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Two-layer fluid formation and propagation of periodic solitons induced by (3+1)-dimensional KP equation

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ABSTRACT

In this paper, multi-wave solutions of Kadomtsev–Petviashvili KP equation with time variable coefficient are obtained. It is found that the formation of two-layer with bright and dark soliton which are symmetric. These solitons are transversely periodic as periodic of two-soliton and two-antisoliton in time with cusps localized in space. These two types of waves propagate in an opposite direction relative the higher layer. the autonomous behavior may be argued to the fact that the KP is composed of the derivative and the anti-derivative terms.

1. Introduction

In recent years, the nonlinear evolution equations (NLEEs) with variable coefficients have attracted much attention of research works in different physical fields such as optical fiber [1–3], fluid dynamics [4–10] and in plasma physics [11,12].

A variety of methods were developed to find different types of exact solutions to (NLEEs), such as traveling wave solutions (TWS) [13–19], (or semi-self-similar and self-similar) wave solutions has been considered in [20–23]. Some powerful methods are used to find multi-soliton solutions of (NLEEs) [24–29]. Further, multi-solitons and multi-pulses waves that are propagating in a medium are interesting in fluid mechanics [30–35]. Also the properties of mixed-type soliton and (elastic or inelastic) collisions between them in the bright–dark solitons are investigated in [36–40].

In this paper, we study the behavior of semi-self-similar solutions of the $(3+1)$-dimensional Kadomtsev–Petviashvili KP equation with time variable-coefficients;

$$(v_t - \frac{1}{2}\alpha(t)\, v v_x + \beta(t)\, u_{xxx})_y - \gamma(t)\, v_{xz} - \delta(t)\,(v_x \partial_x^{-1} v_y)_x = 0\,, \tag{1}$$

where x, y and z are scaled space coordinates and t is the time coordinate and $\partial_x^{-1} f = \int f\, dx$. In general, the solution of Eq. (1) shows the propagation of shallow water waves in dispersive medium [41,42]. In Eq. (1) $\alpha(t)$, $\beta(t)$, $\gamma(t)$ and $\delta(t)$ are convective, dispersity, diffusive and transverse longitudinal coefficients respectively. Thus to study the behavior of the solutions of Eq. (1), it is transformed to coupled partial differential equation, Thus we use $v = u_x$, we have

$$(v_t - \frac{1}{2}\alpha(t)\, v v_x + \beta(t)\, u_{xxx})_y - \gamma(t)\, v_{xz} - \delta(t)\,(v_x u_y)_x = 0\,, \tag{2}$$

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by integrating the first equation in (2) with respect to x with taking the integral constant equal to zero. It reduces to the closed form

$$(u_t - \frac{1}{2}\alpha(t)\, u_x^2 + \beta(t)u_{xxx})_y - \gamma(t)\, v_{xz} - \delta(t)\, u_{xx}u_y = 0\,. \tag{3}$$

The aim of this work is to construct the two-soliton solutions of Eq. (3) by using the generalized unified method (GUM) [43–45] and for computing the solutions an algorithm is presented in the Appendix. The behaviors of the propagation of waves in the x, y and $z-$directions in the two-layers are investigated and illustrated. Some important properties of collisions ad similariton collisions are visualized by choosing appropriately the dispersion coefficient in Eq. (1).

We consider the general evolution equation

$$F\,(u,\ u_t, u_x, u_y, u_z, u_{xx}, u_{yy}, u_{xz}\,\ldots) = 0\,, \tag{4}$$

where F is polynomial in its argument, x and t are missing. Eq. (3) has (TWS) (or semi-self-similar solutions). Thus Eq. (4) reduces to

$$G(U,\ U', U'', \ldots) = 0,\ U' = \frac{dU}{d\xi}\,, \tag{5}$$
$$\xi = \kappa\, x + \eta\, y + \lambda\, z + \int_0^t \omega(t_1)\, dt_1\,.$$

The organization of this paper is as follows. In Section 2, the (GUM) is discussed. In Section 3,the multi-soliton propagations are constructed in the (3+1)-dimensional KP equation with a variable coefficients. The effects of varying dispersion on the amplitude wave of two-solitons and two-anti-solitons are shown with figures. The conclusions are presented in Section 4. Finally, the Appendix is given.

2. The method

In this section, we present the outlines of the (GUM). Which can be found in the forms of a polynomial or a rational function solutions in an auxiliary function together with appropriate auxiliary equation.

Here, we confine our self to find rational solutions.

Rational function solutions

The single rational function solution can be written as follows:

$$u(\xi, t) = \frac{\sum_{i=0}^{n} a_i(t)\, \varphi^i(\xi)}{\sum_{i=0}^{r} q_i(t)\, \varphi^i(\xi)}, \tag{6}$$

$$(\varphi'(\xi))^p = \sum_{i=0}^{k} c_j\, \varphi^i(\xi),\ p = 1, 2$$

where $a_i(t),\ q_i(t)$ and $c_j,\ i, j = 0, 1.$ are unknown parameters.

Two soliton solution can be obtained by accounting for two ordinary differential equations. The rational function solutions (two-nonautonomous or similariton–solitons solutions) take the following expression

$$u(\varphi_1, \varphi_2) = \frac{a_0(t) + \sum_{i=1}^{2} a_i(t)\, \varphi_i + a_3(t)\, \varphi_1\varphi_2}{q_0(t) + \sum_{i=1}^{2} q_i(t)\, \varphi_i + q_3(t)\, \varphi_1\varphi_2}\,, \tag{7}$$

$$\varphi_i'(\xi_j) = c_i\, \varphi_j(\xi_j) + c_{j0}\,,\ i = j = 1, 2.$$

We substitute Eqs. (5) and (7) into (4) and solve the ordinary equations. As a result of this substitution with (*Mathematica*), we get set of a polynomial of φ_i. We calculate all the coefficients of same power of φ_i to zero.

The conditions of variable coefficients and the explicit solutions of Eq. (4) solution are computing.

Further, ensure that the solutions satisfy Eq. (2).

3. Multi-soliton solutions

In this section, we search to find one and two mixed soliton, namely (soliton and antisoliton) solutions of Eq. (1) that describe the propagation of waves in two layers fluid.

These solutions are obtained for the rational functions solutions of Eq. (1).

Here, we write $u(x, t) = U(\xi_j)$ and $\xi_j = \kappa_j x + \eta_j y + \lambda_j z + \int_0^t \omega_j(t_1)\, dt_1$, where $j = 1$ or $j = 1,\ 2$ in the cases of single soliton and two-solitons solutions.

3.1. Single-soliton solution

To get an exact single soliton solution, we take $k = 1$, $n = N = r = 1$, into the rational function solutions, which is given in Eq. (6) and we have:

$$U(\xi_1, t) = \frac{a_0(t) + a_1(t)\,\varphi(\xi_1)}{q_0(t) + q_1(t)\,\varphi(\xi_1)}\,, \tag{8}$$
$$\varphi_{\xi_1} = c_1 + c_0\varphi(\xi_1)$$

where ξ_1 is defined before. By using the algorithm presented in the Appendix, we find solution of Eq. (2) as

$$U(\xi_1, t) = \frac{1}{q_0(t)\,\delta(t)}\,(-\frac{6\kappa_1 q_1(t)\,\beta(t)\,(c_1 q_0(t) - c_0 q_1(t))\left(c_1 e^{\xi_1 c_1} - c_{10}\right)}{c_1\left(q_1(t)e^{\xi_1 c_1} + q_0(t)\right) - c_0 q_1(t)} + a_0(t)\delta(t)\,),$$
$$v(x, y, z, t) = u_x(x, y, z, t)\,, \tag{9}$$

where

$$a_1(t) = \frac{q_1(t)\,(a_0(t)\delta(t) + 6\kappa_1\beta(t)\,(c_{10}q_1(t) - c_1 q_0(t)))}{q_0(t)\delta(t)}\,,$$
$$\alpha(t) = \delta(t),\ \ \omega_1(t) = \frac{\kappa_1\lambda_1\gamma(t)}{\eta_1} - \kappa_1^3 {c_1}^2\beta(t)\,, \tag{10}$$

and bearing in mind that $a_0(t),\ q_0(t),\ q_1(t),\ \kappa_1\,\eta_1,\ c_0,\ c_1,\ \gamma(t),\ \beta(t)$ and $\delta(t)$ are arbitrary functions.

3.2. Two-soliton solutions

For two-soliton solutions, we write transformation $u(x, y, z, t) = U(\xi_1, \xi_2, t),\ \xi_j = \kappa_j x + \eta_j y + \lambda_j z + \int_0^t \omega_j(t_1)\,dt_1$ and $j = 1,\ 2$.

Substituting Eq. (7) into Eq. (2), and using *Mathematica*, then equating to zero each coefficient of the same order power of the exponential functions yields a set of equations as follows and the solution of Eq. (1) is

$$U(\xi_1, \xi_2, t) = A_2(\varphi_i)/Q_2(\varphi_i),\ v = u_x\,,$$
$$A_2(\varphi_i) = a_0(t) + \sum_{i=1}^{2} a_i(t)\,\varphi_i + a_3(t)\,\varphi_1\varphi_2\,, \tag{11}$$
$$\varphi_i'(\xi_j) = c_j\,\phi_i(\xi_j) + c_{j_0}(t)\,, i = j = 1, 2.$$

By the same way $Q_2(\varphi_i)$ is given as in $A_2(\varphi_i)$ in Eq.(9), but $a_i(t) \to q_i(t),\ i = 1,\ 2.$

By substituting from Eq. (11) into (3), and by the same way as in Section 3.1, we get

$$\begin{aligned}
A_2(\varphi_i) = {} & \big(\eta_2\eta_1\left(\kappa_2\left(3\eta_1\kappa_1^2 {c_1}^2\beta(t) - \lambda_1\gamma(t)\right) + 3\eta_1\kappa_1\kappa_2^2 c_1 c_2\beta(t) - \kappa_1\lambda_2\gamma(t)\right) + \eta_1^2\kappa_2\lambda_2\gamma(t) \\
& + \eta_2^2\kappa_1\left(3\eta_1\kappa_2 c_2\beta(t)\left(\kappa_1 c_1 + \kappa_2 c_2\right) + \lambda_1\gamma(t)\right)\big)\, q_3(t)\left(\eta_2^2\kappa_1\lambda_1\gamma(t)\right. \\
& \left(-6\kappa_1\sigma_1(t)q_1(t)\beta(t)\left(c_1 e^{c_1\xi_1} - c_{10}\right)\left(c_2\left(q_3(t)e^{c_2\xi_2} + q_1(t)\right) - c_{20}q_3(t)\right)\right. \\
& + \left(c_1\left(q_3(t)e^{c_1\xi_1} + q_2(t)\right) - c_{10}q_3(t)\right)\left(-6\kappa_2 q_2(t)\beta(t)\left(c_2 e^{c_2\xi_2} - c_{20}\right)\right. \\
& a_0(t)\delta(t)\left(c_2\left(q_3(t)e^{c_2\xi_2} + q_1(t)\right) - c_{20}q_3(t)\right) + \sigma_2(t)\big)\big) + \eta_1\eta_2\left(\left(\kappa_2\lambda_1 + \kappa_1\lambda_2\right)\gamma(t)\right. \\
& \left(6\kappa_1\sigma_1(t)q_1(t)\beta(t)\left(c_1 e^{c_1\xi_1} - c_{10}\right)\left(c_2\left(q_3(t)e^{c_2\xi_2} + q_1(t)\right) - c_{20}q_3(t)\right)\right. \\
& + \left(c_1\left(q_3(t)e^{c_1\xi_1} + q_2(t)\right) - c_{10}q_3(t)\right)\left(6\kappa_2\sigma_2(t)\beta(t)\left(c_2 e^{c_2\xi_2} - c_{20}\right)\right. \\
& q_2(t) - a_0(t)\delta(t)\left(c_2\left(q_3(t)e^{c_2\xi_2} + q_1(t)\right) - c_{20}q_3(t)\right)\big)\big)\big) + 3\eta_2\kappa_1\kappa_2\beta(t) \\
& \left(6\kappa_1^2\sigma_1\beta(t)c_1\left(\sigma_2(t)\left(\sigma_2(t)c_1 e^{c_1\xi_1} - c_2 c_{10}q_1(t)\right) + c_2 e^{c_2\xi_2}\right.\right. \\
& q_3(t)\left(c_2 c_{10}q_1(t) + \sigma_2(t)c_1 e^{c_1\xi_1}\right)\big) + \kappa_2 {c_2}^2\left(c_1 - c_{10}q_3(t)\right. \\
& q_3(t)e^{c_1\xi_1} + q_2(t)\big)\left(a_0(t)\delta(t) - 6\kappa_2\sigma_2(t)q_2(t)\beta(t)\left(c_2 e^{c_2\xi_2} - c_{20}\right)\right. \\
& c_2\left(q_3(t)e^{c_2\xi_2} + q_1(t)\right) - c_{20}q_3(t)\big) + \kappa_1 c_2\left(c_1 e^{c_1\xi_1}\right.
\end{aligned}$$

$$
\begin{aligned}
&\left(6\kappa_2\beta(t)\left(\sigma_2(t)c_2c_{10}q_1(t)q_3(t)-\sigma_2^2(t)c_1q_2(t)-c_2e^{c_2\xi_2}\right.\right.\\
&q_3(t)^2\left(c_2c_{10}q_1(t)+\sigma_{12}(t)c_{20}\right)\big)+\delta(t)a_0(t)c_1q_3(t)\left(c_2-c_{20}q_3(t)\right.\\
&q_3(t)e^{c_2\xi_2}+q_1(t)\big)\big)+\sigma_1(t)\left(a_0(t)c_1\delta(t)\left(c_2\left(q_3(t)e^{c_2\xi_2}-q_1(t)\right)+c_{20}q_3(t)\right)\right.\\
&+6\kappa_2\beta(t)\left(c_2e^{c_2\xi_2}\left(\sigma_2c_1q_2(t)-\sigma_{21}(t)c_{10}q_3(t)\right)+\sigma_2(t)\left(c_2c_{10}q_1(t)-c_1c_{20}q_2(t)\right)\right)\big)\big)\big)\\
&+\eta_1^2\kappa_2\left(\lambda_2\gamma(t)\left(-6\kappa_1\sigma_1(t)\beta(t)\ q_1(t)\left(c_1e^{c_1\xi_1}-c_{10}\right)\left(c_2\left(q_3(t)e^{c_2\xi_2}+q_1(t)\right)-c_{20}q_3(t)\right)\right.\right.\\
&+\left(c_1\left(q_3(t)e^{c_1\xi_1}+q_2(t)\right)-c_{10}q_3(t)\right)\left(-6\kappa_2\sigma_2(t)q_2(t)\beta(t)\left(c_2e^{c_2\xi_2}-c_{20}\right)\right.\\
&a_0(t)+\delta(t)\left(c_2\left(q_3(t)e^{c_2\xi_2}+q_1(t)\right)-c_{20}q_3(t)\right)\big)\big)\big)-3\eta_2\kappa_1\beta(t)\left(6\kappa_1^2\sigma_1(t)c_1^2\beta(t)\right.\\
&q_1(t)\left(c_1e^{c_1\xi_1}-c_{10}\right)\left(c_2\left(q_3(t)e^{c_2\xi_2}+q_1(t)\right)-c_{20}q_3(t)\right)+\kappa_2c_2\\
&\left(\sigma_2(t)c_1\left(c_1\left(q_2(t)-q_3(t)e^{c_1\xi_1}\right)-c_{10}q_3(t)\right)\left(a_0(t)\delta(t)+6\kappa_2c_{20}q_2(t)\beta(t)\right)\right.\\
&c_2\left(-e^{c_2\xi_2}\right)\left(c_1\left(q_3(t)e^{c_1\xi_1}+q_2(t)\right)-c_{10}q_3(t)\right)\left(a_0(t)c_1q_3(t)\delta(t)+6\kappa_2\sigma_2(t)\sigma_{12}(t)\beta(t)\right)\big)\\
&+\kappa_1c_1\left(a_0(t)c_1(-\delta(t))\left(c_1\left(q_3(t)e^{c_1\xi_1}+q_2(t)\right)-c_{10}q_3(t)\right)\right.\\
&c_2\left(q_3(t)e^{c_2\xi_2}+q_1(t)\right)-c_{20}q_3(t)++6\kappa_2\beta(t)\left(\sigma_1(t)\left(\sigma_2(t)\left(c_2c_{10}q_1(t)-c_1c_{20}q_2(t)\right)\right.\right.\\
&c_2+e^{c_2\xi_2}\left(\sigma_2(t)c_1q_2(t)-c_2c_{10}q_1(t)q_3(t)\right)\big)+c_1e^{c_1\xi_1}\\
&c_2q_3(t)^2\left(c_1c_{20}q_2(t)+c_{10}\sigma_{21}(t)\right)e^{c_2\xi_2}+\sigma_2(t)\left(\sigma_{21}(t)c_{10}q_3(t)-\sigma_2(t)c_1q_2(t)\right)\big)\big)\big)\big)\big)\ ,\\
&Q_2(\varphi_i)=\delta(t)\left(\left(\eta_1^2\kappa_2q_1(t)\left(3\eta_2\kappa_1c_1\beta(t)\left(\kappa_2c_2-\kappa_1c_1\right)-\lambda_2\gamma(t)\right)-\eta_2^2\kappa_1\lambda_1q_1(t)\gamma(t)\right.\right.\\
&+\eta_1\eta_2\left(3\eta_2\kappa_1\kappa_2\beta(t)\left(\kappa_1\sigma_{21}(t)c_1-\kappa_2c_2^2q_1(t)\right)+\left(\kappa_2\lambda_1+\kappa_1\lambda_2\right)q_1(t)\gamma(t)\right)\big)q_2(t)\\
&\left(-\eta_2^2\kappa_1\lambda_1\gamma(t)\left(c_1\left(q_3(t)e^{c_1\xi_1}+q_2(t)\right)-c_{10}q_3(t)\right)\left(c_2\left(q_3(t)e^{c_2\xi_2}+q_1(t)\right)-c_{20}q_3(t)\right)\right.\\
&+\eta_1\eta_2\left(\left(\kappa_2\lambda_1+\kappa_1\lambda_2\right)\gamma(t)\left(c_1\left(q_3(t)e^{c_1\xi_1}+q_2(t)\right)-c_{10}q_3(t)\right)\right.\\
&c_2\left(q_3(t)e^{c_2\xi_2}+q_1(t)\right)-c_{20}q_3(t)+3\eta_2\kappa_1\kappa_2c_2\beta(t)\left(\sigma_1(t)\left(\sigma_2(t)\left(\kappa_1c_1-\kappa_2c_2\right)\right.\right.\\
&c_2q_3(t)\left(\kappa_1c_1+\kappa_2c_2\right)\left(-e^{c_2\xi_2}\right)\big)-c_1q_3(t)\left(\kappa_1c_1+\kappa_2c_2\right)e^{c_1\xi_1}\\
&c_2\left(q_3(t)e^{c_2\xi_2}+q_1(t)\right)-c_{20}q_3(t)\big)\big)+\eta_1^2\kappa_2\left(-\lambda_2\gamma(t)\left(c_1-c_{10}q_3(t)\right.\right.\\
&q_3(t)e^{c_1\xi_1}+q_2(t)\big)c_2\left(q_3(t)e^{c_2\xi_2}+q_1(t)\right)-c_{20}q_3(t)+3\eta_2\kappa_1c_1\beta(t)\\
&\left(c_2q_3(t)\left(\kappa_1c_1+\kappa_2c_2\right)\left(-e^{c_2\xi_2}\right)\left(c_1\left(q_3(t)e^{c_1\xi_1}+q_2(t)\right)-c_{10}q_3(t)\right)\right.\\
&+\sigma_2(t)\left(c_1q_3(t)\left(\kappa_1c_1+\kappa_2c_2\right)\left(-e^{c_1\xi_1}\right)-\sigma_1(t)\left(\kappa_1c_1-\kappa_2c_2\right)\right)\big)\big)\big)\big)\ ,
\end{aligned}
\tag{12}
$$

and

$$
\begin{aligned}
&\omega_1(t)=\tfrac{\kappa_1\lambda_1\gamma(t)}{\eta_1}-\kappa_1^3c_1^2\beta(t),\ \omega_2(t)=\tfrac{\kappa_2\lambda_2\gamma(t)}{\eta_2}-\kappa_2^3c_2^2\beta(t)\,,\alpha(t)=\delta(t)\,,\\
&\sigma_1(t)=c_1q_2(t)-c_{10}q_3(t),\ \sigma_2(t)=c_2q_1(t)-c_{20}q_3(t)\,,\\
&\sigma_{12}(t)=c_1q_2(t)-2c_{10}(t)q_3(t),\ \sigma_{21}=c_2q_1(t)-2c_{20}q_3(t).
\end{aligned}
\tag{13}
$$

when $a_0(t),\ q_i(t),\ i=1,2,3,\ \kappa_j\,\eta_j,\ c_j,\ c_{j_0},\ j=1,2,\ \gamma(t),\ \delta(t)$ and $\beta(t)$ are free functions.

By substituting from results in Eqs. (12) and (13) into (11) we get the function $u(x,y,z,t)$. Then results for the function $v(x,y,z,t)$ which is the solution of Eq. (1) is obtained by using the second equation in (2).

The results obtained for $v(x,\ y,\ z,\ t)$ are shown numerically in many figures in what follows. We mention that when the solution which are given in Eq. (12) are displayed against z and t produce the same wave structure as in the case of x and t. So that not produced here. In Fig. 1a, b and c the results for $v(x,y,z,t)$ are displayed against $(x,\ t)$, $(y,\ t)$ and $(z,\ t)$ respectively by taking the values of the pairs $(y,\ z)$, $(z,\ x)$ and (x,y) fixed. We take in Fig. 1, 2 and 3 are the following functions, namely $a_0(t)=q_i(t)=c_{j_0}=1,\ i=1,2,\ j=1,2,\ c_1=2,\ c_2=1.5,\ \gamma(t)=s\beta(t),\ \delta(t)=2$ and $s=0.5$.

Fig. 1a, shows the propagation of periodic two-soliton and two-antisoliton in time with cusps localized in space. Fig. 1b, shows a train of solution which is periodic in time.

Fig. 2a shows the propagation of two-arrays of solitons. These waves see a wave train in space and semi periodic in time in the upper layer. Both vectors are propagating in parallel. Fig. 2b shows mixed distribution for two-freak waves under transmission with higher amplitudes at $t=0$. The deformation of two-soliton in Fig. 2 is greater than that in Fig. 1.

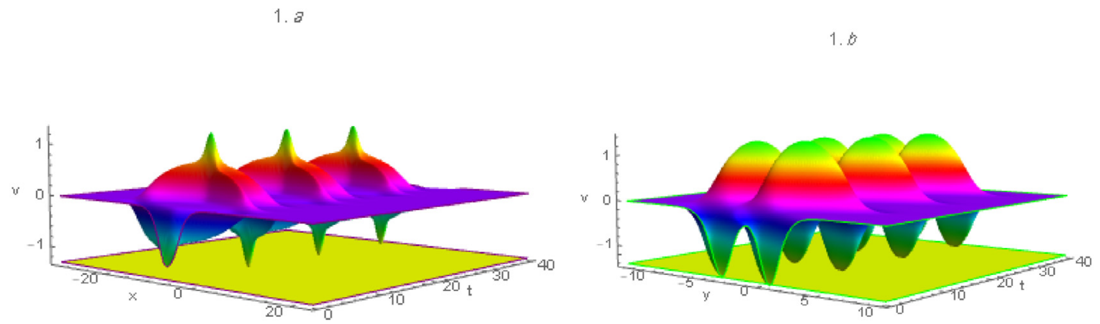

Fig. 1. (a), (b) and (c) the solution $v(x, y, z, t)$ of Eqs. (12) are displayed against (x, t), (y, t) and (z, t) respectively, when $\beta(t) = \cos(0.5t)$,. In 1(a) $\kappa_1 = 0.5, \kappa_2 = -0.6$, $\eta_1 = 0.05$, $\eta_2 = 0.1$, $\lambda_1 = 0.6$, $\lambda_2 = -0.85$ and $y = z = 5$. In 1(b) $\kappa_1 = 0.7$, $\kappa_2 = -0.9$, $\eta_1 = 1, \eta_2 = 1$, $\lambda_1 = 0.55$, $\lambda_2 = 0.7$ and $x = z = 5$. In 1(c) the same caption Fig. 1(a), except $x = y = 5$.

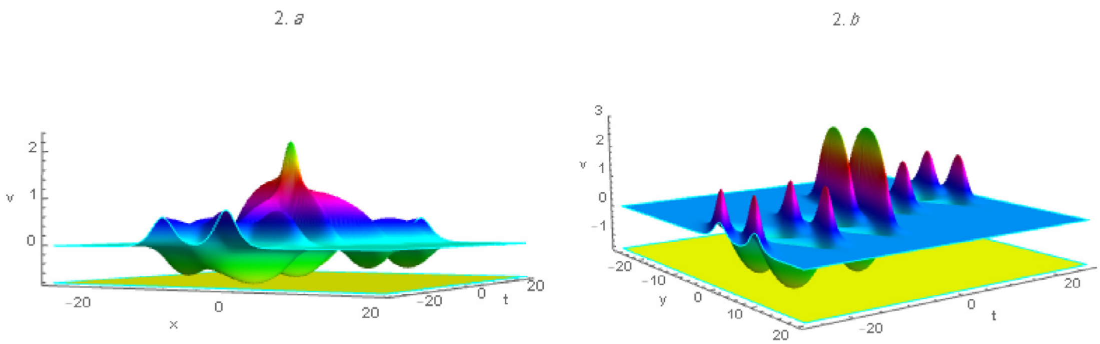

Fig. 2. (a) and (b) the solution $v(x, y, z, t)$ of Eqs. (12), (13) are displayed against (x) and t or $(y$ and $t)$ when $\beta(t) = Sech(0.5t) + \cos(0.5t)$ and $\delta(t) = -2$, with the same values in Fig. 1 except in 2(a) $\eta_1 = 0.1$, $\eta_2 = 0.3$ and 2(b) $\eta_1 = 0.6$, $\eta_2 = 0.7$, $\lambda_2 = 0.85$.

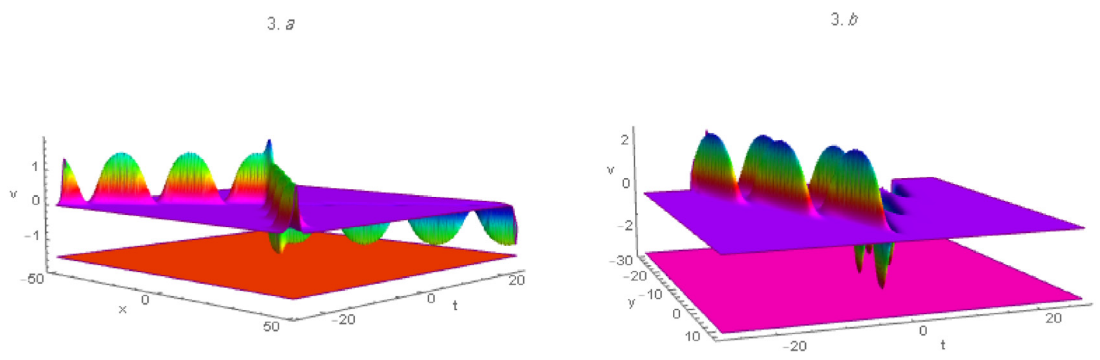

Fig. 3. (a) and (b) are solutions of Eqs. (12), (13) at the same values in Fig. 2 except $\beta(t) = \mathrm{sn}(t,\ 0.5) + \tanh(t)$.

Fig. 3a illustrated propagation of isolated periodic waves in time and localized in space. Fig. 3b see the propagation of progressive waves with cascade waves in the upper layer. In the lower layer, trace of cascade may be observed.

4. Conclusions

It is found that, the solutions of KP equation show two-layer fluid formation. Also, the propagation of periodic soliton anti-solitons with cusps in the upper and lower layers are seen. It is observed that in vector waves in the upper and lower layers may propagate in longitudinal and transverse directions. Also, it is shown that, that arrays in the upper and lower layers may propagate in directions parallel or perpendicular to each other.

Appendix

Here, the equations in Section 3 are used.

The algorithm for two soliton waves is

Input 1: {Eq. (10)}

Simplify[LHS of (3)]

Output 1: $\{ \sum_{i,j}^{s} A_{ij}(a_k(t), q_k(t), c_j(t), \alpha(t), \ldots)\varphi_1^i(\xi_1)\varphi_2^j(\xi_2)\}$

Input 2 := output 1

CoeffcientList [Input2, $\{\varphi_1(\xi_1), \varphi_2(\xi_2)\}$]

Output 2: $\{h_0[a_i(t), q_{j(t)}, c_j(t)], h_1(a_i(t); q_i(t), c_j(t)), \ldots, h_r(a_i(t); q_i(t), c_j(t))\}$

Input 3: $\{h_r(a_i(t); q_i(t), c_j(t))\}$

Solve $[h_r(a_i(t); q_i(t), c_j(t))] = 0]$

Simplify [output 2]

Output 3 $a_1(t) =$ a1(t)$[q_i(t), c_j(t))]$

Input: 4 = Output 3}

Solve [$h_{-1}(a_i(t); q_i(t), c_j(t))] == 0]$

Output {4}

Rpeat:

Output(r+1): $\{a_i(t),\ \ q(t),\ c_i(t),\}$:= Input

Input r+1: = outputs 3: 1, outputs 4: 1, …, outputs r+1 1,

Simplify [{Eqs. (10)}]

Ouutput r: {Eqs. (12), (13)}

DSolve [auxiliary Eqs in (11).]

Output r+2

Input r+3 := {Output1, …, Output r+2}

Simplify [RHS of Eq. (10)]

Output r+3: {The solution}

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