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HEAT AND MASS TRANSFER AND PHYSICAL GASDYNAMICS

Unsteady Magnetohydrodynamic Flow with Heat and Mass Transfer between Two Parallel Walls under Variable Magnetic Field

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Abstract—This paper presents a semi-analytical solution using homotopy perturbation method of magnetohydrodynamic unsteady flow of fluids between two inclined parallel permeable walls with the consideration of heat and mass transfer. The flow is subjected to variable magnetic field normal to the walls. The solution for the governing nonlinear partial differential equations coupled with equations of motion, concentration, and energy equation including the viscous and Joule dissipations is adopted. The effect of Prandtl and Eckart numbers, Reynolds magnetic number, Schmidt and Grashof numbers, and modified Grashof number, and magnetic field strength on velocity, temperature, concentration, and induced magnetic field are discussed. Results are represented graphically for the considered problem.

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INTRODUCTION

Applied mathematical and physical problems in different fields of engineering and science are described by complex systems of nonlinear partial differential equations. Perturbation method is one of the well-known analytical methods to solve nonlinear partial differential equations, which is based on using a small parameter called perturbation quantity [1]. But most nonlinear problems do not involve such perturbation quantity at all. Therefore, many new efficient non-perturbation techniques have been developed to eliminate the shortcomings arising in the small parameter assumption [2], such as a domain decomposition method, tanh method, differential transform method, variational iteration method, and homotopy analysis method [3–12].

Non-perturbation methods cannot guarantee the convergence of the solution series, but the homotopy analysis method provides certain rules for the construction of the solution to control and adjust the convergence rate and the convergence region of the solution series [1]. The homotopy perturbation method (HPM) is a coupling of the traditional perturbation method and homotopy technique for solving various linear, nonlinear initial and boundary value problems [13]. Recently, it has been implemented by many researchers to find an analytical solution of nonlinear partial differential equations. In [14], HPM was used to solve the Couette flow problem of an incompressible conducting elastico-viscous for Maxwell model fluid in a parallel flat channel packed withporous material. In [15], the analytical solution by the use of HPM for fully developed flow in a parallel plates channel under the action of Lorentz force was presented.

The study of the flow with heat and mass transfer has received considerable attention by researchers due to its wide variety of engineering and scientific applications, such as electronic cooling and solar collectors [16, 17]. Salama [18] studied the flowof an elastic-viscous, incompressible, and electrically conducting fluid with heat and mass transfer between two infinite parallel walls, one of them moving with a uniform velocity. The unsteady Couette flow with heat transfer of a dusty viscous incompressible electrically conducting fluid through a porous medium with the consideration of both Hall current and ion slip was presented in [19, 20]. Authors of [21] analyzed the flow of heat transfer and entropy generation for non-Newtonian fluid with power-law and asymmetric convective cooling between two parallel plates. The study of MHD flows becomes very important in recent years because of its possible applications in many branches of science and technology. There are comprehensive investigations about this topic in [22–27].

The aim of this paper is to introduce a semi-analytical solution using HPM for fully developed unsteady flow of a fluid between two inclined parallel walls. Considering heat and mass transfer under the action of a transverse variable magnetic field.

MATHEMATICAL FORMULATION

In this study, we solve the coupled nonlinear equations that govern the unsteady, incompressible fluid flow between two inclined parallel plates with consideration of the effects of heat and mass transfer and variable magnetic field. The two plates are positioned at $y = 0$ and $y = d$. A constant pressure gradient is imposed in the axial x direction. The variable magnetic field $\mathbf{H} = \langle 0, H_y, 0\rangle$ is applied on the plates to produce an induced magnetic field h and an induced electric field E (which is not in the scope of this study). The temperature and the species concentration at the lower walls are T_0 and C_0 and at the upper wall are T_1 and C_1, respectively. All the considered functions depend on the distancebetween the plates y and the time t only, except the pressure gradient P.

The electro-magnetic quantities satisfy Maxwell's equations [27]

$$\nabla \mathbf{H} = 0, \tag{1}$$

$$\nabla \times \mathbf{H} = \mathbf{J}, \tag{2}$$

$$\nabla \times \mathbf{E} = -\mu_e \frac{\partial \mathbf{H}}{\partial t}, \tag{3}$$

$$\nabla \mathbf{E} = 0, \tag{4}$$

where $\mathbf{J}$ is the electric current density and μ_e is the magnetic permeability. It can be easily seen from the above Eqs. (1)–(4) that the induced magnetic field has the components:

$$\mathbf{h} = \langle h_x, H_0, h_z\rangle.$$

The electric current density $\mathbf{J}$ will have non-vanishing components in x and z directions and is related to the induced magnetic field and electric field, which is negligible in this study based on Ohm's law:

$$\mathbf{J} = \langle J_x, 0, J_z\rangle,$$

$$\mathbf{J} = \sigma_f \left(\mathbf{V} \times \mathbf{H} - \xi(\mathbf{J} \times \mathbf{H})\right),$$

where σ_f is the conductivity of the nanofluid and ξ is the Hall factor.

The governing equations for continuity, momentum, energy, and concentration of MHD unsteady, incompressible fluid flow are the following:

$$\nabla \cdot \mathbf{V} = 0,$$

$$\rho_f \left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V}\right) = -\nabla \mathbf{P} + \nabla \cdot \boldsymbol{\tau} + \mu_e(\mathbf{J} \times \mathbf{H}) + (\rho\beta_T)_f \mathbf{g}(T - T_0) + (\rho\beta_C)_f \mathbf{g}(C - C_0),$$

$$(\rho C_p)_f \left(\frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla)T\right) = k_f \nabla^2 T + \frac{\mathbf{JJ}}{\sigma_f},$$

$$\frac{\partial C}{\partial t} + (\mathbf{V} \cdot \nabla)C = D\nabla^2 C.$$

Here, the subscript f denotes fluid, ρ, g, β_T, and β_C denote the density, gravitational acceleration, the thermal expansion coefficient, and the concentration expansion coefficient, respectively, T is the temperature of the fluid, k_f is the thermal conductivity of the fluid, C is the mass concentration, and D is the diffusion coefficient.

From the continuity equation, one can easily deduce that the velocity of the flow will have the following components:

$$\mathbf{V} = \langle u, v_0, w\rangle.$$

Under the usual assumptions, the governing partial differential equations of the conservation of mass, momentum, energy, and concentration for the boundary layer in the presence of magnetic field can be expressed as:

$$\frac{\partial u}{\partial y} = 0, \tag{5}$$

$$\rho_f \left(\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial y} + \mu_f \frac{\partial^2 u}{\partial y^2} + (\rho\beta_T)_f g \sin\alpha(T - T_0) + (\rho\beta_C)_f g \sin\alpha(C - C_0) - \frac{\sigma_f H_0^2}{1 + m^2\left(1 + \frac{H}{H_0}\right)}(u + mwH), \tag{6}$$

$$\rho_f \left(\frac{\partial w}{\partial t} + v_0 \frac{\partial w}{\partial y}\right) = \mu_f \frac{\partial^2 w}{\partial y^2} + \frac{\sigma_f H_0^2}{1 + m^2(1 + \frac{H}{H_0})}(w - muH), \tag{7}$$

$$\frac{\partial h}{\partial t} + v_0 \frac{\partial h}{\partial y} = \frac{1}{\mu_e \sigma_f} \frac{\partial^2 h}{\partial y^2} + H_0 \frac{\partial u}{\partial y}, \tag{8}$$

$$(\rho C_P)_f \left(\frac{\partial T}{\partial t} + v_0 \frac{\partial T}{\partial y}\right) = k_f \frac{\partial^2 T}{\partial y^2} + \mu_f \left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right) + \frac{\sigma_f H_0^2}{1 + m^2\left(1 + \frac{H}{H_0}\right)}(u^2 + w^2), \tag{9}$$

$$\frac{\partial C}{\partial t} + v_0 \frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}, \tag{10}$$

where $m = \sigma_f \xi H_0$ is the Hall current parameter.

The corresponding boundary conditions are:

$$y = 0 : u = 0,\quad w = 0,\quad h = 0,\quad T = T_0,\quad C = C_0;$$
$$y = d : u = 0,\quad w = 0,\quad h = H_0,\quad T = T_1,\quad C = C_1,$$

where μ is the viscosity of the fluid.

Below we introduce non-dimensional variables to be used:

$$(\hat{x},\hat{y},\hat{z}) = \frac{(x,y,z)}{h},\quad \hat{t} = \frac{\mu_f}{\rho_f h^2}t,\quad \hat{u} = \frac{\rho_f h}{\mu_f}u,$$
$$\hat{w} = \frac{\rho_f h}{\mu_f}w,\quad \hat{h} = \frac{h}{H_0},\quad \hat{P} = \frac{\rho_f h^2}{\mu_f^2}P,$$
$$\theta = \frac{T - T_0}{T_1 - T_0},\quad \Gamma = \frac{C - C_0}{C_1 - C_0}.$$

The governing equations (5)–(10) reduce to the following non-dimensional form after dropping the cap:

$$\frac{\partial u}{\partial t} + s\frac{\partial u}{\partial y} = -G + \frac{\partial^2 u}{\partial y^2} - \frac{\mathrm{Ha}^2}{1 + m^2(1 + H)}(u + mwH) + \mathrm{Gr}\sin\alpha\theta + \mathrm{Gc}\sin\alpha\Gamma, \quad (11)$$

$$\frac{\partial w}{\partial t} + s\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} + \frac{\mathrm{Ha}^2}{1 + m^2(1 + H)}(w - muH), \quad (12)$$

$$\frac{\partial h}{\partial t} + s\frac{\partial h}{\partial y} = \frac{1}{\mathrm{Re}_m}\frac{\partial^2 h}{\partial y^2} + \frac{\partial u}{\partial y}, \quad (13)$$

$$\frac{\partial \theta}{\partial t} + s\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} + \mathrm{Ec}\left(\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2\right) + \frac{\mathrm{Ha}^2\mathrm{Ec}}{1 + m^2(1 + H)}H^2(u^2 + w^2), \quad (14)$$

$$\frac{\partial \Gamma}{\partial t} + s\frac{\partial \Gamma}{\partial y} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 \Gamma}{\partial y^2}. \quad (15)$$

Here, $G = \frac{\partial P}{\partial y}$ is the pressure gradient which is assumed to be constant, $\mathrm{Gr} = g\nu\beta_T(T_1 - T_0)/v_0^3$ is the Grashof number, ν is the kinematic fluid viscosity, $\mathrm{Gc} = g\nu\beta_C(C_1 - C_0)/v_0^3$ is the modified Grashof number, $\mathrm{Ha}^2 = \frac{\mu_f H_0^2\sigma_f}{\rho_f^2 U_0^2}$ is the Hartmann number squared, $\mathrm{Pr} = \nu c_{P_f}/k_f$ is the Prandtl number, $\mathrm{Ec} = U_0^2/[C_{Pf}(T_1 - T_0)]$ is the Eckart number, $\mathrm{Sc} = \nu/D$ is the Schmidt number, and $\mathrm{Re}_m = \nu/d$ is the magnetic Reynolds number.

The boundary conditions are reduced with the same scales:

$$y = 0: u = 0,\quad w = 0,\quad h = 0,\quad \theta = 0,\quad \Gamma = 0;$$
$$y = d: u = 0,\quad w = 0,\quad h = 1,\quad \theta = 1,\quad \Gamma = 1.$$

BASIC IDEA OF HPM

HPM was proposed in [28]. The method, which is a combination of homotopy in topology and classic perturbation techniques, provides a convenient way to obtain analytical or approximate solutions for a wide variety of problems arising in different fields. To describe this method we first assume a general partial differential equation in the following form:

$$\mathrm{A}(y(r,t)) - f(r,t) = 0. \quad (16)$$

Here, A is a differential operator, $y(r,t)$ is an unknown function, r and t denote spatial and temporal independent variables, respectively, and $f(r,t)$ is a known analytic function. Generally speaking, A can be divided into two parts, L and N:

$$\mathrm{A} = \mathrm{L} + \mathrm{N},$$

where L is a simple part, which is easy to handle, and N contains the remaining parts of A. Using homotopy technique one can construct a homotopy $\varphi(r,t;q)$ satisfying:

$$\mathrm{H}(\varphi(r,t;q)) = (1 - q)\mathrm{L}(\varphi(r,t;q)) - \mathrm{L}(v_0(r,t)) + q\mathrm{A}(\varphi(r,t;q)) - f(r,t) = 0, \quad (17)$$

where $q, \in [0,1]$ is an embedding parameter and $v_0(r,t)$ is an initial guess for equation (16), which satisfies initial/boundary condition(s). Equation (17) is called homotopy equation.

Clearly we have:

$$q = 0;\ \mathrm{H}(\varphi(r,t;0),0) = \mathrm{L}(\varphi(r,t;q) - \mathrm{L}(v_0(r,t) = 0; \quad (18)$$

$$q = 1;\ \mathrm{H}(\varphi(r,t;1),1) = \mathrm{A}(\varphi(r,t;q) - f(r,t) = 0. \quad (19)$$

The latter is actually equation (17) with solution $y(r,t)$. Equation (18) has $v_0(r,t)$ as one of its solutions and in the case where L is assumed to be linear, $v_0(r,t)$ is the only solution. So we have:

$$\varphi(r,t;1) = y(r,t),\quad \varphi(r,t;0) = v_0(r,t).$$

The changing process of q from zero to unity is just that of $\varphi(r,t;q)$ from $v_0(r,t)$ to $y(r,t)$ and is called deformation. If the embedding parameter $q(0 \le q \le 1)$ is considered as small, applying the classic perturbation technique, we can naturally assume that the solution of Eqs. (18) and (19) can be given as a power series in q, i.e,

$$\varphi(r,t;q)) = u_0(r,t) + u_1(r,t)q + u_2(r,t)q^2 + \cdots. \quad (20)$$

Using equation (20) for $q = 1$, one has

$$y(r,t)) = u_0(r,t) + u_1(r,t) + u_2(r,t) + \cdots. \quad (21)$$

Equation (21) is the approximate solution of (16). In most cases series (21) is a convergent one, which leads to the exact solution of (16). One can take the closed form or truncate the series for obtaining approximate solutions.

SOLUTION OF THE PROBLEM

According to equation (17), we first introduce the initial guess approximation and linear operators for each equation as:

$$u_0 = w_0 = h_0 = y(1-y)\left(1-e^{-\alpha t}\right),$$
$$\theta_0 = \Gamma_0 = y^2\left(1-e^{-\alpha t}\right),\quad L(u) = \frac{\partial^2 u}{\partial y^2},$$
$$L(w) = \frac{\partial^2 w}{\partial y^2},\ L(h) = \frac{\partial^2 h}{\partial y^2},\ L(\theta) = \frac{\partial^2 \theta}{\partial y^2},\ L(\Gamma) = \frac{\partial^2 \Gamma}{\partial y^2}.$$

Applying HPM to governing equations (11)–(15) of the problem, we get the following homotopy equations:

$$\begin{aligned} H(u;q) &= q(-L(u)) + (1-q)\left(\frac{\partial u}{\partial t} + s\frac{\partial u}{\partial y} + G\right. \\ &+ \frac{\mathrm{Ha}^2}{1+m^2(1+H)}(u + mwH) \\ &\left. - \mathrm{Gr}\sin\alpha\theta - \mathrm{Gc}\sin\alpha\Gamma\right) = 0, \end{aligned} \tag{22}$$

$$\begin{aligned} H(w;q) &= q(-L(w)) + (1-q) \\ &\times\left(\frac{\partial w}{\partial t} + s\frac{\partial w}{\partial y} + \frac{\partial P}{\partial y} + \frac{\mathrm{Ha}^2}{1+m^2(1+H)}(w - muH)\right) = 0, \end{aligned} \tag{23}$$

$$\begin{aligned} H(h;q) &= q\left(-\frac{1}{\mathrm{Re}_m}L(h)\right) \\ &+ (1-q)\left(\frac{\partial h}{\partial t} + s\frac{\partial h}{\partial y} - \frac{\partial u}{\partial y}\right) = 0, \end{aligned} \tag{24}$$

$$\begin{aligned} H(\theta;q) &= q\left(-\frac{1}{\mathrm{Pr}}L(\theta)\right) + (1-q) \\ &\times\left(\frac{\partial \theta}{\partial t} + s\frac{\partial \theta}{\partial y} - \mathrm{Ec}\left[\left(\frac{\partial u}{\partial y}\right)^2 - \left(\frac{\partial w}{\partial y}\right)^2\right]\right. \\ &\left. + \frac{\mathrm{Ha}^2\mathrm{Ec}}{1+m^2(1+H)}H^2\left(u^2 + w^2\right)\right) = 0, \end{aligned} \tag{25}$$

$$H(\Gamma;q) = q\left(-\frac{1}{\mathrm{Sc}}L(\Gamma)\right) + (1-q)\left(\frac{\partial \Gamma}{\partial t} + s\frac{\partial \Gamma}{\partial y}\right) = 0. \tag{26}$$

The solution of the equations (22)–(26), according to equation (21) will be assumed in the following forms:

$$u(y,t) = \sum_{n=0}^{\infty} u_n(t)y^n = u_1(t)y + u_2(t)y^2 + u_3(t)y^3 + u_4(t)y^4 + \ldots,$$

$$w(y,t) = \sum_{n=0}^{\infty} w_n(t)y^n = w_1(t)y + w_2(t)y^2 + w_3(t)y^3 + w_4(t)y^4 + \ldots,$$

$$h(y,t) = \sum_{n=0}^{\infty} h_n(t)y^n = h_1(t)y + h_2(t)y^2 + h_3(t)y^3 + h_4(t)y^4 + \ldots,$$

$$\theta(y,t) = \sum_{n=0}^{\infty} \theta_n(t)y^n = \theta_1(t)y + \theta_2(t)y^2 + \theta_3(t)y^3 + \theta_4(t)y^4 + \ldots,$$

$$\Gamma(y,t) = \sum_{n=0}^{\infty} \Gamma_n(t)y^n = \Gamma_1(t)y + \Gamma_2(t)y^2 + \Gamma_3(t)y^3 + \Gamma_4(t)y^4 + \ldots.$$

RESULTS AND DISCUSSION

In this section, the results for unsteady MHD flow between two infinite inclined parallel plates with considering the heat and mass transfer and variable magnetic field are presented graphically and discussed for various values of different parameters. The numerical calculations were performed by taking values of some constant parameters as, $G = 5$, $\mathrm{Ha} = 1$, $m = 1$.

Figures 1–3 illustrate the influence of changing the applied magnetic field on velocity components u and w and on θ. The fluid velocity components decrease with H increasing (Figs. 1 and 2). This is because of the fact that the applied magnetic field is considered as some sort of resistance that opposes the flow of the fluid. Also, it is obvious that the velocity distributions have parabolic profiles and the peaks of the profiles indicate the minimum at the central region. In addition, Fig. 3 shows that θ profile is also reduced substantially with H increasing, which indicates the cooling regime with stronger magnetic fields.

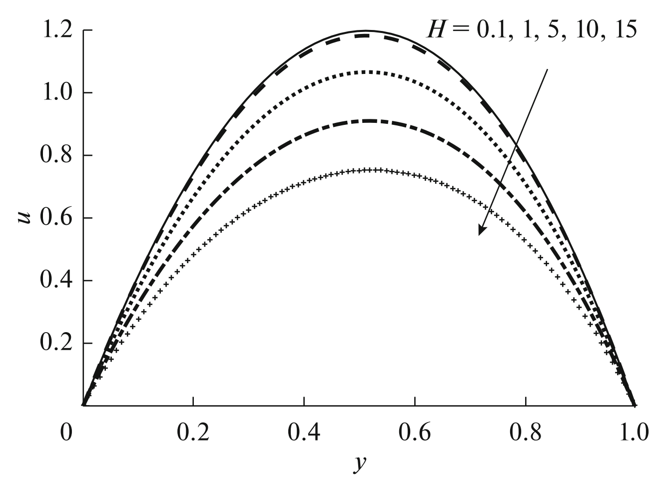

Fig. 1. The influence of applied magnetic field H on velocity component u at $G = 5$, $\mathrm{Gc} = 1$, $\mathrm{Gr} = 1$, $\alpha = \pi/2$, $\mathrm{Pr} = 0.7$, $\mathrm{Sc} = 0.3$, $\mathrm{Ec} = 0.2$.

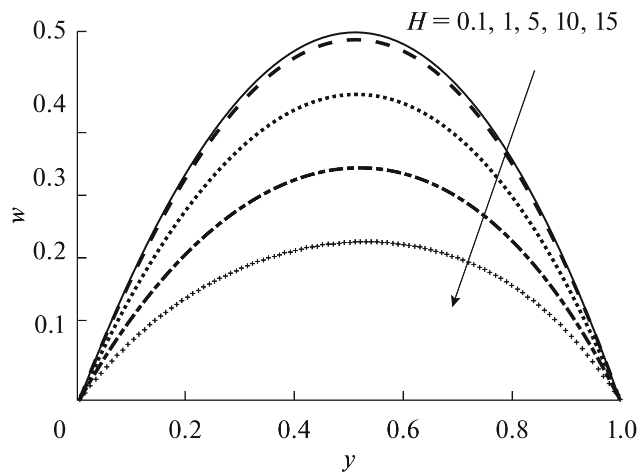

Fig. 2. The influence of applied magnetic field parameter H on velocity component w at $G = 5$, Gc = 1, Gr = 1, $\alpha = \pi/2$, Pr = 0.7, Sc = 0.3, Ec = 0.2.

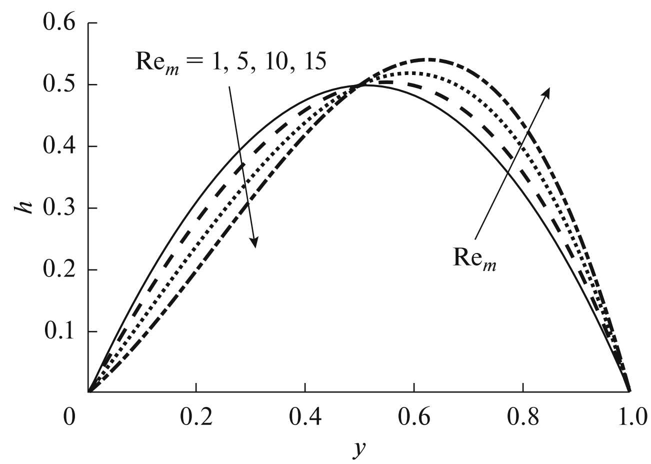

Fig. 4. The effects of the magnetic Reynolds number Re_m on the induced magnetic field profile h at $G = 5$, Gc = 1, Gr = 1, $\alpha = \pi/2$, Pr = 0.7, Sc = 0.3, Ec = 0.3.

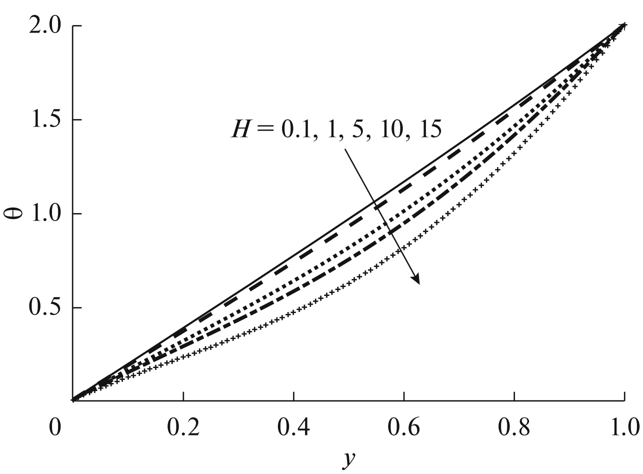

Fig. 3. The influence of H on temperature profile θ at $G = 5$, Gc = 1, Gr = 1, $\alpha = \pi/2$, Pr = 0.7, Sc = 0.3, Ec = 0.2.

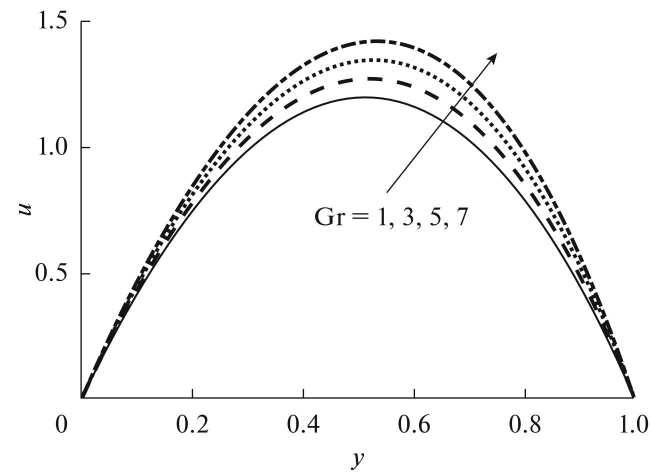

Fig. 5. The effect of Grashof number Gr on the velocity component u at $G = 5$, Gc = 1, $\alpha = \pi/2$, Pr = 0.7, Sc = 0.3, Ec = 0.2.

Figure 4 represents the effect of the magnetic Reynolds number Re_m on the induced magnetic field profile h. It is noted that h profile falls with the increase in Re_m till its peak point then the effect is reversed. Physically, Re_m gives an estimate of the relative effects of advection or induction of a magnetic field by the motion of a conducting medium, often a fluid, to magnetic diffusion. Flux lines of the magnetic field are then adverted with the fluid flow, until the time when gradients are concentrated into regions of short enough length scale that diffusion can balance advection.

Figures 5 and 6 indicate the effect of Gr and Gc on u. It is observed that with Gr increasing u increases. It is due to the fact that increase in Gr increases temperature gradients, which leads to the increase in u (Fig. 5). The the effect of Gc on u is shown in Fig. 6. it can be seen that with an increase in Gc u increases, as a result of the increase of the drag forces.

Figures 7 and 8 present the time progression of θ at the center of the channel ($y = 0$) for different values of Pr and Ec. Figure 7 indicates that with Pr increasing the temperature distribution falls. This is because the thermal boundary layer thickness decrease with Pr increasing. On other hand, Fig. 8 depicts that with Ec increasing the temperature distribution rises.

Figure 9 illustrates the effect the of Sc on the concentration profile Γ. The concentration distribution

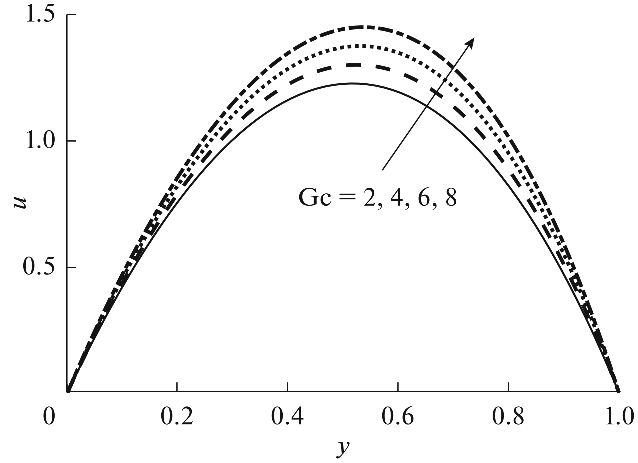

Fig. 6. The effect of modified Grashof number Gc on the velocity component u at $G = 5$, $\mathrm{Gr} = 1$, $\alpha = \pi/2$, $\mathrm{Pr} = 0.7$, $\mathrm{Sc} = 0.3$, $\mathrm{Ec} = 0.2$.

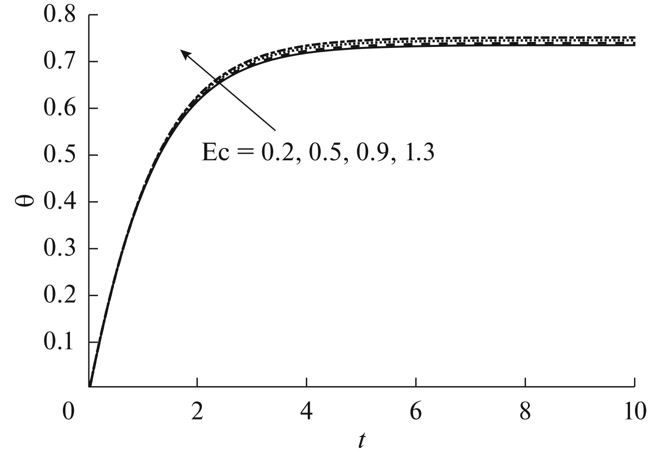

Fig. 8. The effect of Eckart number E on temperature distribution θ at $G = 5$, $\mathrm{Gc} = 1$, $\mathrm{Gr} = 1$, $\alpha = \pi/2$, $\mathrm{Pr} = 0.7$, $\mathrm{Sc} = 0.3$.

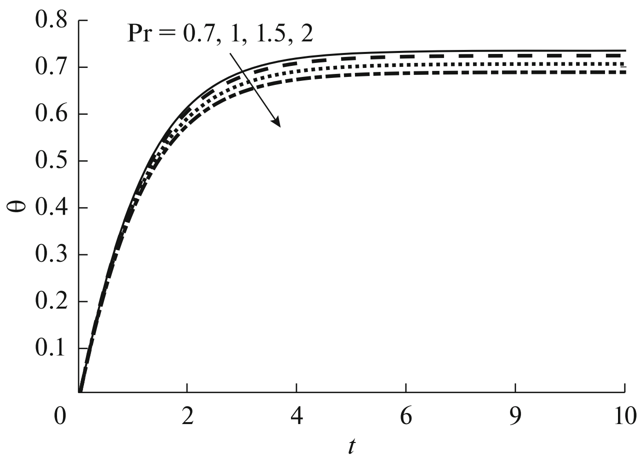

Fig. 7. The effect of Prandtl number Pr on temperature distribution θ at $G = 5$, $\mathrm{Gc} = 1$, $\mathrm{Gr} = 1$, $\alpha = \pi/2$, $\mathrm{Sc} = 0.3$, $\mathrm{Ec} = 0.2$.

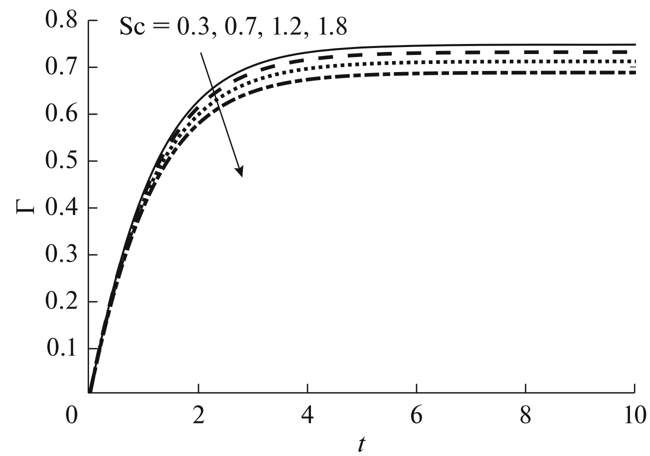

Fig. 9. The effect of the Schmidt number Sc on the concentration profile Γ at $G = 5$, $\mathrm{Gc} = 1$, $\mathrm{Gr} = 1$, $\alpha = \pi/2$, $\mathrm{Pr} = 0.7$, $\mathrm{Ec} = 0.2$.

decreases with Sc increasing. It is due to the fact that the increase in Sc means decrease in molecular diffusivity D that results in decrease in concentration in the boundary layer.

CONCLUSIONS

This study investigated the magnetohydrodynamic unsteady fluid flow between two inclined parallel permeable walls with the consideration of heat and mass transfer using homotopy perturbation method. The flow is subjected to a variable magnetic field normal to the walls. The effects of various parameters, such as applied magnetic field parameter H, Grashof number Gr, modified Grashof number Gc, Prandtl number Pr, Eckart number Ec and Schmidt number Sc were studied. It is found that, the main and secondary fluid velocity components u and w increases with the decrease in the applied magnetic field parameter H. It was observed that with increasing the value of Grashof number Gr increase of temperature gradients, which leads to the increase of the velocity component u. On the other hand, it was found that the velocity distribution increase with increasing the modified Grashof number Gc. In addition, the concentration distribution decreases with an increase in the value of the Schmidt number Sc.

APPENDIX

$$u_1(t)=\frac{\left(s-se^{-\alpha t}-2+G+2e^{-\alpha t}\right)}{2}+\frac{e^{-\alpha t}\left(\alpha+2s-2se^{\alpha t}-\eta+\eta e^{\alpha t}\right)}{6}$$
$$-\frac{\alpha e^{-\alpha t}+\eta-\eta e^{-\alpha t}-\eta m+\mathrm{Gc}\sin(\theta)+\mathrm{Gr}\sin(\theta)}{12}-\frac{\eta m e^{-2\alpha t}\left(e^{\alpha t}-1\right)^2}{15}$$
$$-\frac{\mathrm{Gr}e^{-\alpha t}\sin(\theta)+2\eta m e^{-\alpha t}-\eta m e^{-2\alpha t}-\mathrm{Gc}e^{-\alpha t}\sin(\theta)}{12},$$

$$u_2(t)=\frac{\left(s-se^{-\alpha t}-2+G+2e^{-\alpha t}\right)}{2},\quad u_3(t)=\frac{e^{-\alpha t}\left(\alpha+2s-2se^{\alpha t}-\eta+\eta e^{\alpha t}\right)}{6},$$

$$u_4(t)=-\frac{\alpha e^{-\alpha t}+\eta-\eta e^{-\alpha t}-\eta m+\mathrm{Gc}\sin(\theta)+\mathrm{Gr}\sin(\theta)}{12}$$
$$-\frac{\mathrm{Gc}e^{-\alpha t}\sin(\theta)-\mathrm{Gr}e^{-\alpha t}\sin(\theta)+2\eta m e^{-hat}-\eta m e^{-2\alpha t}}{12}.$$

$$w_1(t)=-\frac{e^{-\alpha t}\left(\alpha+2s-2se^{\alpha t}-\eta+\eta e^{\alpha t}\right)}{6}+\frac{e^{-\alpha t}(s-2)\left(e^{\alpha t}-1\right)}{2}$$
$$+\frac{\eta m e^{-2\alpha t}(e^{\alpha t}-1)^2}{15}-\frac{\left(\alpha e^{-\alpha t}+\eta-\eta e^{-\alpha t}+\eta m-2\eta m e^{-\alpha t}+\eta m e^{-2\alpha t}\right)}{12},$$

$$w_2(t)=\frac{e^{-\alpha t}(s-2)\left(e^{\alpha t}-1\right)}{2},$$

$$w_3(t)=\frac{e^{-\alpha t}\left(\alpha+2s-2se^{\alpha t}-\eta+\eta e^{\alpha t}\right)}{6},$$

$$w_4(t)=-\frac{\left(\alpha e^{-\alpha t}+\eta-\eta e^{-\alpha t}+\eta m-2\eta m e^{-\alpha t}+\eta m e^{-2\alpha t}\right)}{12}.$$

$$h_1(t)=-\left(e^{-\alpha t}-1\right)-\frac{\mathrm{Re}_m e^{-\alpha t}\left(e^{-\alpha t}-1\right)\left(2\mathrm{Re}_m+12e^{\alpha t}-2\mathrm{Re}_m e^{\alpha t}-2\mathrm{Re}_m s+\alpha\mathrm{Re}_m+2\mathrm{Re}_m s e^{\alpha t}-12\right)}{12\mathrm{Re}_m}.$$

$$h_2(t)=\left(e^{-\alpha t}-1\right)+\frac{\mathrm{Re}_m e^{-\alpha t}\left(e^{-\alpha t}-1\right)\left(2\mathrm{Re}_m+12e^{\alpha t}-2\mathrm{Re}_m e^{\alpha t}-2\mathrm{Re}_m s+\alpha\mathrm{Re}_m+2\mathrm{Re}_m s e^{\alpha t}-12\right)}{12\mathrm{Re}_m}$$
$$-\frac{\mathrm{Re}_m e^{-\alpha t}\left(e^{-\alpha t}-1\right)\left(4\mathrm{Re}_m e^{\alpha t}-4\mathrm{Re}_m+4\mathrm{Re}_m s+\alpha\mathrm{Re}_m-4\mathrm{Re}_m s e^{phat}\right)}{12\mathrm{Re}_m},$$

$$h_3(t)=\frac{\alpha^2\mathrm{Re}_m e^{-\alpha t}(e^{-\alpha t}-1)}{12}+\frac{{}^2\mathrm{Re}_m e^{-\alpha t}(e^{-\alpha t}-1)(4\mathrm{Re}_m e^{\alpha t}-4\mathrm{Re}_m+4\mathrm{Re}_m s+\alpha\mathrm{Re}_m-4\mathrm{Re}_m s e^{\alpha t})}{12\mathrm{Re}_m},$$

$$h_4(t)=-\frac{\alpha^2\mathrm{Re}_m e^{-\alpha t}\left(e^{-\alpha t}-1\right)}{12}.$$

$$\theta_1(t)=\frac{e^{-2\alpha t}\left(e^{\alpha t}-1\right)\left(e^{\alpha t}-\mathrm{EcPr}+\mathrm{EcPr}e^{\alpha t}\right)}{}$$
$$-\frac{\mathrm{Pr}e^{-2\alpha t}\left(\alpha e^{\alpha t}-8\mathrm{Ec}+16\mathrm{Ec}e^{\alpha t}-8\mathrm{Ec}e^{2\alpha t}\right)}{12}$$
$$-\frac{\mathrm{Pr}e^{-2\alpha t}\left(e^{\alpha t}-1\right)\left(se^{\alpha t}-4\mathrm{Ec}+4\mathrm{Ec}e^{\alpha t}\right)}{3}+\frac{\eta\mathrm{EcPr}e^{-3\alpha t}\left(e^{\alpha t}-1\right)^3}{140}+1,$$

$$\theta_2(t) = -\frac{e^{-2\alpha t}\left(e^{\alpha t}-1\right)\left(e^{\alpha t}-\mathrm{EcPr}+\mathrm{EcPr}e^{\alpha t}\right)}{} + \frac{\mathrm{Pr}y^4e^{-2\alpha t}\left(\alpha e^{\alpha t}-8\mathrm{Ec}+16\mathrm{Ec}e^{\alpha t}-8\mathrm{Ec}e^{2\alpha t}\right)}{12},$$

$$\theta_3(t) = \frac{\mathrm{Pr}e^{-2\alpha t}\left(e^{\alpha t}-1\right)\left(se^{\alpha t}-4\mathrm{Ec}+4\mathrm{Ec}e^{\alpha t}\right)}{3},$$

$$\theta_4(t) = -\frac{\left(\alpha e^{-\alpha t}+\eta-\eta e^{-\alpha t}+\eta m-2\eta m e^{-t}+\eta m e^{-2\alpha t}\right)}{12}.$$

$$\Gamma_1(t) = \left(\frac{\mathrm{Sc}se^{-\alpha t}}{3}-\frac{\mathrm{Sc}s}{3}-\frac{\alpha\mathrm{Sc}e^{-\alpha t}}{12}-e^{-\alpha t}+2\right),\quad \Gamma_2(t) = \left(e^{-\alpha t}-1\right),$$

$$\Gamma_3(t) = \left(\frac{\mathrm{Sc}s}{3}-\frac{\mathrm{Sc}se^{-\alpha t}}{3}\right),\quad \Gamma_4(t) = \frac{\alpha\mathrm{Sc}e^{-\alpha t}}{12}.$$

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