PROCEEDINGS OF THE IASTED ENERGY SYMPOSIA

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EFFECT OF Dispersion ON THE FLOW
BEHAVIOR OF THREE INMISCIBLE FLUIDS
IN POROUS MEDIA

by
M.H. Sayyouh*
Petroleum Engineering Department
Faculty of Engineering
Cairo University

ABSTRACT

This paper presents an idealized model of a bundle of capillary tubes to
detect the effect of dispersion on the flow behavior of three immiscible
fluids (oil, water and gas). An effort has been made to establish a set of
relationships between dispersion and both static phenomenon (capillary pres-
sure) and flow phenomena (relative permeability) that may prove useful, from
a practical point of view, in the interpretation of the fluid flow mechanism
within the pore space. In addition, the study provides an accurate mean to
calculate relative permeabilities of the three-phase system.

INTRODUCTION

Among factors that play an important role in the movement of oil, water and
gas through the porous media are surface and capillary effects (1,2). Capil-
larv effects include phenomena due to the presence of curved surface bound-
aries at the interface between phases. Such boundaries will adjust their
curvatures to changes in capillary pressure and interfacial tension.

Since the porous medium may contain all three possible phases, and mutual
dispersion of the oil, water and gas may occur as they move through the
the porous medium, an infinite number of menisci arise at the boundary between
the three phases and at the curved surface boundaries of the two-phase inter-
faces. Therefore, the role of capillary effects in the processes of move-
ment of oil, water and gas through a porous medium is very great and the more
so, the greater the curvature of the interfaces. Under actual conditions
surface phenomena and capillary effects go together with flow phenomena and
mutually affect one another. It is believed that dispersion and coalescence
are of the major factors which depend upon both hydrodynamic and physico-
chemical conditions affecting the mechanism of immiscible flow in porous
medium (3).

This paper presents a theoretical study of the effect of dispersion on the
mechanism of movement of oil, water and gas in a porous medium. For simpli-
city an idealized model consisting of a bundle of capillary tubes is used.

MODEL ASSUMPTIONS

1. The capillary tubes are of the same radii and length.
2. Dispersed phase is equally distributed in the dispersed media.
3. Fluids are immiscible and no adsorption on the surface of the tube.
4. Flow is laminar and isothermal.
5. Dispersed phase consists of gas and dispersed media is water and oil.
6. Hydrodynamic gradients are the same through the length of the tubes.

* Now on leave
RELATIONSHIP BETWEEN DISPERSION AND CAPILLARY PRESSURE

Consider a bubble of gas (as a dispersed phase) is bounded on one side by oil and on the other by water (4), as shown in Figure 1. The situation occurs in many instances due to the existence of three immiscible phases. The capillary pressure is defined as (see Figure 2)

\[ P_c = \frac{2 \sigma_1 \cos \theta_1 - \sigma_2 \cos \theta_2}{r} \]  

(1)

Where \( n \) is the number of drops of the dispersed phase or the number of the capillary tubes.

If the total surface area of the dispersed phase is \( S_p \), its volume is \( V_p \), and the total surface area of the dispersed media is \( S_m \), its volume \( V_m \), the degree of dispersion is given by

\[ K_d = \frac{S_p}{V_p} + \frac{S_m}{V_m} \]  

(2)

With some simple mathematics it can be shown that

\[ K_d = \left( \frac{r_d}{r} \right) \frac{L}{s_p} s_m \]  

(3)

Where:

\[ X_0 = \left[ \frac{2 \sin \theta_1}{(1 + \sin \theta_1) \sin \theta_2 / (1 + \sin \theta_2)} \right] / (\sigma_1 \cos \theta_1 - \sigma_2 \cos \theta_2) \]

\[ Y_0 = \left[ (1 + \sin \theta_1)^2 \cos \theta_1 + (1 + \sin \theta_2)^2 \cos \theta_2 \right] / (\sigma_1 \cos \theta_1 - \sigma_2 \cos \theta_2) \]

Equation 3 shows that capillary pressure-saturation relationship is in a direct proportional with dispersion.

EFFECT OF DISPERSION ON THE THREE PHASE RELATIVE PERMEABILITIES

To overcome the effect of capillary pressure, a high hydrodynamic pressure must be applied. According to Darcy the velocity of the dispersed phase is given by

\[ v_p = \frac{1}{\mu_p} \left( \frac{d^2}{L} \right) \]  

(4)

Where \( k_p \) is the effective permeability of the dispersed phase.

Poisson's formula states that:

\[ v_p = \frac{1}{\mu_p} \left( \frac{d^2}{L} \right) \]  

(5)

Using \( r = \sqrt{8k/\mu} \) and substituting the value of \( p_c \) in Equation 3, then we have:

\[ k_{rg} = a_g \left( \frac{r_d^4}{r^4} \right) \frac{L}{s_g s_1} / (rX_0 + rY_0 (2s_2 - 1)) / \mu_p \]  

(6)

Similarly:

\[ k_{go} = a_g \left( \frac{r_d^4}{r^4} \right) \frac{L}{s_g s_2} / (rX_0 + rY_0 (2s_1 - 1)) / \mu_p \]  

(7)

\[ k_{rw} = a_w \left( \frac{r_d^4}{r^4} \right) \frac{L}{s_w s_1} / (rX_0 + rY_0 (2s_2 - 1)) / \mu_p \]  

(8)

Where:

\[ X_0 = \left[ \frac{2 \sin \theta_1}{(1 + \sin \theta_1) \sin \theta_2 / (1 + \sin \theta_2)} \right] / (\cos \theta_1 - \cos \theta_2) \]

\[ Y_0 = \left[ (1 + \sin \theta_1)^2 \cos \theta_1 + (1 + \sin \theta_2)^2 \cos \theta_2 \right] / (\cos \theta_1 - \cos \theta_2) / (2s_2 - 1) \]

Equations 6, 7, and 8 represent the three phase relative permeabilities. Table 1 shows hypothetical data that was used to illustrate the role of dispersion on capillary pressure and relative permeabilities.

Figures 3, 4 and 5 show the relative permeability data for gas, water and oil in a three phase system. The data are plotted on a triangular diagram to
define the saturation condition of the porous media. The relative permeability are plotted as lines of constant percentage relative permeability. Amux et al (5) explained the dependency of the oil or water relative permeability on the saturations of the other phases in terms of wettability and interfacial tension. It is seen, for example, from Figure 5 that by changing water saturation the flow characteristics of the oil are changed.

For three phase flow the regions in which single phase, two-phase, and three-phase fluids will occur are indicated in Figure 6. The important feature of this figure is that the region of the three phase flow highly increased with a slight increase in dispersion. This region is illustrated in Figure 6 by the "hatched" area. From these data it is evident that dispersion is an important factor in affecting the flow behavior of three immiscible phases.

CONCLUSIONS

It was established that a slight increase in dispersion caused an increase in capillary pressure and markedly decrease in relative permeabilities. The region of three phase flow on a triangular diagram highly increased with a slight decrease in dispersion. This work can provide an accurate mean to calculate relative permeabilities of the three phase system from the available data.

REFERENCES


Table 1. Hypothetical Data

<table>
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<th>Property</th>
<th>Value</th>
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<tr>
<td>Interfacial tension between oil and gas (γ)</td>
<td>35 dynes/cm</td>
</tr>
<tr>
<td>Interfacial tension between water and gas (γ')</td>
<td>25 dynes/cm</td>
</tr>
<tr>
<td>Contact angle between oil and gas (θ_o)</td>
<td>40°</td>
</tr>
<tr>
<td>Contact angle between water and gas (θ_w)</td>
<td>56°</td>
</tr>
<tr>
<td>Length of capillary tube (L)</td>
<td>50 cm</td>
</tr>
<tr>
<td>Radius of capillary tube (r)</td>
<td>0.33 \times 10^{-3} cm</td>
</tr>
<tr>
<td>Absolute permeability (k)</td>
<td>0.5 darcy</td>
</tr>
<tr>
<td>Porosity (φ)</td>
<td>35 %</td>
</tr>
<tr>
<td>External pressure (ap)</td>
<td>7.09 \times 10^4 dyne/cm^2</td>
</tr>
</tbody>
</table>

Figure 1. Bubbles of gas are bounded by water and oil

Figure 2. A bubble of gas is bounded by water and oil (after Calhoun6)
Figure 3. $k_{rg}$, relative permeability to gas as a function of saturation.

Figure 4. $k_{rw}$, relative permeability to water as a function of saturation.

Figure 5. $k_{ro}$, relative permeability to oil as a function of saturation.

Figure 6. Effect of dispersion on the three phase relative permeabilities.