coupling constants.

The lowest order cross sections for the subprocesses $q\bar{q}, gg \rightarrow \tilde{t}_1\tilde{t}_1 h$ in the MSSM are obtained by using the following formula:

$$
\frac{d\sigma^{ij}_{\text{LO}}}{d\Omega^*_t} = \frac{1}{2s} \kappa_{ij}^{av} \frac{1}{(2\pi)^4} \frac{k(s_{34},m_t^2,m_h^2)}{8 s_{34}} \frac{\kappa(s,s_{34},m_h^2)}{8s} \Theta(\sqrt{s} - 2m_t - m_h) \Theta(s_{34} - 4m_t^2) \times \\
\times \Theta(|\sqrt{s} - m_h|^2 - s_{34}) \sum_{\text{spins, colours}} |M_{\text{LO}}^{ij}|^2 \, ds_{34} \, d\Omega^*_t \, d\cos(\theta^\text{CM}) ,
$$

(4.32)
with

\[ \kappa(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2xy - 2xz - 2yz}. \]  

(4.33)

The labels \( ij \) indicate the specific initial state of the partonic process. The solid angle \( \Omega^* \) of the stop quark is defined in the rest frame of the “\( \tilde{t}_1 \tilde{t}_1 \)-state”, using the direction of flight of the “\( \tilde{t}_1 \tilde{t}_1 \)-state” in the initial-state centre-of-mass frame as \( z \)-axis. The angle \( \theta^\text{CM}_h \) denotes the polar angle of the Higgs boson in the initial-state centre-of-mass frame, using the direction of the initial-state beams as \( z \)-axis. The corresponding azimuthal angle is trivially integrated to \( 2\pi \) in view of rotational invariance about the initial-state beams. The factor \( k_{av}^{ab} \) results from the average over the initial-state spins and colours:

\[
k_{av}^{q\bar{q}} = \frac{1}{36}, \quad k_{av}^{gg} = \frac{1}{256}, \quad k_{av}^{qg} = k_{av}^{g\bar{q}} = \frac{1}{96},
\]  

(4.34)

where \( k_{av}^{q\bar{q}} \) and \( k_{av}^{gg} \) are given for future use. The LO total cross section of \( pp \rightarrow \tilde{t}_1 \tilde{t}_1 h \) can be expressed as:

\[
\sigma_{LO}(pp \rightarrow \tilde{t}_1 \tilde{t}_1 h) = \sum_{ij} \int dx_1 dx_2 F^p_i(x_1, \mu) F^p_j(x_2, \mu) \hat{\sigma}_{LO}^{ij}(x_1, x_2, \mu).
\]  

(4.35)

where \( \hat{\sigma}_{LO}^{ij}(ij = q\bar{q}, gg) \) is the LO parton-level total cross section for incoming \( i \) and \( j \) partons, \( F^p_i \)’s are the LO parton distribution functions (PDF) with parton \( i \) in a proton/antiproton.
5.1 Virtual Corrections

The virtual corrections consist of self-energy, vertex, box, and pentagon diagrams. If we denote by $\mathcal{M}_{D_i}$ the amplitude associated with each virtual diagram $D_i$, the virtual amplitude squared can then be written as:

$$\sum |\mathcal{M}_{\text{virt}}|^2 = \sum_i \sum (\mathcal{M}_{\text{LO}}^d \mathcal{M}_{D_i}^* + \mathcal{M}_{\text{LO}}^{d*} \mathcal{M}_{D_i}) = \sum_i \sum 2 \Re (\mathcal{M}_{\text{LO}}^d \mathcal{M}_{D_i}^*),$$

(5.1)

where the index $i$ runs over the set of all virtual diagrams, and $\mathcal{M}_{\text{LO}}^d$ denotes the tree-level amplitude calculated in $d=4-2\epsilon$ dimensions.

Self-energy and vertex diagrams contain both IR and UV divergences. Box and pentagon diagrams are ultraviolet finite, but have infrared singularities. The IR poles in the virtual corrections are eventually canceled by analogous singularities in the real corrections to the tree-level cross section. The real corrections will be the subject of Sec. 5.3.

The calculation of the NLO SUSY-QCD corrections has been performed in the framework of the MSSM. We adopt the 't Hooft-Feynman gauge, and use dimensional regularization method in $D = 4 - 2\epsilon$ dimensions to isolate ultraviolet (UV), infrared (IR) and collinear singularities. The masses have been renormalized in the on-shell scheme. Renormalization and factorization are performed in the modified minimal subtraction ($\overline{MS}$) scheme. The Feynman graphs and the relevant amplitude have been generated by using \texttt{FeynArts} [78]. These amplitudes have subsequently been reduced in terms of SME with the help of \texttt{Mathematica} and \texttt{Form} [79]. The algebraic output of the calculations has been implemented in \texttt{Fortran} for numerical evaluation. The phase space integration is implemented by using Monte Carlo technique.

In the calculations of one-loop diagrams we adopt the definitions of one-loop integral...
functions as in Ref.[80]. For later use, we introduce the following definitions for the scalar and tensor integrals in analogy to Refs. [81, 82, 83, 84, 85],

\[
B_{\{0,\mu,\nu,\ldots\}^N}(q_1, m_0, m_1) = \frac{(2\pi\mu)^{2\epsilon}}{i\pi^2} \int d^D q \frac{\{1, q_\mu, q_\mu q_\nu, \ldots\}}{d_0 d_1},
\]

\[
C_{\{0,\mu,\nu,\ldots\}^N}(q_1, q_2, m_0, m_1, m_2) = \frac{(2\pi\mu)^{2\epsilon}}{i\pi^2} \int d^D q \frac{\{1, q_\mu, q_\mu q_\nu, \ldots\}}{d_0 d_1 d_2},
\]

\[
D_{\{0,\mu,\nu,\ldots\}^N}(q_1, q_2, q_3, m_0, m_1, m_2, m_3) = \frac{(2\pi\mu)^{2\epsilon}}{i\pi^2} \int d^D q \frac{\{1, q_\mu, q_\mu q_\nu, \ldots\}}{d_0 d_1 d_2 d_3},
\]

\[
E_{\{0,\mu,\nu,\ldots\}^N}(q_1, q_2, q_3, q_4, m_0, m_1, m_2, m_3, m_4) = \frac{(2\pi\mu)^{2\epsilon}}{i\pi^2} \int d^D q \frac{\{1, q_\mu, q_\mu q_\nu, \ldots\}}{d_0 d_1 d_2 d_3 d_4},
\]

with \(d_0 = q^2 - m_0^2 + i0\), \(d_j = (q + q_j)^2 - m_j^2 + i0\), \(j = 1, \ldots, 4\). (5.2)

Here \(+i0\) is the infinitesimal imaginary part as required by causality, \(D = 4 - 2\epsilon\) is the space-time dimension, and \(\mu\) is the arbitrary reference scale of dimensional regularization, which we may identify with the renormalization scale. The UV and IR divergences appear as poles in \(\epsilon\); it is convenient to use the abbreviations

\[
\Delta_1(\mu) = \frac{\Gamma(1 + \epsilon)}{\epsilon} \left(\frac{4\pi\mu^2}{m_t^2}\right)^\epsilon, \quad \Delta_2(\mu) = \frac{\Gamma(1 + \epsilon)}{\epsilon^2} \left(\frac{4\pi\mu^2}{m_t^2}\right)^\epsilon \tag{5.3}
\]

for these divergences. If we distinguish between UV and IR divergences, we explicitly write \(\Delta_{\mu}^{UV,IR}(\mu)\). Following standard techniques of one-loop calculations Refs. [81, 82, 83, 84, 85], the tensor integrals are algebraically reduced to scalar integrals. The numerical calculations of the IR-infinite integral functions are implemented by using the methods described in Ref. [86]. The method is given by:

\[
T_{\mu_1 \ldots \mu_P}^{(N)D} = T_{\mu_1 \ldots \mu_P}^{(N)D}|_{sing} + T_{\mu_1 \ldots \mu_P}^{(N)4} - T_{\mu_1 \ldots \mu_P}^{(N)4}|_{sing}, \tag{5.4}
\]

where \(T_{\mu_1 \ldots \mu_P}^{(N)D}\) and \(T_{\mu_1 \ldots \mu_P}^{(N)D}|_{sing}\) are the \(N\)-point integral functions and their complete mass-singular parts in \(D = 4 - 2\epsilon\) dimensions, respectively. \(T_{\mu_1 \ldots \mu_P}^{(N)4}\) and \(T_{\mu_1 \ldots \mu_P}^{(N)4}|_{sing}\) are in 4 dimensions. \(T_{\mu_1 \ldots \mu_P}^{(N)4}\) can be calculated by using mass renormalization scheme. \(T_{\mu_1 \ldots \mu_P}^{(N)D}|_{sing}\) can be calculated by:

\[
T_{\mu_1 \ldots \mu_P}^{(N)}(p_0, \ldots, p_{N-1}, m_0, \ldots, m_{N-1})|_{sing} = \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} \sum_{k \neq n, n+1} A_{nk} C_{\mu_1 \ldots \mu_P}(p_n, p_{n+1}, p_k, m_n, m_{n+3}, m_k). \tag{5.5}
\]
where $C_{\mu_1...\mu_P}$ is the corresponding 3-point integral functions. $C_{\mu_1...\mu_P}$ in 4 dimensions can be calculated by using mass renormalization scheme. The explicit expressions of the $L_{\mu_1...\mu_P}$ in $D = 4 - 2\epsilon$ dimensions can be found in Ref.[81].

5.1.1 Counterterms and renormalization constants

SUSY-QCD corrections modify the stop mass matrix and the fields, necessitating the renormalization of the masses $m^2_{\tilde{t}_j} \rightarrow m^2_{\tilde{t}_j} + \delta m^2_{\tilde{t}_j}$ and of the wave functions $\tilde{t}_i \rightarrow Z^{1/2}_{\tilde{t}_j}; Z^{1/2} = 1 + \delta Z/2$ (see Sec. 3.3). The UV divergences are renormalized by introducing a suitable set of counterterms for the external wave–function renormalization of the gluons, of the light quarks, and of the stop quark, and the renormalization of the coupling constants. Since Higgs-squark interaction involves the squark masses, the squark mixing angles and the trilinear squark couplings, a counterterm for each of these parameters will be needed.

We perform the renormalization programme in the on–shell scheme where the quark and squark masses are defined as the poles of their respective propagators $\delta m_{\tilde{q}_i} = \Sigma_{\tilde{q}_i}^q (m_{\tilde{q}_i}^2)$ and $\delta m_q = \Sigma_{\tilde{q}_i}^q (m_q^2)$. The squark wave-function renormalization constants $\delta Z_{\tilde{q}_i}$ are defined as usual in such a way that the residues at the poles are equal to one. The renormalization of the strong coupling constants $g_s$ and $\hat{g}_s$ are carried out in the $\overline{MS}$ renormalization scheme at the charge-renormalization scale $\mu_R$; a finite shift between the bare Yukawa coupling $\hat{g}_s$ and the bare gauge coupling $g_s$ restores supersymmetry at the one-loop level in the $\overline{MS}$ scheme:

$$\hat{g}_s = g_s \left[ 1 + \frac{\alpha_s}{8\pi} \left( \frac{4}{3} N_c - C_F \right) \right] \tag{5.6}$$

The strong coupling renormalization for the SUSY-QCD corrections is given by equation (3.12). The Yukawa coupling counterterms for the neutral MSSM Higgs bosons production are given by:

$$M_{Yuk} = -\frac{\delta m_q}{m_q}. \tag{5.7}$$

Here we list the counter terms for renormalization of vertices and propagators in the one-loop amplitudes for stop-pair production. The Feynman rules for the counter terms can be expressed in terms of the field renormalization constants of quarks, squarks, gluons, and gluinos, defined from the relation between bare and renormalized fields,

$$\Psi_{q_a}^{bare} = \Psi_{q_a}^{ren} \left( 1 + \frac{1}{2} \delta Z_{q_a} \right), \quad \Phi_{\tilde{Q}_a}^{bare} = \Phi_{\tilde{Q}_a}^{ren} \left( 1 + \frac{1}{2} \delta Z_{\tilde{Q}_a} \right),$$

$$C_{\mu}^{bare} = G_{\mu}^{ren} \left( 1 + \frac{1}{2} \delta Z_G \right), \quad \Psi_{\tilde{g}}^{bare} = \Psi_{\tilde{g}}^{ren} \left( 1 + \frac{1}{2} \delta Z_{\tilde{g}} \right), \tag{5.8}$$
together with the renormalization constants for the strong coupling $g_s$, for the strong Yukawa coupling $\hat{g}_s$, and for the squark masses, which are defined according to

$$g_{s\text{ bare}} = g_{s\text{ ren}}(1 + \delta Z_g), \quad \hat{g}_{s\text{ bare}} = \hat{g}_{s\text{ ren}}(1 + \delta Z_{\hat{g}}), \quad m_{Q,a\text{ bare}} = m_{Q,a\text{ ren}} + \delta m_{Q,a}. \quad (5.9)$$

The actual expressions of the counterterms that are relevant for stop-pair production processes are given below.

- Vertex counter terms involving gauge bosons:

\[
\begin{align*}
\hat{g}_i \hat{g}_j & = -ig_s(\delta Z_{\tilde{Q},a} + \frac{\delta Z_G}{2} + \delta Z_g)T_C(k + k')_\mu \\
\end{align*}
\]

\[
\begin{align*}
\hat{g}_i \hat{g}_j & = ig_s^2 \delta Z_{\tilde{Q},a} (\frac{1}{3} \delta C_1 C_2 + f^{C_1 C_2 A} T^A) g_{\mu \nu} \\
\end{align*}
\]

\[
\begin{align*}
B = -ig_s[(\frac{\delta Z_G}{2} + \delta Z_y + \delta Z_{qL})\gamma_\mu \omega_- + (\frac{\delta Z_G}{2} + \delta Z_y + \delta Z_{qR})\gamma_\mu \omega_+]T_C \\
k \text{ and } k' \text{ are the momenta of the squark and the antisuqark, and they are fixed according to the arrow. } T_C \text{ are the color matrices and } f^{ABC} \text{ the structure constants of the color group. We omit the color indices of fermions and sfermions.}
\end{align*}
\]

- Self energy counter terms:

\[
\begin{align*}
\hat{g}_i \hat{g}_j & = i[(p^2 - m_{Q,a}^2)\delta Z_{\tilde{Q},a} - \delta m_{Q,a}^2]
\end{align*}
\]
\[ g \gamma \gamma = i(p_\mu p_\nu - g_\mu_\nu p^2)\delta Z_G \]

- Higgs-Squark counter term:

\[
\begin{align*}
htL^tL : C &= \frac{-igm}{\cos\theta_w}(\frac{1}{2} - \frac{2}{3}\sin^2\theta_w)\sin(\beta + \alpha)(1 + \delta Z_{tL}) - \frac{igm^2\cos\alpha}{m_w\sin\beta}(1 + \delta Z_{tR} + \frac{2\delta m_t}{m_t}) \\
htR^tR : C &= \frac{-igm^2}{\cos\theta_w}(\frac{2}{3}\sin\theta_w)\sin(\beta + \alpha)(1 + \delta Z_{tR}) - \frac{igm^2\cos\alpha}{m_w\sin\beta}(1 + \delta Z_{tL} + \frac{2\delta m_t}{m_t}) \\
htL^tR : C &= \frac{-igm}{2\cos\theta_w}A_t(1 + \frac{1}{2}(\delta Z_{tL} + \delta Z_{tR}) + \frac{\delta m_t}{m_t} + \frac{\delta A_t}{A_t})
\end{align*}
\]

The renormalization constants of the squark sector are fixed by on-shell conditions (Sec. 3.3),

\[
\delta Z_{\tilde{Q},a} = -Re \left\{ \frac{\partial \Sigma_{\tilde{Q},a}(p^2)}{\partial p^2} \right\} \bigg|_{p^2=m^2_{\tilde{Q},a}} \quad \delta m^2_{\tilde{Q},a} = Re \left\{ \Sigma_{\tilde{Q},a}(m^2_{\tilde{Q},a}) \right\},
\]

(5.10)

The field renormalization constants of the quarks are obtained via on-shell conditions as
follows,

\[ \delta Z_{qa} = -Re \left\{ \Sigma_{qa}(m_\tilde{q}_a^2) \right\} - m_\tilde{q}_a^2 Re \left\{ \frac{\partial}{\partial p^2} \left( \Sigma_{qL}(p^2) + \Sigma_{qR}(p^2) + 2\Sigma_{qS}(p^2) \right) \right\}_{p^2 = m_\tilde{q}_a^2} \quad (a = L, R) \]  

(5.11)

with the scalar coefficients in the Lorentz decomposition of the self energy,

\[ \Sigma_q(p^2) = \eta_0 \Sigma_{qL}(p^2) + \eta_+ \Sigma_{qR}(p^2) + m_q \Sigma_{qS}(p^2). \]  

(5.12)

Also in the gluino sector we determine the renormalization constants by on-shell conditions,

\[ \begin{align*} 
\delta m_\tilde{g} &= \frac{1}{2} Re \left\{ m_\tilde{g} \left( \Sigma_{\tilde{g}L}(m_\tilde{g}^2) + \Sigma_{\tilde{g}R}(m_\tilde{g}^2) + 2\Sigma_{\tilde{g}S}(m_\tilde{g}^2) \right) \right\} \\
\delta Z_\tilde{g} &= -Re \left\{ \Sigma_{\tilde{g}L}(m_\tilde{g}^2) \right\} - m_\tilde{g}^2 Re \left\{ \frac{\partial}{\partial p^2} \left( \Sigma_{\tilde{g}L}(p^2) + \Sigma_{\tilde{g}R}(p^2) + 2\Sigma_{\tilde{g}S}(p^2) \right) \right\}_{p^2 = m_\tilde{g}^2}. 
\end{align*} \]  

(5.13)

The renormalization of the strong coupling deserves some particular care. As mentioned in Section 3.1.2, the strong coupling \( g_s \) is renormalized in the \( \overline{\text{MS}} \) scheme and is given by equation (3.12).

### 5.1.2 Vertex corrections

![Fig. 5.1: SUSY-QCD vertex corrections to squark-squark-Higgsboson.](image)

The graph with the gluino–quark–quark loop in Fig. 5.1 leads to

\[
\delta G_{h_0 \tilde{t}_1 \tilde{t}_1}^{(v, \tilde{g})} = - \frac{2}{3} \frac{\alpha_s}{\pi} m_\tilde{g} \cos 2\theta_t \left( - \frac{g}{2 m_W \sin \beta} \cos \alpha \right) \left[ B_0(m_{\tilde{t}_1}^2, m_{\tilde{g}}^2, m_t^2) + B_0(m_{\tilde{t}_1}^2, m_{\tilde{g}}^2, m_t^2) \right] \]

\[ + \left( 4 m_t^2 - m_{h_0}^2 \right) C_0(m_{\tilde{t}_1}^2, m_{\tilde{h}_0}^2, m_{\tilde{t}_1}^2; 0, m_{\tilde{t}_1}^2, m_{\tilde{t}_1}^2) \]  

(5.14)
The vertex correction due to the gluon-stop-stop loop in Fig. 5.1 is
\[
\delta G^{(v,g)}_{h\tilde{t}_i\tilde{t}_i} = \frac{\alpha_s}{3\pi} G^{h\tilde{t}_i\tilde{t}_i} \left[ B_0(m^2_{\tilde{t}_i},0,m^2_{\tilde{t}_i}) + B_0(m^2_{\tilde{t}_i},0,m^2_{\tilde{t}_i}) - B_0(m^2_h,m^2_{\tilde{t}_i},m^2_{\tilde{t}_i}) + 2 \left( m^2_{\tilde{t}_i} + m^2_{\tilde{t}_i} - m^2_h \right) C_0(m^2_{\tilde{t}_i},m^2_h,m^2_{\tilde{t}_i},0,m^2_{\tilde{t}_i},m^2_{\tilde{t}_i}) \right] \quad (5.15)
\]

The vertex correction due to the four–squark interaction in Fig. 5.1 is
\[
\delta G^{(v,\tilde{q})}_{h\tilde{t}_i\tilde{t}_i} = -\frac{\alpha_s}{3\pi} \sum_{i,j=1,2} D_{1i} D_{1j} G^{h\tilde{q}_i\tilde{q}_j} B_0(m^2_h,m^2_{\tilde{q}_j},m^2_{\tilde{q}_i}) \quad (5.16)
\]

where \( D_{11} = \cos 2\theta_{\tilde{q}} \) and \( D_{12} = -\sin 2\theta_{\tilde{q}} \).

5.1.3 Stop Self Energy (Squark wave function correction)

The squark self energy to \( \mathcal{O}(\alpha_s) \) gets contributions from gluon, gluino, and squark loops. The relevant Feynman digrams are shown in Fig. 5.2.

The gluon–squark loop leads to
\[
\Sigma^{(g)}_{ii}(m^2_{\tilde{t}_i}) = -\frac{2}{3} \frac{\alpha_s}{\pi} m^2_{\tilde{t}_i} \{2B_0(m^2_{\tilde{t}_i},0,m^2_{\tilde{t}_i}) + B_1(m^2_{\tilde{t}_i},0,m^2_{\tilde{t}_i}) \}. \quad (5.17)
\]

The gluon loop due to the \( \tilde{t}\tilde{t}gg \) interaction gives no contribution. The contribution due to the gluino–quark loop is
\[
\Sigma^{(\tilde{g})}_{ii}(m^2_{\tilde{t}_i}) = -\frac{4}{3} \frac{\alpha_s}{\pi} \{ A_0(m^2_{\tilde{t}_i}) + m^2_{\tilde{t}_i} B_1(m^2_{\tilde{t}_i},m^2_{\tilde{g}},m^2_t) + [m^2_{\tilde{g}} + (-)^i m_\tilde{g} m_t \sin 2\theta_{\tilde{q}}] B_0(m^2_{\tilde{t}_i},m^2_{\tilde{g}},m^2_{\tilde{t}_i}) \} \quad (5.18)
\]
\[
\Sigma^{(\tilde{g})}_{12}(m^2_{\tilde{t}_i}) = \Sigma^{(\tilde{g})}_{21}(m^2_{\tilde{t}_i}) = \frac{4}{3} \frac{\alpha_s}{\pi} m\tilde{g} m_t \cos 2\theta_{\tilde{q}} B_0(m^2_{\tilde{t}_i},m^2_{\tilde{g}},m^2_{\tilde{t}_i}), \quad (5.19)
\]
and the squark bubble leads to \((i \neq i')\)

\[
\Sigma_{ii}^{(\tilde{q})}(m_{\tilde{t}}^2) = \frac{\alpha_s}{3\pi} \left\{ \cos^2 2\theta_{\tilde{q}} A_0(m_{\tilde{t}}^2) + \sin^2 2\theta_{\tilde{q}} A_0(m_{\tilde{t}}^2) \right\},
\]

\[ (5.20) \]

\[
\Sigma_{i2}^{(\tilde{q})}(m_{\tilde{t}}^2) = \frac{\alpha_s}{6\pi} \sin 4\theta_{\tilde{q}} \{ A_0(m_{\tilde{q}}^2) - A_0(m_{\tilde{q}}^2) \} = \Sigma_{21}^{(\tilde{q})}(m_{\tilde{t}}^2).
\]

\[ (5.21) \]

Note, that \(\Sigma_{ii'}^{(\tilde{q})}(m_{\tilde{q}}^2) = \Sigma_{ii'}^{(\tilde{g})}(m_{\tilde{q}}^2)\).

\(\delta m_{\tilde{t}},\) the shift from the bare to the on-shell squark mass, is given by:

\[
\delta m_{\tilde{t}} = \text{Re} \left[ \Sigma_{ii}^{(\tilde{g})}(m_{\tilde{t}}^2) + \Sigma_{ii}^{(\tilde{g})}(m_{\tilde{t}}^2) + \Sigma_{ii}^{(\tilde{g})}(m_{\tilde{t}}^2) \right].
\]

\[ (5.22) \]

or,

\[
m_{\tilde{t}} \delta m_{\tilde{t}} = \left( \frac{\alpha_s}{3\pi} \right) \left[ -(m_t^2 + m_g^2 - m_{\tilde{t}}^2) B_0(m_{\tilde{t}}^2, m_t^2, m_g^2) - 2m_{\tilde{t}}^2 B_0(m_{\tilde{t}}^2, 0, m_{\tilde{t}}^2) - A_0(m_g^2) - A_0(m_{\tilde{t}}^2) + \frac{1}{2} \left( 1 + \cos^2 2\theta_{\tilde{q}} \right) A_0(m_{\tilde{t}}^2) + \sin^2 2\theta_{\tilde{q}} A_0(m_{\tilde{t}}^2) \right] - 2(-1)^i \sin 2\theta_{\tilde{q}} m_g m_{\tilde{t}} B_0(m_{\tilde{t}}^2, m_t^2, m_g^2)
\]

\[ (5.23) \]

The stop wave–function renormalization constants \(\tilde{Z}_{ii}(\tilde{t})\) are:

\[
\delta \tilde{Z}_{ii}^{(\tilde{g},\tilde{g})} = \text{Re} \left\{ \Sigma_{ii}^{(\tilde{g},\tilde{g})}(m_{\tilde{t}}^2) \right\}, \quad \delta \tilde{Z}_{ii}^{(\tilde{g},\tilde{t})} = \frac{\text{Re} \left\{ \Sigma_{ii}^{(\tilde{g},\tilde{t})}(m_{\tilde{t}}^2) \right\}}{m_{ii'}^2 - m_{ii}^2}, \quad i \neq i'
\]

\[ (5.24) \]

with \(\Sigma_{ii}(m^2) = \partial \Sigma_{ii}(p^2)/\partial p^2 |_{p^2 = m^2}:

\[
\tilde{\Sigma}_{ii}^{(\tilde{g})}(m_{\tilde{q}}^2) = -\frac{2\alpha_s}{3\pi} \left[ B_0(m_{\tilde{t}}^2, 0, m_{\tilde{t}}^2) + 2m_{\tilde{t}}^2 B_0(m_{\tilde{t}}^2, 0, m_{\tilde{t}}^2) \right],
\]

\[ (5.25) \]

\[
\tilde{\Sigma}_{ii}^{(\tilde{q})}(m_{\tilde{t}}^2) = \frac{2\alpha_s}{3\pi} \left[ B_0(m_{\tilde{t}}^2, m_{\tilde{t}}^2, m_{\tilde{g}}^2) + (m_{\tilde{t}}^2 - m_{\tilde{t}}^2 - m_{\tilde{g}}^2) B_0(m_{\tilde{t}}^2, m_{\tilde{t}}^2, m_{\tilde{g}}^2)
\]

\[ (5.26) \]

or

\[
\frac{1}{2} \delta Z_{ii} = \left( \frac{\alpha_s}{3\pi} \right) \left[ (m_t^2 + m_g^2 - m_{\tilde{t}}^2) B_0(m_{\tilde{t}}^2, m_t^2, m_g^2) - B_0(m_{\tilde{t}}^2, m_t^2, m_g^2) + B_0(m_{\tilde{t}}^2, 0, m_{\tilde{t}}^2) \right] + 2(-1)^i \sin 2\theta_{\tilde{q}} m_g m_{\tilde{t}} B_0(m_{\tilde{t}}^2, m_t^2, m_g^2) + 2m_{\tilde{t}}^2 B_0(m_{\tilde{t}}^2, 0, m_{\tilde{t}}^2)
\]

\[ (5.27) \]
The four–squark interaction does not contribute to $\delta \tilde{Z}_{ii}$ because $\tilde{\Sigma}_{ii}^{(t)} = 0$. On the other hand, the off–diagonal squark wave–function renormalization constant $\delta \tilde{Z}_{ii'}$ gets no contribution from gluon exchange diagrams because they do not mix $\tilde{q}_1$ and $\tilde{q}_2$. Bare and renormalized squark fields ($\tilde{t}^0_i$ and $\tilde{t}_i$) are related by

$$
\tilde{t}^0_i = (1 + \frac{1}{2} \delta \tilde{Z}_{ii}) \tilde{t}_i + \delta \tilde{Z}_{i'i} \tilde{t}_{i'}, \tag{5.28}
$$

$$
\tilde{t}^0_{i'} = (1 + \frac{1}{2} \delta \tilde{Z}_{ii}) \tilde{t}_{i'} + \delta \tilde{Z}_{i'i} \tilde{t}_i. \tag{5.29}
$$

5.1.4 Renormalization of the Squark Mixing Angle

The stop couplings depend on the mixing angle which therefore must be renormalized. $\delta \theta_{\tilde{q}}$ gets contributions from gluon and gluino exchanges, $\delta \theta_{\tilde{q}} = \delta \theta_{\tilde{q}}^{(g)} + \delta \theta_{\tilde{q}}^{(g)}$. Explicitly:

$$
\delta \theta_{\tilde{q}}^{(g)} = \frac{\alpha_s}{6\pi} \frac{\sin 4\theta_{\tilde{q}}}{m_{\tilde{q}_2}^2 - m_{\tilde{q}_1}^2} \left[ A_0(m_{\tilde{q}_2}^2) - A_0(m_{\tilde{q}_1}^2) \right], \tag{5.30}
$$

$$
\delta \theta_{\tilde{q}}^{(g)} = \frac{4}{3 \pi} \frac{\alpha_s m_{\tilde{q}} m_q}{I_{3L}^q(m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2)} \left[ B_0(m_{\tilde{q}_2}^2, m_{\tilde{q}_1}^2, m_q^2) c_{11} - B_0(m_{\tilde{q}_1}^2, m_{\tilde{q}_2}^2, m_q^2) c_{22} \right]. \tag{5.31}
$$

5.1.5 Renormalization of $m_q A_q$

The squark couplings to Higgs bosons involve the parameter $A_q$. Therefore, also this parameter has to be renormalized. We proceed like for the definition of the on–shell $M^2_{\tilde{Q}}$ and write the expression $m_q A_q$ in terms of on–shell squark masses $m_{\tilde{q}_i}$ and mixing angles $\theta_{\tilde{q}}$. We thus get

$$
\delta(m_q A_q) = \frac{1}{2} (\delta m_{\tilde{q}_1}^2 - \delta m_{\tilde{q}_2}^2) \sin 2\theta_{\tilde{q}} + (m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2) \cos 2\theta_{\tilde{q}} \delta \theta_{\tilde{q}} + \mu \cot \beta \delta m_q \tag{5.32}
$$

5.1.6 Quark Self Energy

The self energy of a quark gets contributions from the gluon and gluino as shown in Fig. 5.3.

Fig. 5.3: Quark self–energy diagrams.
\( \delta m_q \), the shift from the bare to the pole mass of the quark \( q \), thus has two contributions: The gluino contribution is

\[
\delta m_q^{(g)} = -\frac{\alpha_s}{3\pi} \left\{ m_q \left[ B_1(m_q^2, m_g^2, m_{\tilde{q}}^2) + B_1(m_q^2, m_{\tilde{q}}^2, m_g^2) \right] + m_\tilde{q} \sin 2\theta_\tilde{q} \left[ B_0(m_q^2, m_g^2, m_{\tilde{q}}^2) - B_0(m_q^2, m_{\tilde{q}}^2, m_g^2) \right] \right\}.
\]

(5.33)

and the gluon exchange contribution is

\[
\delta m_q^{(g)} = -\frac{2\alpha_s}{3} m_q \left[ B_0(m_q^2, 0, m_q^2) - B_1(m_q^2, 0, m_q^2) \right] - \left[ 2m_q^2 (\dot{B}_0(m_q^2, 0, m_q^2) + \dot{B}_1(m_q^2, 0, m_q^2)) + b/2 \right].
\]

(5.34)

For the quark wave-function renormalization constants due to gluon exchange one gets:

\[
\delta Z_L^{(g)} = \delta Z_R^{(g)} = -\frac{2\alpha_s}{3} m_q \left[ B_0(m_q^2, 0, m_q^2) + B_1(m_q^2, 0, m_q^2) \right] - \left[ 2m_q^2 (\dot{B}_0(m_q^2, 0, m_q^2) + \dot{B}_1(m_q^2, 0, m_q^2)) + b/2 \right]
\]

(5.35)

those due to gluino exchange are:

\[
\delta Z_L^{(g)} = \frac{2\alpha_s}{3} \left\{ \frac{1}{\pi} \left[ \cos^2 \theta_{\tilde{q}} B_1(m_q^2, m_g^2, m_{\tilde{q}}^2) + \sin^2 \theta_{\tilde{q}} B_1(m_q^2, m_{\tilde{q}}^2, m_g^2) \right] + m_q^2 \left[ \dot{B}_1(m_q^2, m_g^2, m_{\tilde{q}}^2) + \dot{B}_1(m_q^2, m_{\tilde{q}}^2, m_g^2) \right] + m_\tilde{q} \sin 2\theta_{\tilde{q}} \left( \ddot{B}_0(m_q^2, m_g^2, m_{\tilde{q}}^2) \right) \right\},
\]

(5.36)

\[
\delta Z_R^{(g)} = \frac{2\alpha_s}{3} \left\{ \frac{1}{\pi} \left[ \sin^2 \theta_{\tilde{q}} B_1(m_q^2, m_g^2, m_{\tilde{q}}^2) + \cos^2 \theta_{\tilde{q}} B_1(m_q^2, m_{\tilde{q}}^2, m_g^2) \right] + m_q^2 \left[ \dot{B}_1(m_q^2, m_g^2, m_{\tilde{q}}^2) + \dot{B}_1(m_q^2, m_{\tilde{q}}^2, m_g^2) \right] + m_\tilde{q} \sin 2\theta_{\tilde{q}} \left( \ddot{B}_0(m_q^2, m_g^2, m_{\tilde{q}}^2) \right) \right\},
\]

(5.37)

The relation between the bare quark field \( q^0 \) and the renormalized one \( q \) is

\[
q^0 = (1 + \frac{1}{2} \delta Z_L P_L + \frac{1}{2} \delta Z_R P_R) q,
\]

(5.38)

\[
\bar{q}^0 = \bar{q} (1 + \frac{1}{2} \delta Z_L P_L + \frac{1}{2} \delta Z_R P_R).
\]

(5.39)

The parameter \( b \) in (5.34) and (5.35) exhibits the dependence on the regularization scheme: \( b = 0 \) in DRED while \( b = 1 \) in DREG. In our calculations \( b \) does not cancel.
5.1.7 Gluon Self Energy

The gluon self energy gets contributions from gluino, squark, gluon, and quark loops. The relevant Feynman diagrams are shown in Fig. 5.4.

The contribution of the gluino loop leads to

\[
\frac{3\delta_{ab}\alpha_s}{\pi} \left[ A_0(m_{\tilde{g}}^2) - 2B_{00}(k^2, m_{\tilde{g}}^2, m_{\tilde{g}}^2) - 2k_\mu k_\nu (B_1(k^2, m_{\tilde{g}}^2, m_{\tilde{g}}^2) + B_{11}(k^2, m_{\tilde{g}}^2, m_{\tilde{g}}^2)) \right] \quad (5.40)
\]

and the contributions of the diagrams with squark loop are

\[
\frac{\delta_{ab}\alpha_s}{\pi} \left[ B_{00}(k^2, m_{\tilde{q}_i}^2, m_{\tilde{q}_i}^2) + 2k_\mu k_\nu \left( \frac{1}{4} B_0(k^2, m_{\tilde{q}_i}^2, m_{\tilde{q}_i}^2) + B_1(k^2, m_{\tilde{q}_i}^2, m_{\tilde{q}_i}^2) + B_{11}(k^2, m_{\tilde{q}_i}^2, m_{\tilde{q}_i}^2) \right) \right] \quad (5.41)
\]

for two points squark loop, and

\[
\frac{-\delta_{ab}\alpha_s}{2\pi} \left[ g_{\mu\nu} A_0(m_{\tilde{q}_i}^2) \right] \quad (5.42)
\]

for one point loop. The diagram with gluon bubble gives no contribution. The two points gluon loop gives contribution of

\[
\frac{\delta_{ab}\alpha_s}{2\pi} \left[ -1 - 3B_0(k^2, 0, 0) + 15g_{\mu\nu}(B_{00}(k^2, 0, 0)) + k_\mu k_\nu (B_1(k^2, 0, 0) + B_{11}(k^2, 0, 0)) \right] \quad (5.43)
\]
The quark loop contribution leads to
\[ \frac{\delta_{ab}\alpha_s}{\pi} \left[ g_{\mu\nu}(A_0(m_q^2) - 2B_{00}(k^2, m_q^2, m_q^2)) - k_{\mu}k_{\nu}(B_1(k^2, m_q^2, k^2) + B_{11}(k^2, m_q^2, m_q^2)) \right] \] (5.44)

### 5.2 Scalar integrals appearing in our calculation

In the following we give the minimal set of scalar integrals appearing in our calculation. Most these expressions are presented in [87, 88, 89, 90]. We use the kinematics of the process:

\[ i(p_1) + j(p_2) \rightarrow \tilde{t}_1(p_3) + \tilde{\bar{\tau}}_1(p_4) + h(p_5) , \]

with \( p_5 = p_1 + p_2 - p_3 - p_4 \), and

\[
\begin{align*}
  s &= (p_1 + p_2)^2, \\
  \tau_1 &= m_{\tilde{t}_1}^2 - (p_1 - p_3)^2 = 2p_1 \cdot p_3, \\
  \tau_2 &= m_{\tilde{t}_1}^2 - (p_2 - p_4)^2 = 2p_2 \cdot p_4, \\
  \tau_3 &= m_{\tilde{t}_1}^2 - (p_2 - p_3)^2 = 2p_2 \cdot p_3, \\
  \tau_4 &= m_{\tilde{t}_1}^2 - (p_1 - p_4)^2 = 2p_1 \cdot p_4, \\
  \bar{s}_{\tilde{t}_1\tilde{\bar{\tau}}_1} &= (p_3 + p_4)^2 = 2p_3 \cdot p_4 + 2m_{\tilde{t}_1}^2 .
\end{align*}
\] (5.45)

and the on-shell conditions \( p_1^2 = p_2^2 = 0 \) and \( p_3^2 = p_4^2 = m_{\tilde{t}_1}^2 \). The following results are expressed in terms of the invariants \( s_{ij} = (q_i + q_j)^2 \), where \( q_i \) and \( q_j \) are two of the external momenta. We will always factor out of the integrals the factor:

\[ N_{\tilde{t}_1} = \left( \frac{4\pi\mu^2}{m_{\tilde{t}_1}^2} \right)^\epsilon \Gamma(1 + \epsilon) . \] (5.46)

#### A0 integral

\( A0 \) and \( B0 \) integrals are the only scalar UV-divergent integrals. The 1-point integral is IR-finite, and it is given by:

\[ A0(m_{\tilde{t}_1}) = \frac{i}{16\pi^2} N_{\tilde{t}_1} m_{\tilde{t}_1}^2 \left( \frac{1}{\epsilon_{UV}} + 1 \right) . \] (5.47)
**B0 integrals**

All $B_0$ integrals are UV-divergent and their corresponding UV-pole part is:

$$B_0(p; m_0, m_1) \bigg|_{\text{UV-pole}} = \frac{i}{16\pi^2} \mathcal{N}_{\tilde{t}_1} \frac{1}{\epsilon_{\text{UV}}}.$$

(5.48)

The only IR-divergent $B_0$ integral is the one with zero internal masses and with a light-like ($p_1^2 = 0$) external momentum $B_0(p_1; 0, 0)$. Indeed, there is no invariant available to build this integral, so it should vanish. One can understand this vanishing as a cancellation between the UV and IR behavior of this integral of the form:

$$B_0(p_1; 0, 0) = \frac{i}{16\pi^2} \mathcal{N}_{\tilde{t}_1} \left( \frac{1}{\epsilon_{\text{UV}}} - \frac{1}{\epsilon_{\text{IR}}} \right),$$

(5.49)

where we have made explicitly the UV or IR nature of the two single poles. Notice that the overall constant in front of the integral is a matter of convention.

**IR-divergent C0 integrals**

Let us organize the integrals by the number of internal masses.

**I. All internal masses equal to zero**

1. $C_0(p_1, p_2; 0, 0, 0)$, two on-shell massless legs:

$$C_0(p_1, p_2; 0, 0, 0) = \frac{i}{16\pi^2} \mathcal{N}_{\tilde{t}_1} \frac{1}{s_{12}} \left( \frac{1}{\epsilon^2} + \frac{1}{\epsilon} \ln \frac{m_{t_1}^2}{s_{12}} + \frac{1}{2} \ln^2 \frac{m_{t_1}^2}{s_{12}} - \frac{\pi^2}{6} \right).$$

(5.50)

2. $C_0(p_1, -p_3 - p_4; 0, 0, 0)$, one on-shell massless leg:

$$C_0(p_1, -p_3 - p_4; 0, 0, 0) = \frac{i}{16\pi^2} \mathcal{N}_{\tilde{t}_1} \frac{1}{2 p_1 \cdot (-p_3 - p_4)} \left( \frac{1}{\epsilon} \left[ \ln \frac{m_{t_1}^2}{|s_{12}|} - \ln \frac{m_{t_1}^2}{(-p_3 - p_4)^2} \right] + \frac{1}{2} \left[ \ln^2 \frac{m_{t_1}^2}{|s_{12}|} - \pi^2 - \ln^2 \frac{m_{t_1}^2}{(-p_3 - p_4)^2} \right] \right),$$

(5.51)

notice that for this case, in our kinematics, $s_{12} < 0$ so we have included absolute values in the logarithm arguments to avoid negative arguments.
II. One non-zero internal mass

1. \( C_{0}(p_1, -p_3; 0, 0, m_{t_1}) \), One on-shell massless leg and one on-shell massive leg:

\[
C_{0}(p_1, -p_3; 0, 0, m_{t_1}) = \frac{i}{16\pi^2} N_{t_1} \left\{ \frac{1}{s_{12} - m_{t_1}^2} \ln \frac{m_{t_1}^2}{s_{12} - m_{t_1}^2} + \frac{1}{2} \ln \left( 1 - \frac{s_{12}}{m_{t_1}^2} \right) + \ln \left( 1 - \frac{m_{t_1}^2}{s_{12}} \right) \ln \left( 1 - \frac{s_{12}}{m_{t_1}^2} \right) - \frac{\pi^2}{6} \right\}.
\]

(5.52)

where the dilogarithm \( \text{Li}_2(x) \) is defined as

\[
\text{Li}_2(x) = -\int_0^1 dt \frac{\ln(1 - xt)}{t}.
\]

(5.53)

2. \( C_{0}(p_1, p_2 - p_3; 0, 0, m_{t_1}) \), One on-shell massless leg:

\[
C_{0}(p_1, p_2 - p_3; 0, 0, m_{t_1}) = \frac{i}{16\pi^2} N_{t_1} \left\{ \frac{1}{s_{12} - (p_2 - p_3)^2} \left\{ \frac{1}{\epsilon} \left( \ln \frac{m_{t_1}^2}{s_{12} - m_{t_1}^2} - \ln \frac{m_{t_1}^2}{(p_2 - p_3)^2 - m_{t_1}^2} \right) + \ln^2 \frac{m_{t_1}^2}{s_{12} - m_{t_1}^2} - \ln^2 \frac{(p_2 - p_3)^2 - m_{t_1}^2}{m_{t_1}^2} + \text{Li}_2 \left( 1 + \frac{s_{12} - m_{t_1}^2}{m_{t_1}^2} \right) \right. \\
- \ln \left( \frac{m_{t_1}^2}{(p_2 - p_3)^2 - m_{t_1}^2} \right) \ln \left( 1 - \frac{(p_2 - p_3)^2 - m_{t_1}^2}{m_{t_1}^2} \right) \left. + \text{Li}_2 \left( \frac{m_{t_1}^2}{m_{t_1}^2} - (p_2 - p_3)^2 \right) - \frac{7}{6} \pi^2 \right\}. \]

(5.54)

III. Two non-zero internal masses

1. \( C_{0}(-p_3, p_3 + p_4; 0, m_{\tilde{t}_1}, m_{t_1}) \), Two on-shell massive legs:

\[
C_{0}(-p_3, p_3 + p_4; 0, m_{\tilde{t}_1}, m_{t_1}) = \frac{i}{16\pi^2} N_{\tilde{t}_1} \frac{1}{\beta(p_3 + p_4)^2} \left\{ \frac{1}{\epsilon} \ln \left( \frac{1 - \beta}{1 + \beta} \right) - \ln \left( \frac{(p_3 + p_4)^2}{m_{\tilde{t}_1}^2} \right) \ln \left( \frac{1 - \beta}{1 + \beta} \right) - \frac{1}{2} I_2 - \pi^2 \right\},
\]

(5.55)

where we have defined:

\[
\beta = \sqrt{1 - \frac{4 m_{t_1}^2}{(p_3 + p_4)^2}},
\]

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\[ I_2 = -\ln^2 \left( \frac{1}{2}(1 + \beta) \right) + \ln^2 \left( \frac{1}{2}(1 - \beta) \right) - \pi^2 + 2 \ln \beta \ln \left( \frac{1 - \beta}{1 + \beta} \right) -2 \text{Li}_2 \left( -\frac{1 - \beta}{2\beta} \right) + 2 \text{Li}_2 \left( \frac{1 + \beta}{2\beta} \right). \]

**IR-divergent D0 integrals**

1. All internal masses equal to zero

We write the massless box expressions following the results of [89].

1. \(D_0(p_1, p_2, -p_3 - p_4; 0, 0, 0, 0)\), two adjacent on-shell massless legs:

\[
D_0(p_1, p_2, -p_3 - p_4; 0, 0, 0, 0) = \frac{i}{16\pi^2 N_t} \frac{1}{s_{12}s_{23}} \left\{ \frac{2}{\epsilon^2} \left[ \left( \frac{m_{t_1}^2}{-s_{12}} \right) - \left( \frac{m_{t_1}^2}{s_{23}} \right) \right] + \frac{1}{\epsilon^2} \left[ \left( \frac{m_{t_1}^2}{s_{12}} \right) - \left( \frac{m_{t_1}^2}{-s_{23}} \right) \right] - 2 \text{Li}_2 \left( 1 - \frac{(p_1 - p_2 + p_3 + p_4)^2}{s_{12}} \right) - 2 \text{Li}_2 \left( 1 - \frac{(p_1 - p_2 + p_3 + p_4)^2}{s_{23}} \right) - \ln^2 \left( \frac{-s_{12}}{-s_{23}} \right) - \frac{\pi^2}{6} \right\}. \tag{5.56}
\]

2. \(D_0(p_1, -p_3 - p_4, p_2; 0, 0, 0, 0)\), two opposite on-shell massless legs:

\[
D_0(p_1, -p_3 - p_4, p_2; 0, 0, 0, 0) = \frac{i}{16\pi^2 N_t} \frac{1}{s_{12}s_{23} - \eta^2\xi^2} \left\{ \frac{2}{\epsilon^2} \left[ \left( \frac{m_{t_1}^2}{-s_{12}} \right) - \left( \frac{m_{t_1}^2}{s_{23}} \right) \right] - \left( \frac{m_{t_1}^2}{-\eta^2} \right) - \left( \frac{m_{t_1}^2}{-\xi^2} \right) \right\} - 2 \text{Li}_2 \left( 1 - \frac{\eta^2}{s_{12}} \right) - 2 \text{Li}_2 \left( 1 - \frac{\eta^2}{s_{23}} \right) - 2 \text{Li}_2 \left( 1 - \frac{\xi^2}{s_{12}} \right) - 2 \text{Li}_2 \left( 1 - \frac{\xi^2}{s_{23}} \right) + 2 \text{Li}_2 \left( 1 - \frac{\eta^2\xi^2}{s_{12}s_{23}} \right) - \ln^2 \left( \frac{-s_{12}}{-s_{23}} \right) \right\}. \tag{5.57}
\]
II. One non-zero internal mass

1. \(D0(p_1, p_2, -p_4; 0, 0, 0, m_{t_1})\), two adjacent on-shell massless legs and one one-shell massive leg:

\[
D0(p_1, p_2, -p_4; 0, 0, 0, m_{t_1}) = \frac{i}{16\pi^2} \mathcal{N}_{t_1} \left\{ \frac{1}{s_{12}(s_{23} - m_{t_1}^2)} \right\} \\
\frac{3}{2\epsilon^2} + \frac{1}{\epsilon} \left[ 2 \ln \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{23}} \right) + \ln \left( \frac{m_{t_1}^2}{s_{12}} \right) - \ln \left( \frac{m_{t_1}^2}{(p_1 + p_2 - p_4)^2 - m_{t_1}^2} \right) \right] \\
+ 2 \ln \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{23}} \right) \ln \left( \frac{m_{t_1}^2}{s_{12}} \right) - \ln^2 \left( \frac{m_{t_1}^2}{(p_1 + p_2 - p_4)^2 - m_{t_1}^2} \right) \\
- 2 \text{Li}_2 \left( 1 + \frac{(p_1 + p_2 - p_4)^2 - m_{t_1}^2}{m_{t_1}^2 - s_{23}} \right) + \frac{\pi^2}{3} \right\}.
\]

2. \(D0(p_1, -p_1 + p_3 + p_4, -p_4; 0, 0, 0, m_{t_1})\), one on-shell massless leg and two one-shell massive legs:

\[
D0(p_1, -p_1 + p_3 + p_4, -p_4; 0, 0, 0, m_{t_1}) = \frac{i}{16\pi^2} \mathcal{N}_{t_1} \left\{ \frac{1}{s_{12}(s_{23} - m_{t_1}^2)} \right\} \\
\frac{1}{2\epsilon^2} + \frac{1}{\epsilon} \left[ \ln \left( \frac{m_{t_1}^2}{s_{12}} \right) + \ln \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{23}} \right) - \ln \left( \frac{m_{t_1}^2}{-\eta^2} \right) \right] \\
\ln^2 \left( \frac{m_{t_1}^2}{s_{12}} \right) - \ln^2 \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{23}} \right) + \frac{\pi^2}{2} \\
+ \ln \left( \frac{-\eta^2 + s_{12}}{m_{t_1}^2 - s_{23}} \right) \ln \left( \frac{m_{t_1}^2}{s_{12}} \right)^2 - 2 \ln \left( 1 - \frac{s_{12}}{-\eta^2} \right) \ln \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{23}} \right) \\
+ 2 \left[ \ln \left( \frac{-\eta^2}{m_{t_1}^2 - s_{23}} \right) \ln \left( 1 - \frac{-\eta^2(-s_{12} + m_{t_1}^2 - s_{23} + \eta^2)}{(m_{t_1}^2 - s_{23})(-s_{12})} \right) \right] \\
- \ln \left( \frac{-\eta^2}{m_{t_1}^2 - s_{23}} \right) \ln \left( \frac{-\eta^2 - m_{t_1}^2 + s_{23}}{-\eta^2} \right) + \ln \left( \frac{-\eta^2 + s_{12}}{m_{t_1}^2 - s_{23}} \right) \ln \left( 1 - \frac{m_{t_1}^2 - s_{23}}{-\eta^2 + s_{12}} \right) \\
- \text{Li}_2 \left( 1 - \frac{m_{t_1}^2 - s_{23}}{-\eta^2 + s_{12}} \right) - \text{Li}_2 \left( 1 - \frac{-\eta^2 + s_{12}}{m_{t_1}^2 - s_{23}} \right) - \text{Li}_2 \left( 1 - \frac{-\eta^2 - m_{t_1}^2 + s_{23}}{-s_{12}} \right) \right\}.
\]
\[ + \text{Li}_2\left( \frac{-\eta^2(-s_{12}+m_{t_i}^2-s_{23}+\eta^2)}{(m_{t_i}^2-s_{23})(-s_{12})} \right) - \text{Li}_2\left( \frac{m_{t_i}^2-s_{23}}{-\eta^2+s_{12}} \right) + \text{Li}_2\left( \frac{m_{t_i}^2-s_{23}}{-\eta^2} \right) \right) \right), \quad (5.59) \]

where we have defined the mass square of the second leg as \( \eta^2 = (-p_1 + p_3 + p_4)^2 \).

3. \( D0(p_1, -p_1 - p_2 + p_3 + p_4, p_2 - p_4; 0, 0, 0, m_{t_i}) \), one on-shell massless leg and one one-shell massive leg:

\[
D0(p_1, -p_1 - p_2 + p_3 + p_4, p_2 - p_4; 0, 0, 0, m_{t_i}) = \frac{i}{16\pi^2} N_{t_i} \frac{1}{s_{12}(s_{23} - m_{t_i}^2)} \left\{ \right.
\]

\[
\frac{1}{2\epsilon^2} + \frac{1}{\epsilon} \left[ \ln \left( \frac{m_{t_i}^2}{-s_{12}} \right) + \ln \left( \frac{m_{t_i}^2}{m_{t_i}^2 - s_{23}} \right) - \ln \left( \frac{m_{t_i}^2}{\eta^2} \right) \right]
\]

\[
+ \ln^2 \left( \frac{-s_{12}}{m_{t_i}^2} \right) + \ln^2 \left( \frac{m_{t_i}^2 - s_{23}}{m_{t_i}^2} \right) - \ln^2 \left( \frac{\eta^2}{m_{t_i}^2} \right) + \frac{4\pi^2}{3}
\]

\[
+ 2\ln \left( \frac{\eta^2}{m_{t_i}^2} \right) \ln \left( \frac{(-s_{12} + \eta^2)(m_{t_i}^2 - s_{23} + \eta^2)}{s_{12}(s_{23} - m_{t_i}^2)} \right) - 2\ln \left( \frac{m_{t_i}^2 - s_{23}}{m_{t_i}^2} \right) \ln \left( \frac{-m_{t_i}^2 - s_{23} + \eta^2}{s_{12}} \right)
\]

\[
+ 2\ln \left( \frac{-s_{12}}{m_{t_i}^2} \right) \ln \left( \frac{-s_{12} + \eta^2}{m_{t_i}^2 - s_{23}} \right) + \ln \left( \frac{m_{t_i}^2 - \zeta^2}{m_{t_i}^2} \right) \ln \left( -\frac{-m_{t_i}^2\eta^2 - (s_{23} - m_{t_i}^2)(\zeta^2 - m_{t_i}^2)}{-m_{t_i}^2\eta^2} \right)
\]

\[
+ 2\text{Li}_2 \left( 1 - \frac{(-s_{12} + \eta^2)(m_{t_i}^2 - s_{23} + \eta^2)}{s_{12}(s_{23} - m_{t_i}^2)} \right) - 2\text{Li}_2 \left( 1 - \frac{m_{t_i}^2 - s_{23} + \eta^2}{s_{12}} \right)
\]

\[
- 2\text{Li}_2 \left( 1 + \frac{-s_{12} + \eta^2}{m_{t_i}^2 - s_{23}} \right) - \text{Li}_2 \left( 1 - \frac{s_{23} - m_{t_i}^2}{\eta^2} \right)
\]

\[
- \text{Li}_2 \left( 1 - \frac{(m_{t_i}^2 - s_{23})\zeta^2}{-m_{t_i}^2\eta^2 - (s_{23} - m_{t_i}^2)(\zeta^2 - m_{t_i}^2)} \right) + \text{Li}_2 \left( 1 - \frac{\eta^2\zeta^2}{m_{t_i}^2\eta^2} + (s_{23} - m_{t_i}^2)(\zeta^2 - m_{t_i}^2) \right)
\]

\[
- \text{Li}_2 \left( 1 - \frac{(-\eta^2 + s_{12})\zeta^2}{m_{t_i}^2(-\eta^2 + s_{12}) - (s_{23} - m_{t_i}^2)(\zeta^2 - m_{t_i}^2)} \right) + \text{Li}_2 \left( 1 - \frac{m_{t_i}^2 - \zeta^2}{m_{t_i}^2} \right)
\]

\[
+ \text{Li}_2 \left( 1 + \frac{(-s_{12} + \eta^2)\zeta^2}{m_{t_i}^2(-\eta^2 + s_{12}) - (s_{23} - m_{t_i}^2)(\zeta^2 - m_{t_i}^2)} \right) \right\}, \quad (5.60)
\]

where we have defined \( \eta^2 = (-p_1 + p_3 + p_4)^2 \) and \( \zeta^2 = (p_2 - p_4)^2 \).

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III. Two non-zero internal masses

1. \( D0(p_1, -p_4, p_2; 0, 0, m_{t_1}, m_{t_1}) \), two opposite on-shell massless legs and one on-shell massive leg:

\[
D0(p_1, -p_4, p_2; 0, 0, m_{t_1}, m_{t_1}) = \frac{i}{16\pi^2} N_{t_1} \frac{1}{(s_{12} - m_{t_1}^2)(s_{23} - m_{t_1}^2)} \left\{ \right.
\]

\[
\frac{1}{2\epsilon^2} + \frac{1}{\epsilon} \left[ \ln \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{23}} \right) + \ln \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{12}} \right) - \ln \left( \frac{m_{t_1}^2}{(p_1 + p_2 - p_4)^2 - m_{t_1}^2} \right) \right]
\]

\[
+ \ln^2 \left( \frac{m_{t_1}^2 - s_{23}}{m_{t_1}^2} \right) + \ln^2 \left( \frac{m_{t_1}^2 - s_{12}}{m_{t_1}^2} \right) - \ln^2 \left( \frac{(p_1 + p_2 - p_4)^2 - m_{t_1}^2}{m_{t_1}^2} \right) + \frac{3}{2} \pi^2
\]

\[
+ 2 \ln \left( \frac{-s_{23} + (p_1 + p_2 - p_4)^2}{m_{t_1}^2 - s_{12}} \right) \ln \left( \frac{m_{t_1}^2 - s_{12}}{m_{t_1}^2 - s_{23} - (p_1 + p_2 - p_4)^2} \right)
\]

\[
+ 2 \ln \left( \frac{-s_{12} + (p_1 + p_2 - p_4)^2}{m_{t_1}^2 - s_{23}} \right) \ln \left( \frac{m_{t_1}^2 - s_{23}}{m_{t_1}^2 - s_{23} - (p_1 + p_2 - p_4)^2} \right)
\]

\[
- 2 \text{Li}_2 \left( \frac{m_{t_1}^2 - s_{23} - s_{12} + (p_1 + p_2 - p_4)^2}{m_{t_1}^2 - s_{12}} \right)
\]

\[
- 2 \text{Li}_2 \left( \frac{m_{t_1}^2 - s_{23} - s_{12} + (p_1 + p_2 - p_4)^2}{m_{t_1}^2 - s_{23}} \right)
\]

\[
- 2 \text{Li}_2 \left( \frac{(-s_{23} + (p_1 + p_2 - p_4)^2)(-s_{12} + (p_1 + p_2 - p_4)^2)}{(m_{t_1}^2 - s_{23})(m_{t_1}^2 - s_{12})} \right) \right\}.
\]

\[
(5.61)
\]

2. \( D0(p_1, -p_4, -p_1 + p_3 + p_4; 0, 0, m_{t_1}, m_{t_1}) \), one on-shell massless legs and two opposite on-shell massive legs:

\[
D0(p_1, -p_4, -p_1 + p_3 + p_4; 0, 0, m_{t_1}, m_{t_1}) = \frac{i}{16\pi^2} N_{t_1} \frac{1}{(s_{12} - m_{t_1}^2)(s_{23} - m_{t_1}^2)} \left\{ \right.
\]

\[
\frac{1}{\epsilon^2} + \frac{1}{\epsilon} \left[ \ln \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{12}} \right) + \ln \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{23}} \right) \right]
\]

\[
+ \ln^2 \left( \frac{m_{t_1}^2 - s_{12}}{m_{t_1}^2 - s_{23}} \right) - \frac{2\pi^2}{3} + 2 \text{Li}_2 \left( \frac{1}{z_+} \right) + 2 \text{Li}_2 \left( \frac{1}{z_-} \right) \right\}.
\]

\[
(5.62)
\]
with

\[ z_\pm = \frac{1}{2} \left( 1 \pm \sqrt{1 - \frac{4 \, m_{t_1}^2}{(-p_1 + p_3 + p_4)^2}} \right). \]

3. \( D0(p_2, -p_4, -p_1 - p_2 + p_3 + p_4; 0, 0, m_{t_1}, m_{t_1}) \), one on-shell massless leg and one on-shell massive leg:

\[
\begin{align*}
D0(p_2, -p_4, -p_1 - p_2 + p_3 + p_4; 0, 0, m_{t_1}, m_{t_1}) &= \frac{i}{16\pi^2} \frac{1}{N_{t_1}} \frac{1}{(s_{12} - m_{t_1}^2)(s_{23} - m_{t_1}^2)} \left\{ \frac{1}{2\epsilon^2} + \frac{1}{\epsilon} \left[ \ln \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{12}} \right) + \ln \left( \frac{m_{t_1}^2}{s_{23} - m_{t_1}^2} \right) - \ln \left( \frac{m_{t_1}^2}{s_{23} - m_{t_1}^2} \right) \right] \\
&+ \text{Re} \left[ -\frac{5}{6} \pi^2 \ln^2 \left( \frac{s_{23} - m_{t_1}^2}{m_{t_1}^2} \right) + \ln^2 \left( \frac{m_{t_1}^2 - s_{12}}{m_{t_1}^2} \right) - \ln^2 \left( \frac{m_{t_1}^2 - s_{12}}{m_{t_1}^2} \right) \right] + 2 \ln \left( \frac{s_{23} - m_{t_1}^2}{m_{t_1}^2} \right) \ln \left( \frac{m_{t_1}^2 - s_{12}}{s_{23} - m_{t_1}^2} \right) - \text{Li}_2 \left( \frac{-s_{12} + m_{t_1}^2 - s_{12}}{s_{23} - m_{t_1}^2} \right) \\
&- 2 \text{Li}_2 \left( \frac{(m_{t_1}^2 - s_{12}) (s_{23} - m_{t_1}^2 + s_{12})}{(s_{23} - m_{t_1}^2)(m_{t_1}^2 - s_{12})} \right) - I_0 \right\} ,
\end{align*}
\]

where \( \xi^2 = (p_1 - p_3)^2 \) is the mass square of the fourth leg,

\[
I_0 = \ln \left( \frac{m_{t_1}^2 - s_{12}}{m_{t_1}^2 - \xi^2} \right) \ln \left( \frac{\xi^2}{m_{t_1}^2} \right) + \left\{ \text{Li}_2 \left( \frac{1}{\lambda_+} \right) \\
+ \ln \left( \frac{m_{t_1}^2 - s_{12}}{m_{t_1}^2 - \xi^2} \right) \ln \left( -\frac{m_{t_1}^2 - \xi^2}{\lambda_+ (-s_{12} + \xi^2)} \right) - s_{12} + \xi^2 \\
- \text{Li}_2 \left( \frac{m_{t_1}^2 - s_{12}}{\lambda_+ (-s_{12} + \xi^2) + m_{t_1}^2 - \xi^2} \right) + \text{Li}_2 \left( \frac{m_{t_1}^2 - \xi^2}{\lambda_+ (-s_{12} + \xi^2) + m_{t_1}^2 - \xi^2} \right) \\
+ (\lambda_+ \leftrightarrow \lambda_-) \right\} ,
\]

(5.64)
\( \zeta^2 = (-p_1 - p_2 + p_3 + p_4)^2 \) is the mass square of the third leg and
\[
\lambda_\pm = \frac{1}{2} \left( 1 \pm \sqrt{1 - \frac{4m_{t_i}^2}{\zeta^2}} \right) . \tag{5.65}
\]

IV. Three non-zero internal masses

1. \( D0(-p_3, p_2, -p_2 + p_3 + p_4; 0, m_{t_1}, m_{t_1}, m_{t_1}) \), One on-shell massless leg and two adjacent on-shell massive legs:

\[
D0(-p_3, p_2, -p_2 + p_3 + p_4; 0, m_{t_1}, m_{t_1}, m_{t_1}) = \frac{i}{16\pi^2} \mathcal{N}_{t_1} \frac{1}{s_{23}(s_{12} - m_{t_1}^2)\beta} \left\{ \frac{1}{\epsilon} \ln \left( \frac{1 - \beta}{1 + \beta} \right) - \left[ (-2 \ln (\kappa_s) \ln \left( \frac{m_{t_1}^2}{m_{t_1}^2 - s_{12}} \right) + 2 \ln (\kappa_s) \ln (1 - \kappa_s^2) + \pi^2/2 + \text{Li}_2(\kappa_s \kappa_s) + \ln^2(\kappa_3) \right.ight. \\
\left. -2 \left( \text{Li}_2(\kappa_s^2 \kappa_3) + \ln (\kappa_s) \ln (1 - \kappa_s \kappa_3) + \ln (\kappa_3) \ln (1 - \kappa_s \kappa_3) \right) \left. -2 \left( \text{Li}_2(\kappa_s^2 / \kappa_3) + \ln (\kappa_s) \ln (1 - \kappa_s^2 / \kappa_3) - \ln (\kappa_3) \ln (1 - \kappa_s^2 / \kappa_3) \right) \right] \} \right] \right], \tag{5.66}
\]

where, using \( \zeta^2 = (-p_2 + p_3 + p_4)^2 \) the mass square of the third leg, we have defined:
\[ \beta = \sqrt{1 - \frac{4m_{t_1}^2}{s_{23}}}, \quad \beta_3 = \sqrt{1 - \frac{4m_{t_1}^2}{\zeta^2}}, \quad \kappa_s = \frac{1 - \beta}{1 + \beta} \quad \text{and} \quad \kappa_3 = \frac{1 - \beta_3}{1 + \beta_3}. \]

2. \( D0(-p_3, -p_1 - p_2 + p_3 + p_4, p_1 + p_2; 0, m_{t_1}, m_{t_1}, m_{t_1}) \), Two adjacent on-shell massive legs:

\[
D0(-p_3, -p_1 - p_2 + p_3 + p_4, p_1 + p_2; 0, m_{t_1}, m_{t_1}, m_{t_1}) = \frac{i}{16\pi^2} \mathcal{N}_{t_1} \frac{1}{(s_{12} - m_{t_1}^2)s_{23}\beta} \left\{ \frac{1}{\epsilon} \ln \left( \frac{1 - \beta}{1 + \beta} \right) + \kappa_0 \right\} , \tag{5.67}
\]

where we have defined:
\[ \beta = \sqrt{1 - \frac{4m_{t_1}^2}{s_{23}}}. \]

Defining \( \eta^2 = (-p_1 - p_2 + p_3 + p_4)^2 \), \( \zeta^2 = (p_1 + p_2)^2 \), \( \beta_2 = \sqrt{1 - \frac{4m_{t_1}^2}{\eta^2}}, \quad \beta_3 = \sqrt{1 - \frac{4m_{t_1}^2}{\zeta^2}}. \]
\[ \kappa_s = \frac{1 - \beta}{1 + \beta}, \quad \kappa_2 = \frac{1 - \beta_2}{1 + \beta_2}, \quad \text{and} \quad \kappa_3 = \frac{1 - \beta_3}{1 + \beta_3}, \]

we can express the finite piece \( \kappa_0 \) as follows:

\[
\kappa_0 = \frac{\kappa_s s_2 s_3}{m_t^2 (1 - \kappa_s^2)} \left\{ 2 \ln (\kappa_s) \ln (1 - \kappa_s^2) - 2 \ln (\kappa_s) \ln \left( \frac{m_t^2}{m_t^2 - s_{12}} \right) + \frac{\pi^2}{2} + \text{Li}_2(\kappa_s^2) + \ln^2(\kappa_2) + \ln^2(\kappa_3) + \text{Li}_2(\kappa_s^2/\kappa_2/\kappa_3) \right.

+ \ln (1 - \kappa_s/\kappa_2/\kappa_3) \ln (\kappa_s) + \ln (1 - \kappa_s/\kappa_2/\kappa_3) \ln (\kappa_2)

+ \ln (1 - \kappa_s/\kappa_2/\kappa_3) \ln (\kappa_3) + \ln (1 - \kappa_s/\kappa_2/\kappa_3) \ln (\kappa_3)

+ \ln (1 - \kappa_s/\kappa_2/\kappa_3) \ln (\kappa_3) + \ln (1 - \kappa_s/\kappa_3/\kappa_2) \ln (1/\kappa_3)

+ \ln (1 - \kappa_s/\kappa_3/\kappa_2) \ln (\kappa_s) + \ln (1 - \kappa_s/\kappa_3/\kappa_2) \ln (1/\kappa_2)

+ \ln (1 - \kappa_s/\kappa_3/\kappa_2) \ln (\kappa_3) + \ln (1 - \kappa_s/\kappa_2/\kappa_3) \ln (\kappa_3)

+ \ln (1 - \kappa_s/\kappa_2/\kappa_3) \ln (1/\kappa_2) + \ln (1 - \kappa_s/\kappa_2/\kappa_3) \ln (1/\kappa_3) \right\}.

Some of self-energy, vertex, box, and pentagon diagrams for The virtual corrections are plotted in Figs.(5-5)-(5-20).
Fig. 5.5: Virtual Corrections: Self diagrams for the $qq \rightarrow t\bar{t}h^0$. 
Fig. 5.6: Virtual Corrections: Vertex diagrams for the $qq \rightarrow tt h^0$. 
Fig. 5.7: Virtual Corrections (continued): Vertex diagrams for the $qq \rightarrow t\bar{t}h^0$. 
Fig. 5.8: Virtual Corrections: Boxes diagrams for the $qq \rightarrow t\bar{t}h^0$. 

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Fig. 5.9: Virtual Corrections (continued): Boxes diagrams for the $qq \rightarrow \bar{t}t h^0$. 

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Fig. 5.10: Virtual Corrections: Self diagrams for the $gg \rightarrow t\bar{t}h^0$. 
Fig. 5.11: Virtual Corrections: (continued) Self diagrams for the $gg \rightarrow t\bar{t}h^0$. 
Fig. 5.12: Virtual Corrections: (continued) Self diagrams for the \( gg \to \tilde{t} \tilde{h}^0 \).
Fig. 5.13: Virtual Corrections: (continued) Self diagrams for the $gg \rightarrow t\bar{t}h^0$. 
Fig. 5.14: Virtual Corrections: Vertex diagrams for the $gg \rightarrow t\bar{t}h^0$. 
Fig. 5.15: Virtual Corrections: (continued) Vertex diagrams for the $gg \to t\bar{t}h^0$. 
Fig. 5.16: Virtual Corrections: (continued) Vertex diagrams for the $gg \rightarrow t\bar{t}h^0$. 
Fig. 5.17: Virtual Corrections: (continued) Vertex diagrams for the $gg \to t\bar{t}h^0$. 
Fig. 5.18: Virtual Corrections: Boxes and Pentagon diagrams for the $gg \rightarrow t\bar{t}h^0$. 

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Fig. 5.19: Virtual Corrections: (continued) Boxes and Pentagon diagrams for the $gg \rightarrow t\bar{t}h^0$. 

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Fig. 5.20: Virtual Corrections: (continued) Boxes and Pentagon diagrams for the $gg \rightarrow t \bar{t} h^0$. 
5.3 Real Corrections

The corrections due to real parton emission give origin to IR singularities which cancel exactly the analogous singularities present in the $O(\alpha_s)$ virtual corrections. These singularities can be either of soft or collinear nature and can be conveniently isolated by slicing the phase space into different regions defined by suitable cutoffs, a method which goes under the general name of Phase Space Slicing (PSS). In our calculation we consider the real parton emission subprocesses listed below for a consistent and complete mass factorization:

\[
\begin{align*}
q(p_1) + \bar{q}(p_2) & \rightarrow \tilde{t}_1(p_3) + \tilde{t}_1(p_4) + h(p_5) + g(p_6), \\
g(p_1) + g(p_2) & \rightarrow \tilde{t}_1(p_3) + \tilde{t}_1(p_4) + h(p_5) + g(p_6), \\
(q, \bar{q})(p_1) + g(p_2) & \rightarrow \tilde{t}_1(p_3) + \tilde{t}_1(p_4) + h(p_5) + (q, \bar{q})(p_6).
\end{align*}
\]

(5.68)

In the next section we explain in detail how we have applied the PSS method to our case, using the two-cutoff implementation.

5.3.1 Phase Space Slicing method with two cutoffs

In the phase-space slicing approach (see Ref. [91] for a review) the soft and collinear regions are excluded from phase space by appropriate phase-space cuts. The general implementation of the PSS method using two cutoffs proceeds in two steps. First, by introducing an arbitrary small soft cutoff $\delta_s$ we separate the overall integration of the $2 \rightarrow 4$ phase space into two regions, according to whether the energy of the gluon is soft, i.e. $E_6 \leq \delta_s \sqrt{s}/2$, or hard, i.e. $E_6 > \delta_s \sqrt{s}/2$. The partonic real cross section of Eq. (4.2) can then be written as:

\[
\hat{\sigma}_{\text{real}} = \hat{\sigma}_{\text{soft}} + \hat{\sigma}_{\text{hard}},
\]

(5.69)

where $\hat{\sigma}_{\text{soft}}$ is obtained by integrating over the soft region of the gluon phase space, and contains all the IR soft divergences of $\hat{\sigma}_{\text{real}}$. To isolate the remaining collinear divergences from $\hat{\sigma}_{\text{hard}}$, we further split the integration over the hard gluon phase space according to whether the gluon is $\hat{\sigma}_{\text{hard/coll}}$ or is not $\hat{\sigma}_{\text{hard/non-coll}}$ emitted within an angle $\theta$ from the initial-state such that $(1 - \cos \theta) < \delta_c$, for an arbitrary small collinear cutoff $\delta_c$:

\[
\hat{\sigma}_{\text{hard}} = \hat{\sigma}_{\text{hard/coll}} + \hat{\sigma}_{\text{hard/non-coll}}.
\]

(5.70)
The hard non-collinear part of the real cross section, $\hat{\sigma}_{\text{hard/non-coll}}$, is finite and can be computed numerically, using standard Monte-Carlo techniques.

### 5.3.2 Soft gluon emission

For the real gluon emission subprocesses

$$
q(p_1) + \bar{q}(p_2) \rightarrow \tilde{t}_1(p_3) + \tilde{t}_1(p_4) + h(p_5) + g(p_6),
$$

$$
g(p_1) + g(p_2) \rightarrow \tilde{t}_1(p_3) + \tilde{t}_1(p_4) + h(p_5) + g(p_6)
$$

(5.71)

Denoting the gluon energy in the partonic centre-of-mass frame by $E_6$, the soft region of the phase space is defined by requiring that

$$
E_6 > \delta_s \sqrt{s}/2 = \Delta E
$$

(5.72)

for an arbitrary small value of the soft cutoff $\delta_s$. In the limit when the energy of the gluon becomes small, i.e. in the soft limit, gluon radiation is described by an eikonal current (see e.g. Ref. [92, 93]), i.e. it factorizes into the Born matrix element squared times an eikonal factor. Moreover, in the soft region the $2 \rightarrow 3 + g$ phase space also factorizes as:

$$
d(PS_4)(2 \rightarrow 3 + g) \xrightarrow{soft} d(PS_3)(2 \rightarrow 3) d(PS_g)_{soft}
$$

$$
= d(PS_3)(2 \rightarrow 3) \frac{d^{(d-1)}p_6}{(2\pi)^{(d-1)}2E_6} \theta(\delta_s \sqrt{s}/2 - E_6),
$$

(5.73)

where $d(PS_g)_{soft}$ denotes the integration over the phase space of the soft gluon.

### 5.3.3 Soft gluon emission for $q\bar{q}$ collision channel

The parton level soft cross section can be written as:

$$
\hat{\sigma}_{soft} = \hat{\sigma}_{LO} \otimes \frac{\alpha_s}{2\pi} \sum_{i,j=1}^{4} (T_i \cdot T_j) g_{ij}(p_i,p_j),
$$

(5.74)

where $T_i$ are the color operators [93, 94, 95], $g_{ij}$ are the soft integrals defined as:

$$
g_{ij}(p_i,p_j) = \frac{(2\pi\mu)^{2\epsilon}}{2\pi} \int_{E_6 \leq \Delta E} \frac{d^{D-1}p_6}{E_6} \left[ \frac{2(p_ip_j)}{(p_ip_6)(p_jp_6)} - \frac{p_i^2}{(p_ip_6)^2} - \frac{p_j^2}{(p_jp_6)^2} \right].
$$

(5.75)
The integrals are obtained as follows: $g_{12}$ is calculated easily; $g_{34}$ can be taken over from Ref. [96], where this integral is given for an infinitesimal gluon mass $m_g$ that translates into $1/\varepsilon$ via the substitution \( \ln(m^2_g) \rightarrow (4\pi\mu^2)^{\varepsilon} \Gamma(1 + \varepsilon)/\varepsilon \); the remaining $g_{ij}$ can be derived using the auxiliary integrals (C.20) and (C.25) of Ref. [97]. The results are

\[
g_{12}(p_1, p_2) = \left( \frac{4\pi\mu^2}{4\Delta E^2} \right)^{\varepsilon} \Gamma(1 + \varepsilon) \left\{ \frac{2}{\varepsilon^2} - \frac{2\pi^2}{3} \right\}
\]

\[
= 2\Delta^\text{IR}_2(\mu) - 2 \ln \left( \frac{4\Delta E^2}{m_{t_1}^2} \right) \Delta^\text{IR}_1(\mu) + \ln^2 \left( \frac{4\Delta E^2}{m_{t_1}^2} \right) - \frac{2\pi^2}{3},
\]

(5.76)

\[
g_{ij}(p_i, p_j) = \left( \frac{4\pi\mu^2}{4\Delta E^2} \right)^{\varepsilon} \Gamma(1 + \varepsilon) \left\{ \frac{1}{\varepsilon^2} - \frac{2}{\varepsilon} \ln \left( \frac{p_ip_j}{p_{i,0}m_{t_1}} \right) - \frac{\pi^2}{3} - \frac{1}{2} \ln^2 \left( \frac{1 + \beta_j}{1 - \beta_j} \right) \\
- 2Li_2 \left( 1 - \frac{p_{i,0}p_{j,0}(1 - \beta_j)}{p_{i,0}p_{j}} \right) - 2Li_2 \left( 1 - \frac{p_{i,0}p_{j,0}(1 + \beta_j)}{p_{i,0}p_{j}} \right) \\
+ \frac{1}{\varepsilon} - \frac{1}{\beta_j} \ln \left( \frac{1 - \beta_j}{1 + \beta_j} \right) \right\}
\]

\[
= \Delta^\text{IR}_2(\mu) - \ln \left( \frac{4\Delta E^2}{m_{t_1}^2} \right) \Delta^\text{IR}_1(\mu) + \frac{1}{2} \ln^2 \left( \frac{4\Delta E^2}{m_{t_1}^2} \right) - 2 \ln \left( \frac{p_ip_j}{p_{i,0}m_{t_1}} \right) \Delta^\text{IR}_1(\mu) \\
+ 2 \ln \left( \frac{4\Delta E^2}{m_{t_1}^2} \right) \ln \left( \frac{p_ip_j}{p_{i,0}m_{t_1}} \right) - \frac{\pi^2}{3} - \frac{1}{2} \ln^2 \left( \frac{1 + \beta_j}{1 - \beta_j} \right) \\
- 2Li_2 \left( 1 - \frac{p_{i,0}p_{j,0}(1 - \beta_j)}{p_{i,0}p_{j}} \right) - 2Li_2 \left( 1 - \frac{p_{i,0}p_{j,0}(1 + \beta_j)}{p_{i,0}p_{j}} \right) \\
+ \Delta^\text{IR}_1(\mu) - \ln \left( \frac{4\Delta E^2}{m_{t_1}^2} \right) - \frac{1}{\beta_j} \ln \left( \frac{1 - \beta_j}{1 + \beta_j} \right),
\]

for $i = 1, 2$, $j = 3, 4$, with $\beta_j = \sqrt{1 - \frac{m_{t_1}^2}{p_{j,0}^2}}$, 

(5.77)
\[
g_{34}(p_3, p_4) = \left( \frac{4\pi\mu^2}{4\Delta E^2} \right)^\varepsilon \Gamma(1 + \varepsilon) \frac{s_{34} - 2m_{t_1}^2}{s_{34}\beta_{s34}} \left\{ \frac{-2}{\varepsilon} \ln(\alpha) + \frac{1}{2} \ln^2 \left( \frac{u_0 - |u|}{u_0 + |u|} \right) + 2Li_2 \left( 1 - \frac{u_0 + |u|}{v} \right) + 2Li_2 \left( 1 - \frac{u_0 - |u|}{v} \right) \right\}_{u=\alpha p_3}^{u=p_4} + \left( \frac{4\pi\mu^2}{4\Delta E^2} \right)^\varepsilon \Gamma(1 + \varepsilon) \left\{ \frac{2}{\varepsilon} - \frac{1}{\beta_3} \ln \left( \frac{1 - \beta_3}{1 + \beta_3} \right) - \frac{1}{\beta_4} \ln \left( \frac{1 - \beta_4}{1 + \beta_4} \right) \right\}
\]
\[
\frac{s_{34} - 2m_{t_1}^2}{s_{34}\beta_{s34}} \left\{ -2 \ln(\alpha) \Delta_{1}^{\text{IR}}(\mu) + 2 \ln \left( \frac{4\Delta E^2}{m_{t_1}^2} \right) \ln(\alpha) + \left( \frac{1}{2} \ln^2 \left( \frac{u_0 - |u|}{u_0 + |u|} \right) + 2Li_2 \left( 1 - \frac{u_0 + |u|}{v} \right) + 2Li_2 \left( 1 - \frac{u_0 - |u|}{v} \right) \right\}_{u=\alpha p_3}^{u=p_4} + 2\Delta_{1}^{\text{IR}}(\mu) - 2 \ln \left( \frac{4\Delta E^2}{m_{t_1}^2} \right) - \frac{1}{\beta_3} \ln \left( \frac{1 - \beta_3}{1 + \beta_3} \right) - \frac{1}{\beta_4} \ln \left( \frac{1 - \beta_4}{1 + \beta_4} \right),
\]

with \( \alpha = \frac{1 + \beta_{s34}}{1 - \beta_{s34}} = \frac{1}{\rho}, \quad v = \frac{m_{t_1}^2(\alpha^2 - 1)}{2(\alpha p_{3,0} - p_{4,0})} \).

Where
\[
\Delta_{n}^{\text{IR}}(\mu) = \frac{\Gamma(1 + \varepsilon)}{\varepsilon^n} \left( \frac{4\pi\mu^2}{m_{t_1}^2} \right)^\varepsilon
\]

After integration, singular expressions of the form \( \varepsilon^{-i} \) \( (i = 1, 2) \) are generated. The double poles correspond to configurations where IR and collinear singularities coincide. When the differential soft-gluon distribution is added to the virtual corrections, the sum is IR-finite. This sum, however, is not free of divergences until the collinear singularities are removed by means of mass factorization.

Using the definitions of color operators, we get the expressions of \( \sigma_{\text{soft}} \) for \( q\bar{q} \) annihilation channel.

\[
\hat{\sigma}_{\text{soft}}^{\text{q}\bar{q}} = -\frac{\alpha_s}{2\pi} \left[ \frac{1}{6}(g_{12} + g_{34}) - \frac{7}{6}(g_{13} + g_{24}) - \frac{1}{3}(g_{14} + g_{23}) \right] \hat{\sigma}_{LO}^{\text{q}\bar{q}},
\]

5.3.4 Soft gluon emission for gg collision channel

The soft region of the phase space for the gluon emission process

\[
g^a(p_1) + g^b(p_2) \rightarrow \tilde{t}_1(p_3) + \tilde{t}_1(p_4) + h(p_5) + g^c(p_6)
\]
is defined by demanding that the energy of the emitted gluon \((k^0 = E_g)\) satisfies the condition

\[
E_g \leq \delta_s \frac{\sqrt{s}}{2}
\]

(5.82)

for an arbitrary small value of the soft cutoff \(\delta_s\). In the soft limit \((E_g \to 0)\), the amplitude for this process can be written as:

\[
\mathcal{A}_{soft}(gg \to \tilde{t}_1 \tilde{t}_1 h + g) =
\]

\[
t^a t^b t^c \left( p_3 \cdot \epsilon^* - p_1 \cdot \epsilon^* \right) \left( \mathcal{A}_q + \mathcal{A}_{\tilde{t}} + \mathcal{A}_s \right) + t^a t^b t^c \left( p_4 \cdot \epsilon^* - p_2 \cdot \epsilon^* \right) \left( \mathcal{A}_q + \mathcal{A}_{\tilde{t}} + \mathcal{A}_s \right) - t^a t^b t^c \left( p_4 \cdot \epsilon^* - p_1 \cdot \epsilon^* \right) \left( \mathcal{A}_q + \mathcal{A}_{\tilde{t}} + \mathcal{A}_s \right)
\]

(5.83)

where \(a, b,\) and \(c\) are the color indices of the external gluons, while \(\epsilon^*(k, \lambda)\) (for \(\lambda = 1, 2\)) is the polarization vector of the emitted soft gluon. Moreover, in the soft region the \(gg \to \tilde{t}_1 \tilde{t}_1 h + g\) phase space factorizes as:

\[
d(PS_1)(gg \to \tilde{t}_1 \tilde{t}_1 h + g) \xrightarrow{soft} d(PS_3)(gg \to \tilde{t}_1 \tilde{t}_1 h)d(PS_{g,soft})
\]

\[
= d(PS_3)(gg \to \tilde{t}_1 \tilde{t}_1 h) \frac{d^{(d-1)}k}{(2\pi)^{(d-1)}2E_g} \theta \left( \delta_s \frac{\sqrt{s}}{2} - E_g \right)
\]

(5.84)

where \(d(PS_{g,soft})\) denotes the the phase space measure of the soft gluon. Using the definitions of color operators, we get the expressions of \(\hat{\sigma}_{soft}\) for \(gg\) fusion channel.

\[
\hat{\sigma}_{soft}^{gg} = \frac{\alpha_s}{12\pi} \int d\Phi_3 \sum \left[ \left( \frac{256}{3} (9g_{12} + 9g_{13} + 9g_{24} - g_{34}) + 96(g_{12} - g_{14} - g_{23} + g_{34}) \right) |M_1^{gg}|^2 
\]

\[
+ \left( \frac{256}{3} (9g_{12} + 9g_{23} + 9g_{14} - g_{34}) + 96(g_{12} - g_{13} - g_{24} + g_{34}) \right) |M_2^{gg}|^2 
\]

\[
+ \left( -\frac{32}{3} (9g_{12} + 9g_{13} + 9g_{24} - g_{34}) + 96(g_{12} - g_{14} - g_{23} + g_{34}) \right) 2Re(M_1^{gg} \cdot M_2^{gg}) \right],
\]

(5.85)

where \(M_1^{gg}\) and \(M_2^{gg}\) have been expressed in Eqs.(4.10) and (4.11) respectively. The integrals which we have used in calculating the results in Eqs. (5.76)-(5.78) are obtained by
parameterizing the soft gluon $d$-momentum in the $q\bar{q}'$ (gg) rest frame Refs. [91, 98], as:

$$k = E_g(1, \ldots, \sin \theta_1 \sin \theta_2, \sin \theta_1 \cos \theta_2, \cos \theta_1) ,$$  

(5.86)
such that the phase space of the soft gluon in $d=4-2\epsilon$ dimensions can be written as:

$$d(PS_g)_{\text{soft}} = \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)(2\pi)^3} \int_0^{\delta_s \sqrt{\pi}/2} dE_g E_g^{1-2\epsilon} \times \int_0^{\pi} d\theta_1 \sin^{1-2\epsilon} \theta_1 \int_0^{\pi} d\theta_2 \sin^{-2\epsilon} \theta_2 .$$  

(5.87)

Then, all the integrals we need are of the form:

$$I_n^{(k,l)} = \int_0^{\pi} d\theta_1 \sin^{d-3} \theta_1 \int_0^{\pi} d\theta_2 \sin^{d-4} \theta_2 \times 
\frac{(a + b \cos \theta_1)^{-k}}{(A + B \cos \theta_1 + C \sin \theta_1 \cos \theta_2)^l} .$$  

(5.88)

In particular we need the following four cases. When $A^2 \neq B^2 + C^2$, and $b = -a$, we use (dropping terms of order $O((d-4)^2)$):

$$I_n^{(1,1)} = \frac{\pi}{a(A+B)} \left\{ \frac{2}{d-4} + \ln \left[ \frac{(A+B)^2}{A^2 - B^2 - C^2} \right] 
+ \frac{1}{2} (d-4) \ln^2 \left( \frac{A - \sqrt{B^2 + C^2}}{A + B} \right) 
- \frac{1}{2} \ln^2 \left( \frac{A + \sqrt{B^2 + C^2}}{A - \sqrt{B^2 + C^2}} \right) 
+ 2 \text{Li}_2 \left( \frac{-B + \sqrt{B^2 + C^2}}{A + B} \right) 
- 2 \text{Li}_2 \left( \frac{B - \sqrt{B^2 + C^2}}{A + B} \right) \right\} ,$$  

(5.89)
while when \( b \neq -a \) we use:

\[
I_n^{(0,1)} = \frac{\pi}{\sqrt{B^2 + C^2}} \left\{ \ln \left( \frac{A + \sqrt{B^2 + C^2}}{A - \sqrt{B^2 + C^2}} \right) - (d - 4) \left[ Li_2 \left( \frac{2\sqrt{B^2 + C^2}}{A + \sqrt{B^2 + C^2}} \right) \right] + \frac{1}{4} \ln^2 \left( \frac{A + \sqrt{B^2 + C^2}}{A - \sqrt{B^2 + C^2}} \right) \right\}, \tag{5.90}
\]

\[
I_n^{(0,2)} = \frac{2\pi}{A^2 - B^2 - C^2} \times \left[ 1 - \frac{1}{2} (d - 4) \frac{A}{\sqrt{B^2 + C^2}} \ln \left( \frac{A + \sqrt{B^2 + C^2}}{A - \sqrt{B^2 + C^2}} \right) \right]. \tag{5.91}
\]

Finally, when \( A^2 = B^2 + C^2 \), and \( b = -a \), we have:

\[
I_n^{(1,1)} = \frac{2\pi}{aA} \frac{1}{d - 4} \left( \frac{A + B}{2A} \right)^{d/2 - 3} \times \left[ 1 + \frac{1}{4} (d - 4)^2 Li_2 \left( \frac{A - B}{2A} \right) \right]. \tag{5.92}
\]

\[
\int d(P S_g)_{soft} \frac{(p_1 \cdot p_2)}{(p_1 \cdot p_6)(p_2 \cdot p_6)} = \frac{1}{(4\pi)^2} N_{t_1} 2 \left[ \frac{1}{\epsilon^2 - \frac{2}{\epsilon} \ln(\delta_s) - \frac{1}{\epsilon} \Lambda_s} - \frac{\pi^2}{3} + \frac{1}{2} \left( \Lambda_s^2 + 4\Lambda_s \ln(\delta_s) + 4 \ln^2(\delta_s) \right) \right],
\]

\[
\int d(P S_g)_{soft} \frac{(p_1 \cdot p_3)}{(p_1 \cdot p_6)(p_3 \cdot p_6)} = \frac{1}{(4\pi)^2} N_{t_1} \left[ \frac{1}{\epsilon^2 - \frac{2}{\epsilon} \Lambda_t - \frac{2}{\epsilon} \ln(\delta_s) - \frac{\pi^2}{3}} - \frac{1}{2} \Lambda_s^2 + 2\Lambda_t \Lambda_s + 2 \ln^2(\delta_s) + 4\Lambda_t \ln(\delta_s) + F(p_1, p_3) \right],
\]

\[
\int d(P S_g)_{soft} \frac{(p_3 \cdot p_4)}{(p_3 \cdot p_6)(p_4 \cdot p_6)} = \frac{1}{(4\pi)^2} N_{t_1} \left[ \frac{2(p_3 \cdot p_4) - 2m_{t_1}^2}{2(p_3 \cdot p_4)} \right] \left[ \left( -\frac{2}{\epsilon} + 2\Lambda_s + 4 \ln(\delta_s) \right) \frac{1}{\beta_{t_1 t_1}} \Lambda_{t_1 t_1} - \frac{1}{\beta_{t_1 t_1}} \Lambda_{t_1 t_1} + \frac{4}{\beta_{t_1 t_1}} \Li_2 \left( \frac{2\beta_{t_1 t_1}}{1 + \beta_{t_1 t_1}} \right) \right],
\]

\[
\int d(P S_g)_{soft} \frac{p_3^2}{(p_3 \cdot p_6)^2} = \frac{1}{(4\pi)^2} N_{t_1} \left[ -\frac{2}{\epsilon} + 2\Lambda_s + 4 \ln(\delta_s) - 2 \frac{1}{\beta_{t_1 t_1}} \Lambda_{t_1 t_1} \right], \tag{5.93}
\]

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We have introduced the notation $\Lambda_s = \ln(s/m_b^2)$ and $\Lambda_{ti,\Bar{t}i} = \ln(t_i/m_b^2)$, where the invariants have been defined in Eq. (5.45), while $\beta_{ti,\Bar{t}i}$ and $\Lambda_{ti,\Bar{t}i}$ are defined by:

\[
\beta_{ti,\Bar{t}i} = \sqrt{1 - \frac{4m_{ti}^2}{s_{ti,\Bar{t}i}}},
\]

\[
\Lambda_{ti,\Bar{t}i} = \ln\left(\frac{1 + \beta_{ti,\Bar{t}i}}{1 - \beta_{ti,\Bar{t}i}}\right),
\]  

(5.94)

Moreover we have denoted by $F(p_i, p_f)$ the function:

\[
F(p_i, p_f) = \ln^2\left(\frac{1 - \beta_f}{1 - \beta_f \cos \theta_{if}}\right) - \frac{1}{2} \ln^2\left(\frac{1 + \beta_f}{1 - \beta_f}\right)
+ 2\text{Li}_2\left(-\frac{\beta_f(1 - \cos \theta_{if})}{1 - \beta_f}\right) - 2\text{Li}_2\left(-\frac{\beta_f(1 + \cos \theta_{if})}{1 - \beta_f \cos \theta_{if}}\right),
\]

(5.95)

where $\theta_{if}$ is the angle between partons $i$ and $f$ in the center-of-mass frame of the initial state partons, and

\[
\beta_f = \sqrt{1 - \frac{m_{ti}^2}{(p^0_f)^2}}, \quad 1 - \beta_f \cos \theta_{if} = \frac{s_{if}}{p^0_f \sqrt{s}}.
\]

(5.96)

All the quantities in Eq. (5.95) can be expressed in terms of kinematical invariants, once we use $s_{if} = 2p_i \cdot p_f$ and:

\[
p^0_3 = \frac{s - s_{ti,\Bar{t}i}h + m_{ti}^2}{2\sqrt{s}} \quad \text{and} \quad p^0_{\Bar{t}i} = \frac{s - s_{ti,\Bar{t}i}h + m_{\Bar{t}i}^2}{2\sqrt{s}},
\]

(5.97)

with $s_{fh} = (p_f + p_h)^2$.

### 5.3.4.0.1 Collinear gluon emission from the initial state

The hard region of the gluon phase space is defined by requiring that the energy of the emitted gluon is above a given threshold. This is expressed by the condition that

\[
E_6 > \delta_s \sqrt{s}/2 = \Delta E,
\]

(5.98)

for an arbitrary small soft cutoff $\delta_s$, which automatically assures that $\hat{\sigma}_{\text{hard}}$ does not contain soft singularities. However, a hard gluon can still give origin to singularities when it is emitted at a small angle, i.e. collinear, to a massless incoming or outgoing parton. In order to isolate these divergences and compute them analytically, we further divide the hard region of the $q\Bar{q} \rightarrow t\Bar{t}h + g$ phase space into a hard/collinear and a hard/non-collinear region, by
introducing a second small collinear cutoff \( \delta_c \). The hard/non-collinear region is defined by the condition that both

\[
\frac{2p_1 \cdot p_6}{E_6 \sqrt{s}} > \delta_c \quad \text{and} \quad \frac{2p_2 \cdot p_6}{E_6 \sqrt{s}} > \delta_c
\] (5.99)

are verified. The contribution from the hard/non-collinear region, \( \tilde{\sigma}_{\text{hard/non-coll}} \), is finite and we compute it numerically by using standard Monte Carlo integration techniques.

In the hard/collinear region, one of the conditions in Eq. (5.99) is not satisfied and the hard gluon is emitted collinear to one of the incoming partons. In this region, the initial-state parton \( i \) \((i=q, \bar{q}, g)\) is considered to split into a hard parton \( i' \) and a collinear gluon \( g \), \( i \to i'g \), with \( p_{i'} = zp_i \) and \( p_6 = (1 - z)p_i \). The matrix element squared for \( q\bar{q}(gg) \to \tilde{t}_1 \tilde{t}_1 hg \) factorizes into the Born matrix element squared and the Altarelli-Parisi splitting function for \( i \to i'g \), i.e.:

\[
\sum |\mathcal{M}_{HC}(ij \to \tilde{t}_1 \tilde{t}_1 hg)|^2 \xrightarrow{\text{collinear}} 2 \sum_i \sum_j |\mathcal{M}_{LO}(i'j \to \tilde{t}_1 \tilde{t}_1 h)|^2 \frac{2P_{i'i} (z, \epsilon)}{z s_{i6}} , \quad (5.100)
\]

where

\[
s_{i6} = 2p_{i1} \cdot p_6,
\]

\[
P_{i'i} (z, \epsilon) = P_{ii'} (z) + \epsilon P'_{ii'} (z),
\]

\[
P_{gg} (z) = 2N \left[ \frac{z}{1 - z} + \frac{1 - z}{z} + z(1 - z) \right], \quad P'_{gg} (z) = 0,
\]

\[
P_{qq} (z) = C_F \left( \frac{1 + z^2}{1 - z} \right), \quad P'_{qq} (z) = -C_F (1 - z). \quad (5.101)
\]

Using the approximation \( p_i - p_6 \approx zp_i \((i = 1, 2)\)\), the element in the four body collinear phase space region can be written as[99] is the unregulated Altarelli-Parisi splitting function for \( q \to q + g \) at lowest order, including terms of \( \mathcal{O}(\epsilon) \), and \( C_F = (N^2 - 1)/2N \). Moreover, in the collinear limit, the \( q\bar{q} \to t\bar{t}h + g \) phase space also factorizes as:

\[
d(PS_4)(ij \to \tilde{t}_1 \tilde{t}_1 h + g) \xrightarrow{\text{collinear}} d(PS_3)(i'j \to \tilde{t}_1 \tilde{t}_1 h) \frac{z d^{(d-1)} p_6}{(2\pi)^{(d-1)} 2E_6} \theta \left( E_6 - \delta_s \frac{\sqrt{s}}{2} \right) \theta(\cos \theta_{i6} - (1 - \delta_c)) \]

\[
\overset{d=4-2\epsilon}{\mathcal{O}(1-\epsilon) \frac{(4\pi)^{\epsilon}}{\Gamma(1-2\epsilon)} \frac{z dz ds_{i6} [(1 - z)s_{i6}]^{-\epsilon} \theta \left( \frac{1 - z}{z} s' \delta_c - s_{i6} \right)}{16\pi^2} ,
\]

where the integration range for \( s_{i6} \) in the collinear region is given in terms of the collinear cutoff, and we have defined \( s' = 2p_{i'} \cdot p_j \). The integral over the collinear gluon degrees
of freedom can then be performed separately, and this allows us to explicitly extract the collinear singularities of \( \hat{\sigma}_{\text{hard}} \). \( \hat{\sigma}_{\text{hard/coll}} \) turns out to be of the form \([91, 100]\): 

\[
\hat{\sigma}_{\text{hard/coll}} = \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{m_i^2} \right)^\epsilon \right] \left( -\frac{1}{\epsilon} \right) \hat{\sigma}_{\text{LO}}(i'j \to \tilde{t}_1\tilde{t}_1h) + (i \leftrightarrow j) \]  

(5.102)

The upper limit on the \( z \) integration ensures the exclusion of the soft gluon region. As usual, these initial-state collinear divergences are absorbed into the parton distribution functions. In order to factorize the collinear singularity into the parton distribution function, we introduce a scale dependent parton distribution function using the \( \overline{\text{MS}} \) convention:

\[
\mathcal{F}_p^p(x, \mu_f) = \mathcal{F}_q^p(x) \left[ 1 - \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{\mu_f^2} \right)^\epsilon \left( \frac{1}{\epsilon} \right) A_1^{ec}(q \to qg) \right] + \left( -\frac{1}{\epsilon} \right) \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{\mu_f^2} \right)^\epsilon \left( \frac{1}{\epsilon} \right) A_1^{ec}(g \to gg) \left[ \int_x^{1-\delta_s} \frac{dz}{z} P_{qq}(z, \epsilon) \mathcal{F}_q^p(x/z) \right], 

(5.103)

\[
\mathcal{F}_g^p(x, \mu_f) = \mathcal{F}_g^p(x) \left[ 1 - \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{\mu_f^2} \right)^\epsilon \left( \frac{1}{\epsilon} \right) A_1^{ec}(g \to gg) \right] + \left( -\frac{1}{\epsilon} \right) \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu^2}{\mu_f^2} \right)^\epsilon \left( \frac{1}{\epsilon} \right) A_1^{ec}(g \to gg) \left[ \int_x^{1-\delta_s} \frac{dz}{z} P_{gg}(z, \epsilon) \mathcal{F}_g^p(x/z) \right]. 

(5.104)

where

\[
A_1^{ec}(q \to qg) = C_F(2\ln\delta_s + 3/2), \quad C_F = 4/3, \\
A_1^{ec}(g \to gg) = 2N\ln\delta_s + (11N - 2n_{lf})/6. 

(5.105)

By using above expressions, the NLO QCD correction parts of the total cross sections contributed by \( q\bar{q} \) annihilation and \( gg \) fusion subprocesses in the initial state collinear phase space region are obtained as
\[
\sigma_{HC}^{gg} = \int \hat{\sigma}_{LO}^{gg} \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_r^2}{s} \right)^\epsilon \right] \\
\left\{ \tilde{F}_q^p(x_1,\mu_f)F_q^p(x_2,\mu_f) + F_q^p(x_1,\mu_f)\tilde{F}_q^p(x_2,\mu_f) \right. \\
+ \sum_{\alpha=u,\bar{u},d,\bar{d}} \left[ A_1^{sc}(\alpha \rightarrow \alpha g) + A_0^{sc}(\alpha \rightarrow \alpha g) \right] F_q^p(x_1,\mu_f)F^p_q(x_2,\mu_f) + (1 \leftrightarrow 2) \right\} \, dx_1dx_2,
\]

\[
\sigma_{HC}^{gg} = \int \hat{\sigma}_{LO}^{gg} \left[ \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left( \frac{4\pi\mu_r^2}{s} \right)^\epsilon \right] \frac{1}{\epsilon} \left\{ \tilde{F}_g^p(x_1,\mu_f)F_g^p(x_2,\mu_f) + F_g^p(x_1,\mu_f)\tilde{F}_g^p(x_2,\mu_f) \right. \\
+ \left. \left[ A_1^{sc}(g \rightarrow gg) + A_0^{sc}(g \rightarrow gg) \right] F_g^p(x_1,\mu_f)F^p_g(x_2,\mu_f) + (1 \leftrightarrow 2) \right\} \, dx_1dx_2,
\]

where

\[
A_0^{sc} = A_1^{sc} \ln \left( \frac{s}{\mu_r^2} \right),
\]

\[
\tilde{F}_q^p(x,\mu_f) = \int_x^{1-\delta_s} \frac{dy}{y} F_q^p(x/y,\mu_f)\tilde{P}_{\alpha\alpha}(y), \quad (\alpha = u,\bar{u},d,\bar{d},g),
\]

with

\[
\tilde{P}_{\alpha\alpha}(y) = P_{\alpha\alpha}(y) \ln \left( \frac{\delta_c (1-y)^2}{2y} \frac{s}{\mu_r^2} \right) - P'_{\alpha\alpha}(y), \quad (\alpha = u,\bar{u},d,\bar{d},g).
\]

2. Hard Light-quark Emission Subprocesses \((q,\bar{q})g \to \tilde{t}_1\tilde{t}_2h + (q,\bar{q})\)

The method in the calculation of the hard light-quark emission subprocesses \((q,\bar{q})g \to \tilde{t}_1\tilde{t}_2h + (q,\bar{q})\) is similar to that for hard gluon emission subprocesses. In the collinear region, the initial state parton \(i(i = u, d, \bar{u}, \bar{d}, g)\) is considered to split into a hard parton \(i'\) and a collinear light-quark, \(i \to i'q\), with \(p_{i'} = zp_i\) and \(p_6 = (1 - z)p_i\). Let the hard light-quark be emitted collinear to one of the incoming partons, the collinear region is then defined as:

\[
\frac{2p_1 \cdot p_6}{E_6\sqrt{s}} < \delta_c \quad \text{or} \quad \frac{2p_2 \cdot p_6}{E_6\sqrt{s}} < \delta_c.
\]
The collinear singularity of $\hat{\sigma}^{qq}_{\text{real}}$ can be written as:

$$
\hat{\sigma}^{qq}_{\text{HC}} = \left[ \frac{\alpha_s \Gamma(1 - \epsilon)}{2\pi \Gamma(1 - 2\epsilon)} \left( \frac{4\pi \mu_r^2}{s} \right)^\epsilon \right] \left( \frac{-1}{\epsilon} \right) \delta^{-\epsilon} \left\{ \int_0^{1 - \delta_s} dz \left[ \frac{(1 - z)^2}{2z} \right]^{-\epsilon} \times P_{gq}(z, \epsilon)\hat{\sigma}^{qq}_{\text{LO}}(gg \rightarrow \tilde{t}_1 \tilde{t}_1 h) + P_{gq}(z, \epsilon)\hat{\sigma}^{qq}_{\text{LO}}(qg \rightarrow \tilde{t}_1 \tilde{t}_1 h) \right\},
$$

with

$$
P_{ii'}(z, \epsilon) = P_{ii}(z) + \epsilon P'_{ii}(z),
$$

$$
P_{qq}(z) = \frac{1}{2} \left[ z^2 + (1 - z)^2 \right], \quad P'_{qq}(z) = -(1 - z),
$$

$$
P_{gq}(z) = \frac{N^2 - 1}{2N} \left( 1 + (1 - z)^2 \right), \quad P'_{gq}(z) = -\frac{N^2 - 1}{2N} z.
$$

Using the $\overline{MS}$ scheme, the scale dependent distribution function can be written as:

$$
\mathcal{F}_{i}^p(x, \mu_f) = \mathcal{F}_{i}^p(x) + \left( -\frac{1}{\epsilon} \right) \left[ \frac{\alpha_s \Gamma(1 - \epsilon)}{2\pi \Gamma(1 - 2\epsilon)} \left( \frac{4\pi \mu_r^2}{\mu_f^2} \right)^\epsilon \right] \int_x^1 \frac{dz}{z} P_{ii'}(z)\mathcal{F}_{i'}^p(x/z),
$$

And we can get the expression for the initial state collinear contribution at $O(\alpha_s)^3$ order:

$$
\sigma^q_{\text{HC}} = \frac{\alpha_s}{2\pi} \sum_{i=q,\bar{q}} \int dx_1 dx_2 \left\{ \int_x^1 \frac{dz}{z} \mathcal{F}_{i}^p(\frac{x_1}{z}, \mu)\mathcal{F}_{i}^p(x_2, \mu) \times \right.
$$

$$
\hat{\sigma}^{qq}_{\text{LO}}(x_1, x_2, \mu) \left[ P_{gq}(z) \ln \left( \frac{s}{\mu^2} \frac{(1 - z)^2}{z} \delta_c \right) - P'_{gq}(z) \right]
$$

$$
+ \int_x^1 \frac{dz}{z} \mathcal{F}_{g}^p(\frac{x_1}{z}, \mu)\mathcal{F}_{g}^p(x_2, \mu) \times
$$

$$
\hat{\sigma}^{qq}_{\text{LO}}(x_1, x_2, \mu) \left[ P_{gi}(z) \ln \left( \frac{s}{\mu^2} \frac{(1 - z)^2}{z} \delta_c \right) - P'_{gi}(z) \right] + (1 \leftrightarrow 2) \right\}.
$$

### 5.3.5 Total NLO Cross Section

The final result for the NLO QCD corrected cross section part of $qq$ annihilation subprocesses can be written as:

$$
\sigma^{qq}_{\text{NLO}} = \int dx_1 dx_2 \mathcal{F}_{i}^p(x_1, \mu)\mathcal{F}_{i}^p(x_2, \mu) \left[ \hat{\sigma}^{qq}_{\text{LO}}(x_1, x_2, \mu) + \hat{\sigma}^{qq}_{\text{virtual}}(x_1, x_2, \mu) + \hat{\sigma}^{qq}_{\text{soft}}(x_1, x_2, \mu) \right]
$$

$$
+ (1 \leftrightarrow 2) + \sigma^{qq}_{\text{HC}} + \int dx_1 dx_2 \left[ \mathcal{F}_{g}^p(x_1, \mu)\mathcal{F}_{g}^p(x_2, \mu) \hat{\sigma}^{qq}_{\text{PC}}(x_1, x_2, \mu) + (1 \leftrightarrow 2) \right].
$$
And the NLO QCD corrected cross section part for $gg$ fusion subprocess has the expression as:

$$
\sigma_{NLO}^{gg} = \frac{1}{2} \int dx_1 dx_2 \mathcal{F}_g^p(x_2, \mu) \mathcal{F}_g^p(x_2, \mu) \left[ \hat{\sigma}_{LO}^{gg}(x_1, x_2, \mu) + \hat{\sigma}_{virtual}^{gg}(x_1, x_2, \mu) + \hat{\sigma}_{soft}^{gg}(x_1, x_2, \mu) \right] + (1 \leftrightarrow 2)
$$

+ \frac{1}{2} \int dx_1 dx_2 \left[ \mathcal{F}_g^p(x_1, \mu) \mathcal{F}_g^p(x_2, \mu) \hat{\sigma}_{HC}^{gg}(x_1, x_2, \mu) + (1 \leftrightarrow 2) \right].

(5.116)

The cross section of $(q, \bar{q})g \rightarrow \tilde{t}_1 \bar{t}_1 h + (q, \bar{q})$ ($q = u, d$) can be written as:

$$
\sigma_{NLO}^{qq} = \sigma_{HC}^{gg} + \sum_{i=q, \bar{q}} \int dx_1 dx_2 \left[ \mathcal{F}_i^p(x_1, \mu) \mathcal{F}_g^p(x_2, \mu) \hat{\sigma}_{HC}^{gg}(x_1, x_2, \mu) + (1 \leftrightarrow 2) \right].
$$

(5.117)

with the hard-non-collinear partonic cross section given by

$$
\hat{\sigma}_{HC}^{ij} = \int_{HC} \sum_{M} |M(ij \rightarrow \tilde{t}_1 \bar{t}_1 h g(q, \bar{q}))|^2 d\Phi_4.
$$

(5.118)

where $d\Phi_4$ denotes the four-particle phase space element.

Finally, the NLO QCD corrected total cross sections for $pp \rightarrow \tilde{t}_1 \bar{t}_1 h$ can be obtain by using the formula:

$$
\sigma_{NLO} = \sigma_{NLO}^{qq} + \sigma_{NLO}^{gg} + \sigma_{NLO}^{qg}.
$$

(5.119)
6.1 LO- total cross sections

In the following we discuss in detail our results for the LO total cross section for \( pp \rightarrow \tilde{t}_1 \tilde{t}_1 h \). Our numerical results are found using CTEQ6L parton distribution functions \([101]\). The matrix elements squared for the tree level processes have been checked with Madgraph. The results have been obtained by two completely independent calculations, based on a combination of FORM and on the Mathematica. The numerical results have been obtained with two independent Fortran codes. The physical masses of the produced squark mass eigenstates and mixing angles are calculated using the recently updated computer program SUSPECT \([102]\). As a numerical demonstration, in this work we choose the minimal supergravity (mSUGRA) point, SPS1a, as benchmark for our numerical study \([103]\). SPS1a is a typical mSUGRA point with an intermediate value of \( \tan \beta = 10 \), \( \mu > 0 \). It has a model line attached to it, which is specified by \( m_0 = -A_0 = 100 \, \text{GeV} \) and \( m_{1/2} = 250 \, \text{GeV} \). We take the SM parameters as: \( \alpha_{ew}(M_Z)^{-1} = 127.918 \), \( m_W = 80.423 \, \text{GeV} \), \( m_Z = 91.18 \, \text{GeV} \), \( m_t = 175 \, \text{GeV} \), \( m_b = 4.62 \, \text{GeV} \). The numerical analyses of the hadronic cross sections have been performed for the CERN LHC with pp center-of-mass of \( \sqrt{s} = 14 \, \text{TeV} \). The hadronic cross sections are obtained by convoluting the partonic cross sections with the parton distribution functions of the initial states hadrons as specified in Eq. (4.35).

Fig. 6.1 shows the dependence of LO cross section for \( \tilde{t}_1 \tilde{t}_1 h \) on the arbitrary renormalization/factorization scale \( \mu \). The renormalization and factorization scales are set to a common scale which is then varied around the central value \( \mu_0 = m_{\tilde{t}_1} + \frac{M_h}{2} \). Fig. 6.2 shows the cross sections at the leading order as a function of the renormalization/factorization scale \( \mu \) for \( M_h = 100 \, \text{GeV} \) and \( 120 \, \text{GeV} \). For the Higgs mass between 95 and 130 GeV, the cross section varies between about 0.3651 fb and 0.2476 fb, as shown in table (6.1), if the central value \( \mu \rightarrow \mu_0 \) is chosen for the renormalization and factorization scales. For \( \mu \rightarrow 0.5\mu_0 \), the cross
section increases and varies between about 0.490 fb and 0.336 fb. But for $\mu \rightarrow 4\mu_0$, the cross section decreases and varies between about 0.2109 fb and 0.1434 fb. The variation show that the LO prediction for the total cross section is plagued by uncertainties and, therefore, cannot provide a reliable prediction. This underline the need for NLO QCD.

![Graph showing variation of cross section with $M_h$](image)

**Fig. 6.1:** Total cross section for the $pp \rightarrow t\bar{t}h^0$ at the LHC in LO with the renormalization/factorization scales set to $\mu_0 = m_{t_1} + \frac{M_h}{2}$, $0.5\mu_0$ and $4\mu_0$.

![Graph showing variation of cross section with $\mu/\mu_0$](image)

**Fig. 6.2:** Variation of the LO cross sections with the renormalization/factorization scales for the $pp \rightarrow t\bar{t}h^0$ at LHC.
\[
M_h (\text{GeV}) & \sigma_{\text{LO}} (\text{fb}) \\
\mu = 0.5\mu_0 & \mu = \mu_0 & \mu = 4\mu_0 \\
95 & 0.490 & 0.3651 & 0.2109 \\
100 & 0.470 & 0.3499 & 0.2024 \\
105 & 0.448 & 0.3311 & 0.1919 \\
110 & 0.424 & 0.3137 & 0.1817 \\
115 & 0.401 & 0.2963 & 0.1713 \\
120 & 0.379 & 0.2799 & 0.1624 \\
125 & 0.357 & 0.2637 & 0.1529 \\
130 & 0.336 & 0.2476 & 0.1434 \\
\]

Table 6.1: Values $\sigma_{\text{LO}}$ for different renormalization/factorization scales $\mu$.

6.2 Total NLO SUSY-QCD corrected cross sections

In our numerical calculation, we adopt the CTEQ6M[104] parton distribution functions and the 2-loop evolution of $\alpha_s(\mu)$ to evaluate the hadronic NLO SUSY-QCD corrected cross sections with $\alpha_s^{\text{NLO}}(M_Z) = 0.118$. In Fig. 6.3 and Fig. 6.4 we depict the curves for total cross sections at leading-order $\sigma_{\text{LO}}$ and $\sigma_{\text{NLO}}$ including NLO corrections for the processes. They show that the $\sigma_{\text{NLO}}$ in the MSSM is nearly independent on the normalization/factorization scale $\mu/\mu_0$ than $\sigma_{\text{LO}}$ both at different masses of MSSM Higgs. It is clear that the NLO corrections significantly reduce the renormalization/factorization scale dependence and stabilize the theoretical predictions for the cross section at LHC. From Figs.(6.5-6.8) we depict the LO and total NLO QCD corrected cross sections, $\sigma_{\text{LO}}$ and $\sigma_{\text{NLO}}$, in the MSSM as the functions of $M_h$ at the LHC. We can see that the cross sections $\sigma_{\text{NLO}}$ and $\sigma_{\text{LO}}$ decrease as $M_h$ varies in the range from 95GeV to 130GeV at the LHC. From these figures it is appeared that the K factor, $K = \sigma_{\text{NLO}}/\sigma_{\text{LO}}$ is nearly independent of $M_h$ in the relevant Higgs mass range. At $\mu_0$ we obtain $K \sim 1.21$, increasing to $\sim 1.52$ at $2\mu_0$, and increase to $\sim 1.63$ at $4\mu_0$, also, the K factor shows a significant $\mu$ dependence. The K factor becomes lower than one for small values of $\mu$ relative to $\mu_0 = (m_{\tilde{t}_1} + M_h/2)$ and larger than one for higher scales. Fig. 6.9 demonstrates that the main contributions are coming from subprocess $gg \rightarrow \tilde{t}_1\tilde{t}_1h$, which receives positive NLO corrections in the domain of the central renormalization/factorization scale $\mu_0$ at the LHC. For the hard collinear quark emission (qg), the NLO correction is small, essentially vanishing at $\mu = 2\mu_0$ then becomes negative for larger values of $\mu$. However, there is no reason to expect the correction to vanish near the appropriate factorization scale; it is
enough that it is small, indicating a convergent perturbation series.

Fig. 6.3: The total cross sections $\sigma_{LO}$ and $\sigma_{NLO}$ for the processes $pp \to \tilde{t}_1\tilde{t}_1h$ as the functions of the renormalization/factorization scale $Q$ with $M_h = 95 \text{ GeV}$

Fig. 6.4: The total cross sections $\sigma_{LO}$ and $\sigma_{NLO}$ for the processes $pp \to \tilde{t}_1\tilde{t}_1h$ as the functions of the renormalization/factorization scale $Q$ with $M_h = 120 \text{ GeV}$
Fig. 6.5: Total cross section for $pp \rightarrow \tilde{t}_1 \tilde{t}_1 h$ at the LHC in LO and NLO approximation, with the renormalization and factorization scales set to $0.5\mu_0 = 0.5(m_{\tilde{t}_1} + M_h/2)$.

Fig. 6.6: Total cross section for $pp \rightarrow \tilde{t}_1 \tilde{t}_1 h$ at the LHC in LO and NLO approximation, with the renormalization and factorization scales set to $\mu_0 = (m_{\tilde{t}_1} + M_h/2)$. 

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Fig. 6.7: Total cross section for $pp \rightarrow \tilde{t}_1 \tilde{\bar{t}}_1 h$ at the LHC in LO and NLO approximation, with the renormalization and factorization scales set to $2\mu_0 = 2(m_{\tilde{t}_1} + M_h/2)$.

Fig. 6.8: Total cross section for $pp \rightarrow \tilde{t}_1 \tilde{\bar{t}}_1 h$ at the LHC in LO and NLO approximation, with the renormalization and factorization scales set to $4\mu_0 = 4(m_{\tilde{t}_1} + M_h/2)$. 
Fig. 6.9: Total cross section for \( pp \rightarrow \tilde{t}_1 \bar{t}_1 h \) at the LHC in LO and NLO approximation, with the renormalization and factorization scales set to \( 4\mu_0 = 4(m_{\tilde{t}_1} + M_h/2) \).
CHAPTER 7

Conclusions

We have studied neutral Higgs boson production in association with lower mass scalar top quark at the LHC, in the context of the SUGRA inspired MSSM. Our interest in such reaction comes from the fact that they constitute a production mechanism of Higgs particles and also because they carry a strong dependence on the five inputs of the SUSY model, so that they can possibly be used to constrain the latter.

In this thesis we calculated the NLO SUSY-QCD corrections to the processes $pp \rightarrow \tilde{t}_1 \tilde{t}_1 h$ in the MSSM at the the LHC. We investigated the contributions of the NLO QCD corrections to the total cross sections and found the NLO QCD corrections significantly modify the corresponding LO cross sections. We analyzed the dependence of the NLO QCD corrected cross sections and the corresponding relative corrections on the renormalization/factorization scale $\mu$ and MSSM Higgs-boson mass $M_h$ respectively. Our numerical results show that the theoretical NLO SUSY-QCD corrections in the MSSM reduce the dependence of the cross section on the factorization and normalization scales. The NLO SUSY-QCD corrected total cross sections at the LHC are nearly independent of these scales, and the relative correction is obviously related to $M_h$ the LHC. the K factor shows a significant $\mu$ dependence where becomes lower than one for small values of $\mu$ relative to $\mu_0 = (m_{\tilde{t}_1} + M_h/2)$ and larger than one for higher scales.
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