Special Purpose Reservoir Simulators

1-Basic Equations and Concepts

Basic Differential Equations for Flow in Reservoir Engineering

(Note: A similar approach is possible with energy and entropy equations)

Material Balance for One Component in One Phase

$$\frac{\partial}{\partial t}\left(\varepsilon_j \rho_j \omega_{ij}\right) + \vec{\nabla}\cdot\left(\rho_j \omega_{ij} \vec{u}_j - \varepsilon_j \vec{\vec{K}}_{ij}\cdot\vec{\nabla}\rho_j \omega_{ij}\right) = \varepsilon_j r_{ij} + r_{mij} \quad i=1,\ldots,N_C \quad j=1,\ldots,N_P$$

$\sum_{i=1}^{N_P}(\ )$ → Overall Compositional Equations

$\sum_{i=1}^{N_C}(\ )$ → Phase Flow Equations

Overall Compositional Equations

$$\frac{\partial}{\partial t}\left(\phi\sum_{j=1}^{N_P} S_j \rho_j \omega_{ij} + (1-\phi)\rho_s \omega_{is}\right) + \vec{\nabla}\cdot\left(\sum_{j=1}^{N_P}\left(\rho_j \omega_{ij}\vec{u}_j - \phi S_j \vec{\vec{K}}_{ij}\cdot\vec{\nabla}\left(\rho_j \omega_{ij}\right)\right)\right)$$

$$= \phi\sum_{j=1}^{N_P} S_j r_{ij} + (1-\phi) r_{is}, \quad i=1,\ldots,N_C$$

Phase Flow Equations

$$\frac{\partial}{\partial t}\left(\phi S_j \rho_j\right) + \vec{\nabla}\cdot\left(\rho_j \vec{u}_j\right) = \sum_{i=1}^{N_C} r_{mij}, \quad j=1,\ldots,N_P$$

$\sum_{i=1}^{N_C}(\ )$ → Continuity Equation

$\sum_{i=1}^{N_P}(\ )$ → Continuity Equation

Continuity Equation

$$\frac{\partial}{\partial t}\left(\phi\sum_{j=1}^{N_P}\rho_j S_j + (1-\phi)\rho_s\right) + \vec{\nabla}\cdot\left(\sum_{j=1}^{N_P}\rho_j \vec{u}_j\right) = 0$$

Note:

$\varepsilon_j = \phi S_j$ for fluids

$\varepsilon_s = 1-\phi$ for solids

Special Cases of Phase Flow Equations

Phase Flow Equations

$$\frac{\partial}{\partial t}\left(\phi S_j \rho_j\right)+\vec{\nabla}\cdot\left(\rho_j \vec{u}_j\right)=\sum_{i=1}^{N_C} r_{mij}, \quad j=1, \ldots, N_P$$

Buckley-Leverett

$$\varphi \frac{\partial S_w}{\partial t}+u \frac{\partial f_w}{\partial x}=0$$

Diffusivity Equation

$$\nabla \cdot\left(\frac{k\rho}{\mu} \nabla \Phi\right)=\frac{\partial(\phi\rho)}{\partial t}$$

Liquid: $\frac{\partial^2 P}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}=\frac{\phi\mu c_t}{k}\frac{\partial P}{\partial t}$

Gas: $\frac{\partial^2 m(p)}{\partial r^2}+\frac{1}{r}\frac{\partial m(p)}{\partial r}=\frac{\phi\mu c_t}{k}\frac{\partial m(p)}{\partial t}$

Laplace Equation

$$\nabla^2 \Phi=0$$

Poisson Equation

$$\nabla^2 P=\text{constant}$$

Special Cases of Overall Compositional Equations

Overall Compositional Equations

$$\frac{\partial}{\partial t}\left(\phi\sum_{j=1}^{N_P} s_j\rho_j\omega_{ij}+(1-\phi)\rho_s\omega_{is}\right)+\vec{\nabla}\cdot\left(\sum_{j=1}^{N_P}\left(\rho_j\omega_{ij}\vec{u}_j-\phi S_j\vec{\vec{K}}_{ij}\cdot\vec{\nabla}\left(\rho_j\omega_{ij}\right)\right)\right)$$

$$=\phi\sum_{j=1}^{N_P} s_j r_{ij}+(1-\phi)r_{is},\qquad i=1,\ \ldots,\ N_C$$

Convection-Dispersion Equations

$$\phi\frac{\partial C_i}{\partial t}+u\frac{\partial C_i}{\partial x}=\phi K_{li}\frac{\partial^2 C_i}{\partial x^2},\qquad i=1,\ \ldots,\ N_C$$

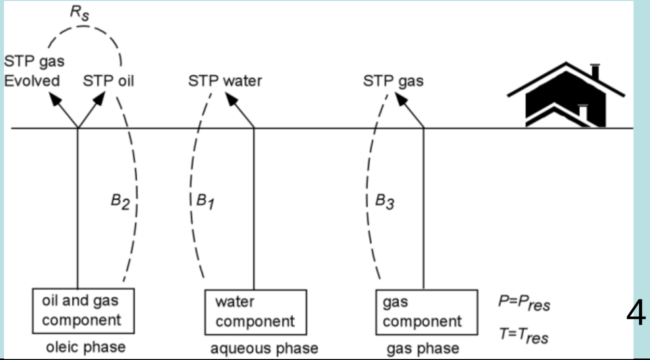
Dispersion-Free Overall Compositional Equations

$$\phi\frac{\partial C_i}{\partial t}+u\frac{\partial F_i}{\partial x}=0,\qquad i=1,\ \ldots,\ N_C$$

Black-Oil Equations

$$\frac{\partial}{\partial t}\left(\frac{\phi S_j}{B_j}\right)+\vec{\nabla}\cdot\left(\frac{\vec{u}_j}{B_j}\right)=0,\qquad j=o,\ w$$

$$\frac{\partial}{\partial t}\left(\phi\left[\frac{S_g}{B_g}+\frac{S_oR_s}{B_o}\right]\right)+\vec{\nabla}\cdot\left(\frac{R_s}{B_o}\vec{u}_o+\frac{\vec{u}_g}{B_g}\right)=0$$



Basic Oil Recovery Equation

$$N_P = \frac{E_V E_D S_{OI} V_P}{B_O}$$

E_V = Volumetric sweep efficiency = $E_A E_I$

N_P = Cumulative oil recovery

E_A = Areal sweep efficiency

E_I = Vertical sweep efficiency

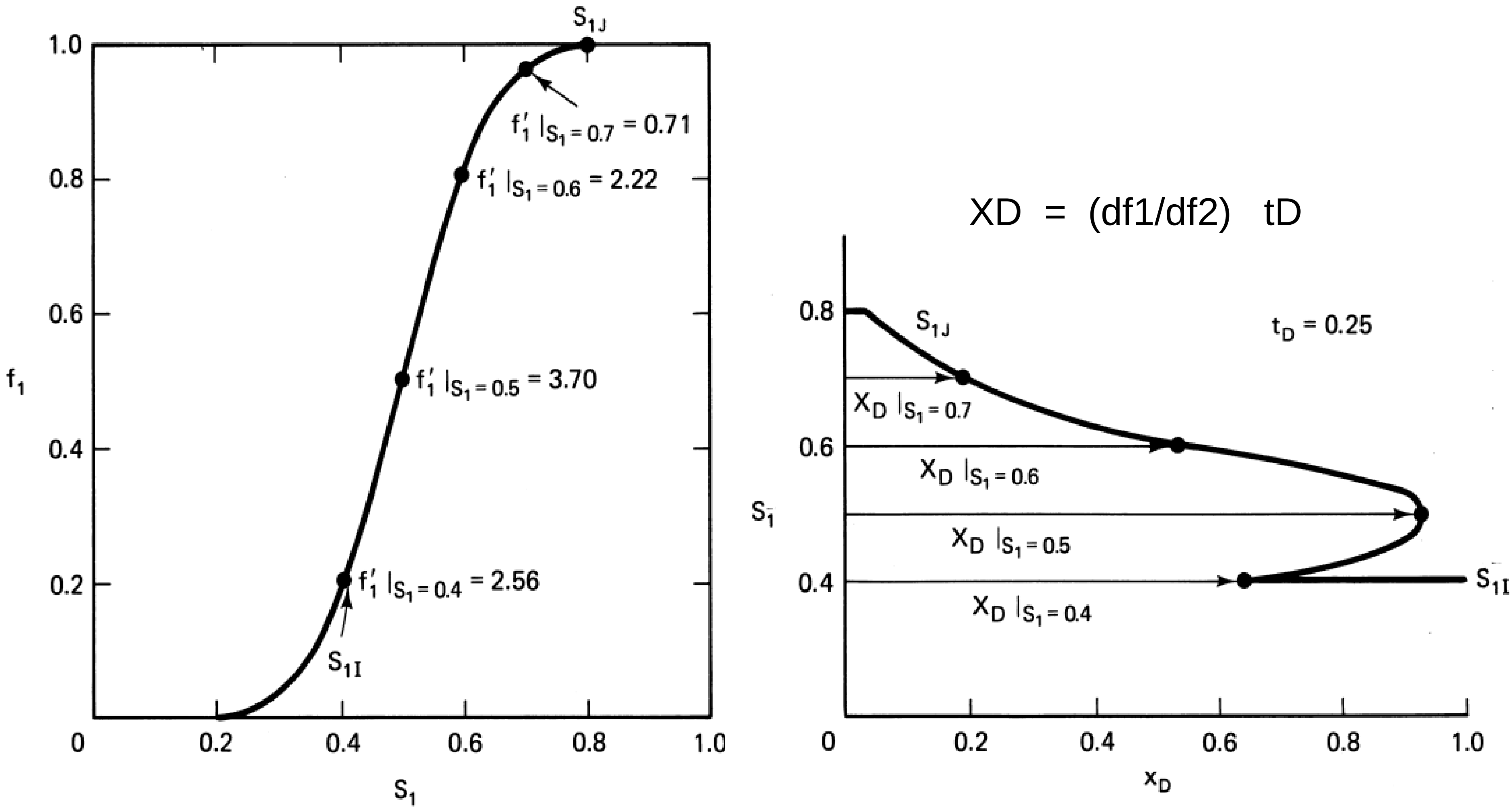
E_D = Displacement (local) sweep efficiency

S_{OI} = Initial oil saturation

V_P = Pore volume

Buckley-Leverett construction of $S_1(x_D, t_D)$

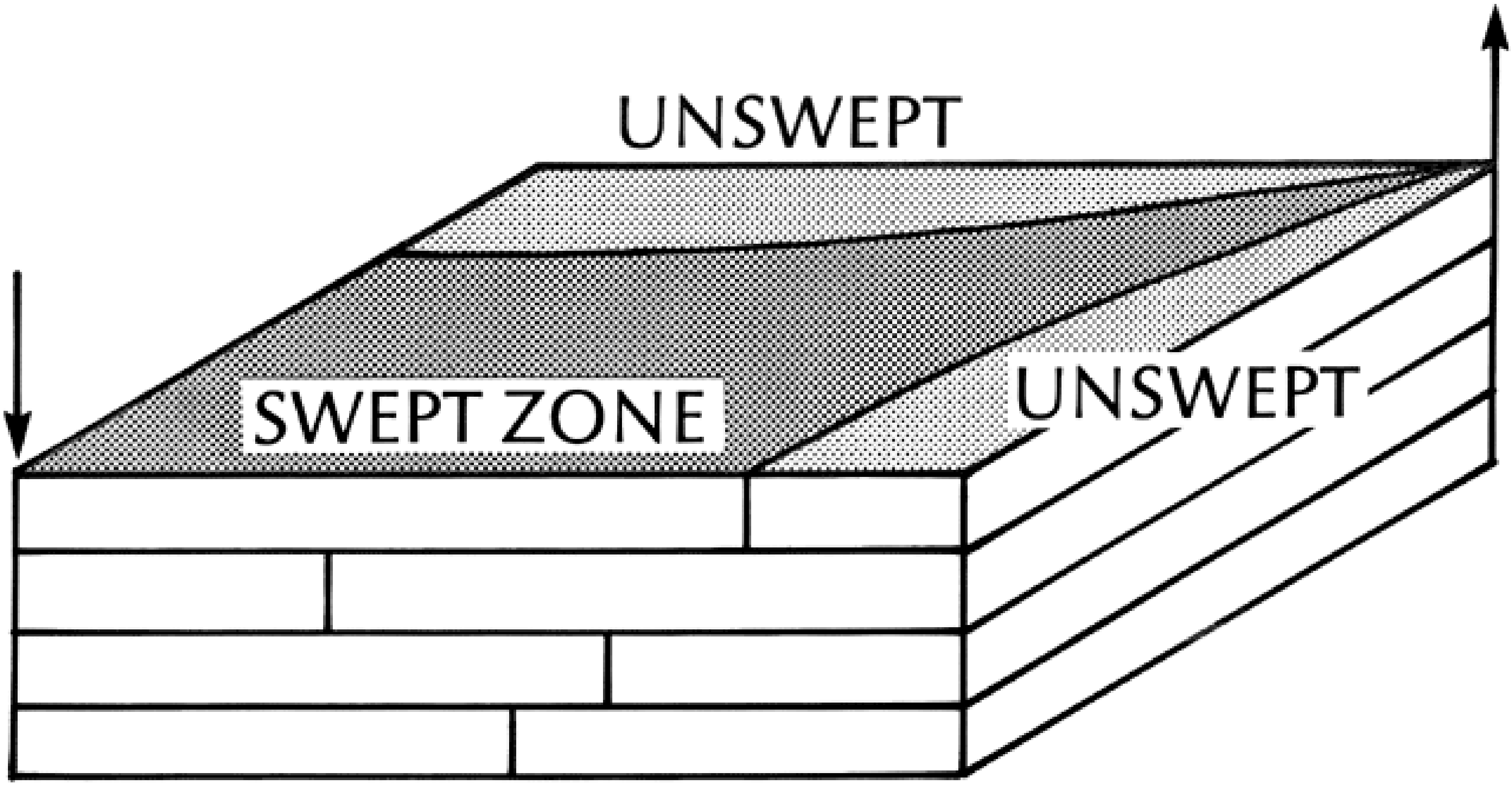
$XD = (df1/df2) \quad tD$

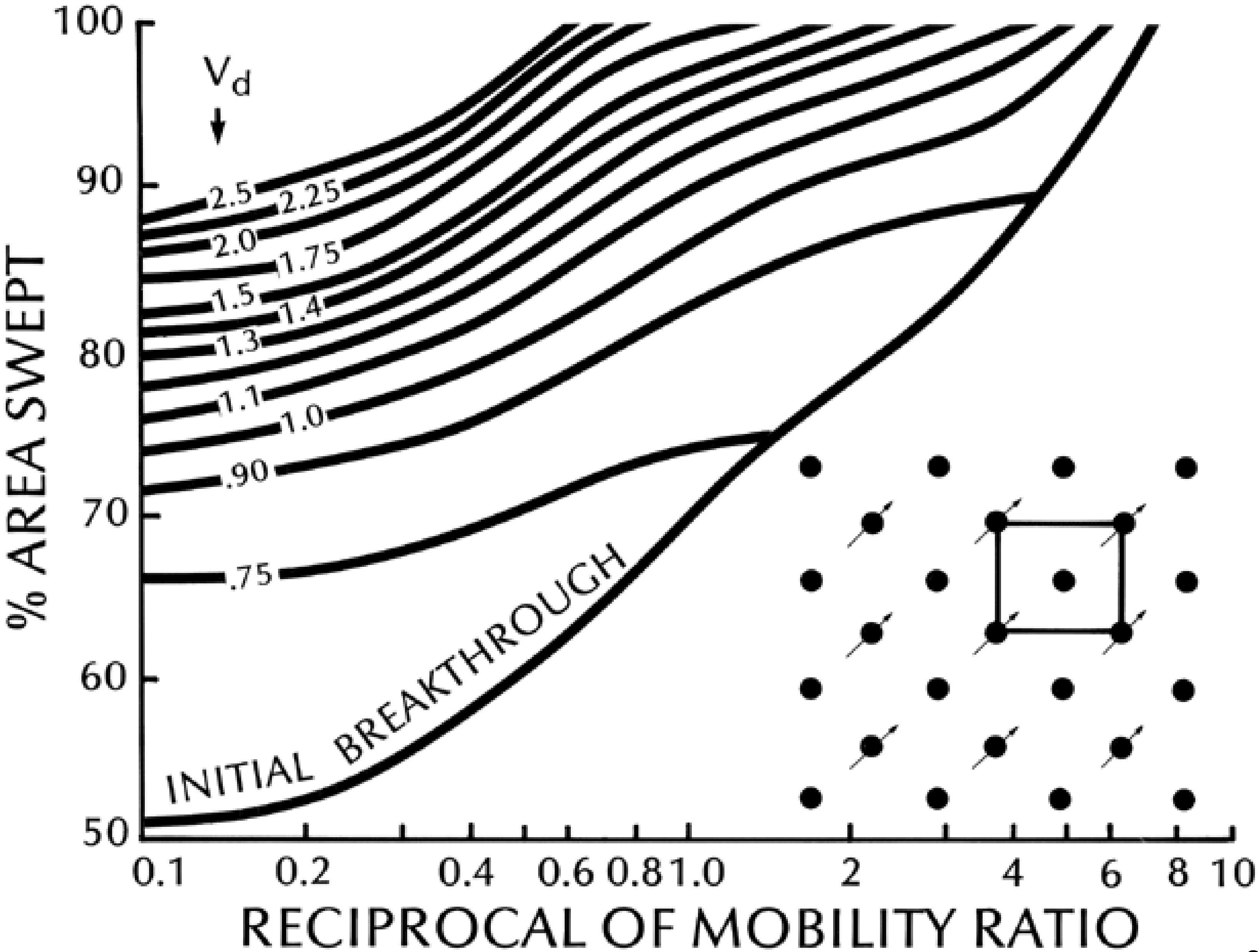


(a) Slopes of a fractional flow curve

(b) Corresponding saturation profile

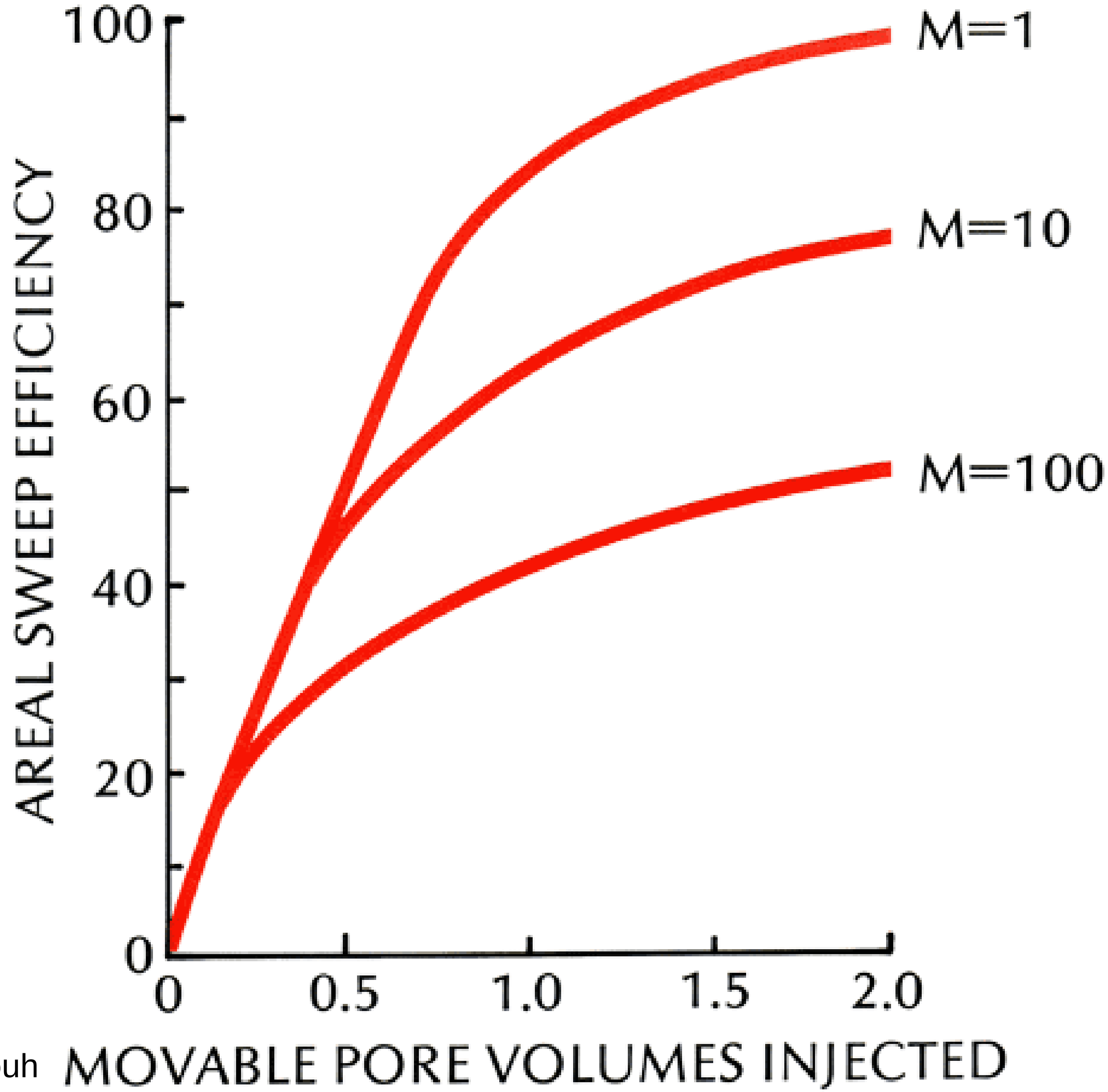
Areal Sweep Efficiency Schematic...

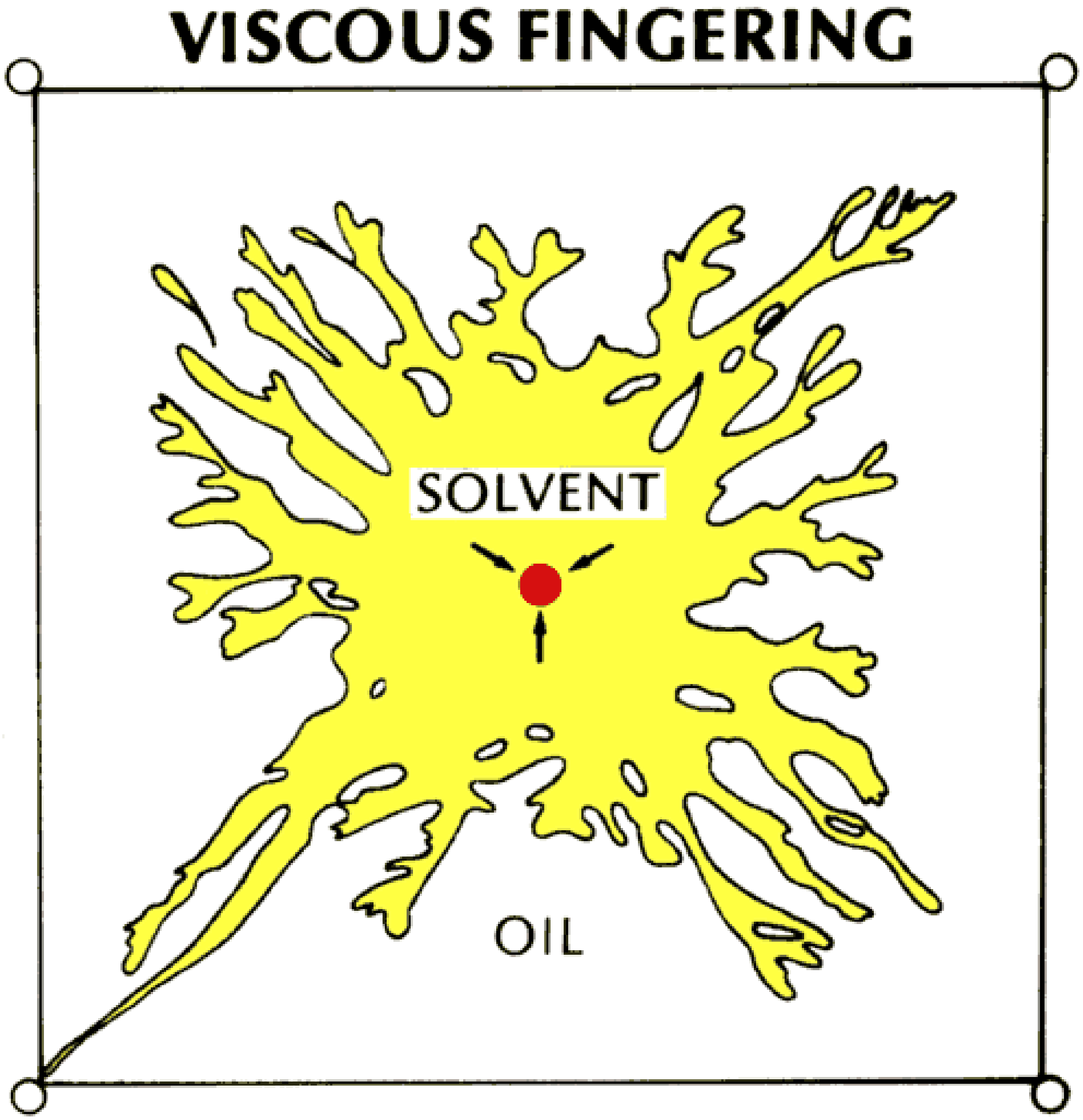




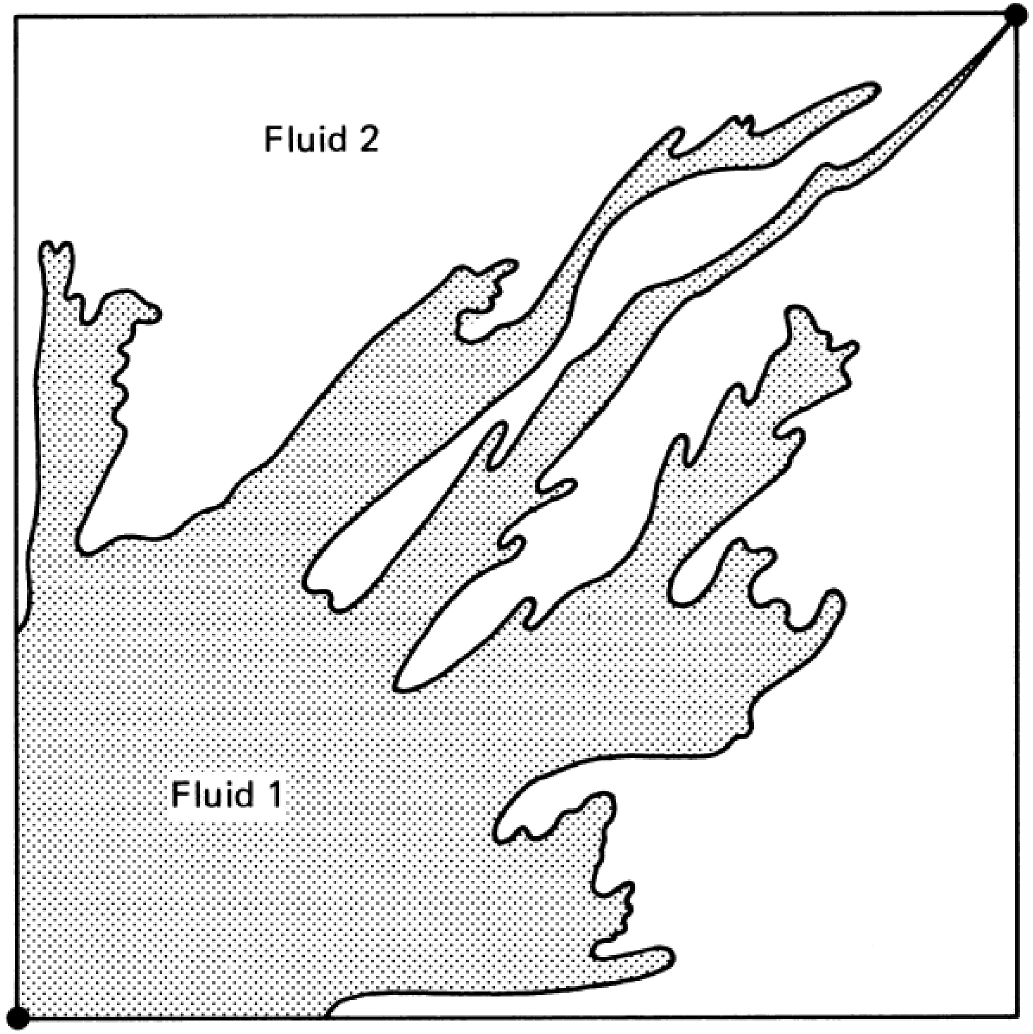
100
90
80
70
60
50
V_d
2.5
2.25
2.0
1.75
1.5
1.4
1.3
1.1
1.0
.90
.75
INITIAL BREAKTHROUGH
% AREA SWEPT
0.1
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10
RECIPROCAL OF MOBILITY RATIO

Five-Spot Areal Sweep...

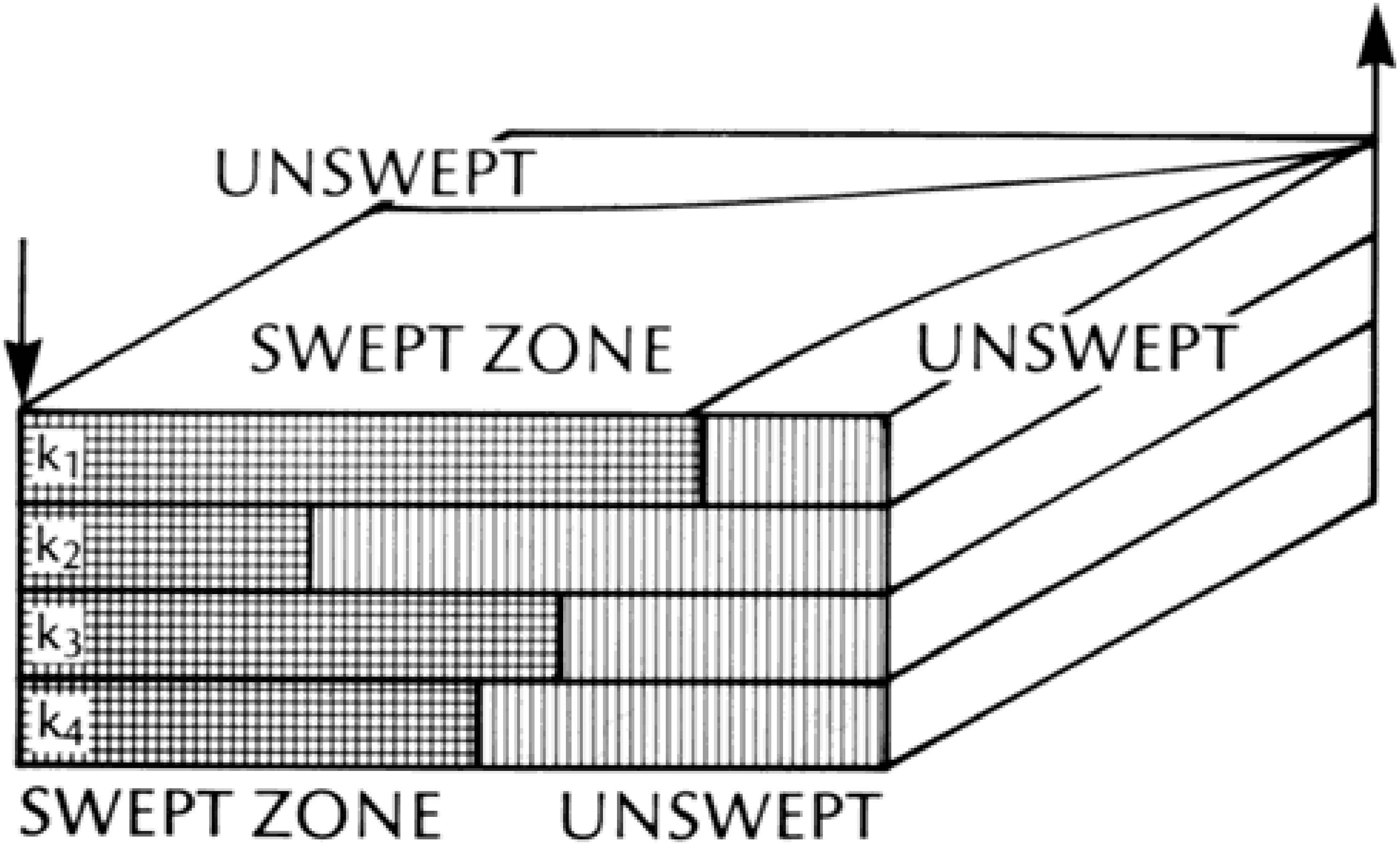



VISCOUS FINGERING
SOLVENT
OIL

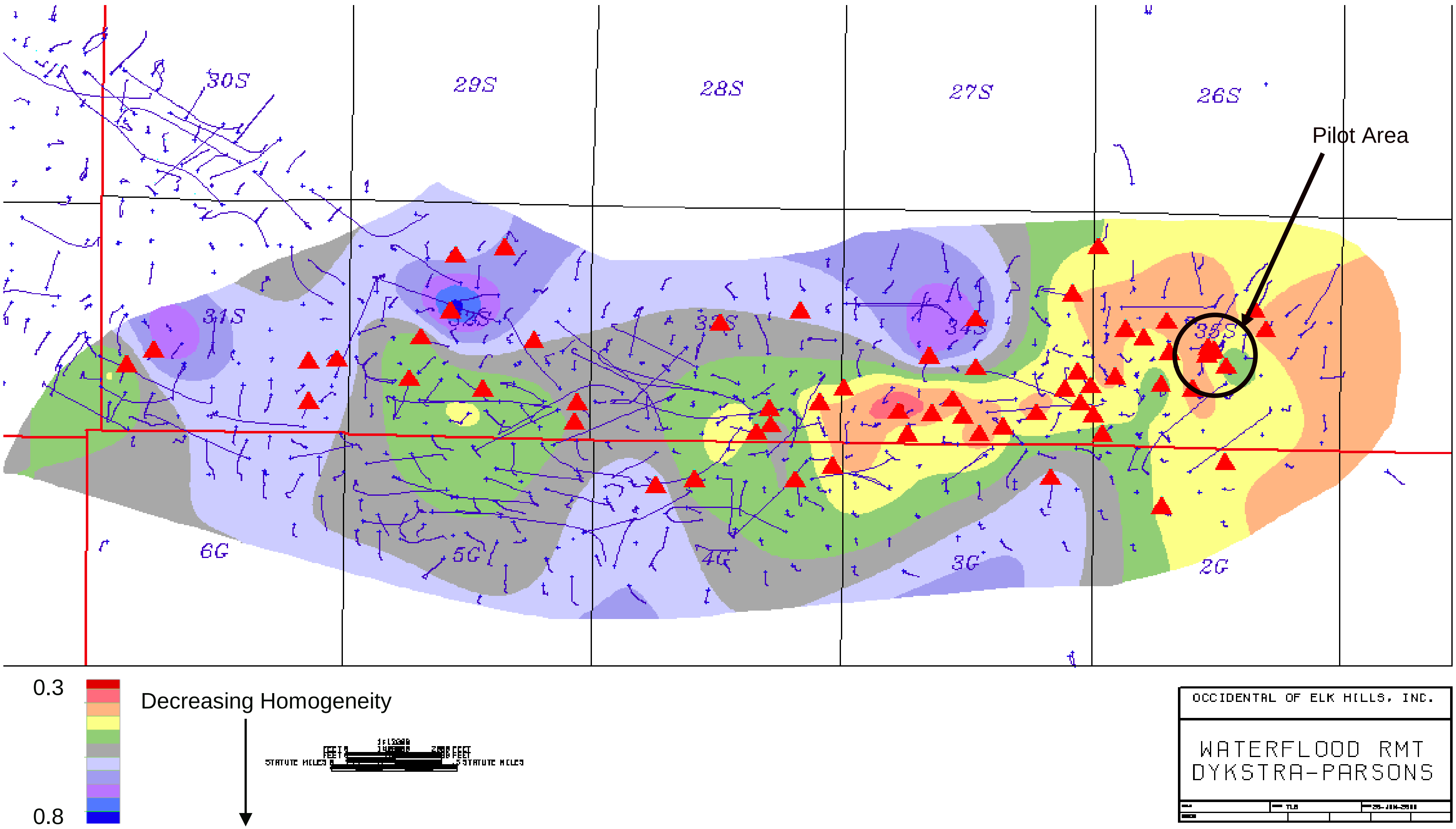
Viscous Fingering in a Quarter Five-spot Model, $M^o = 17$



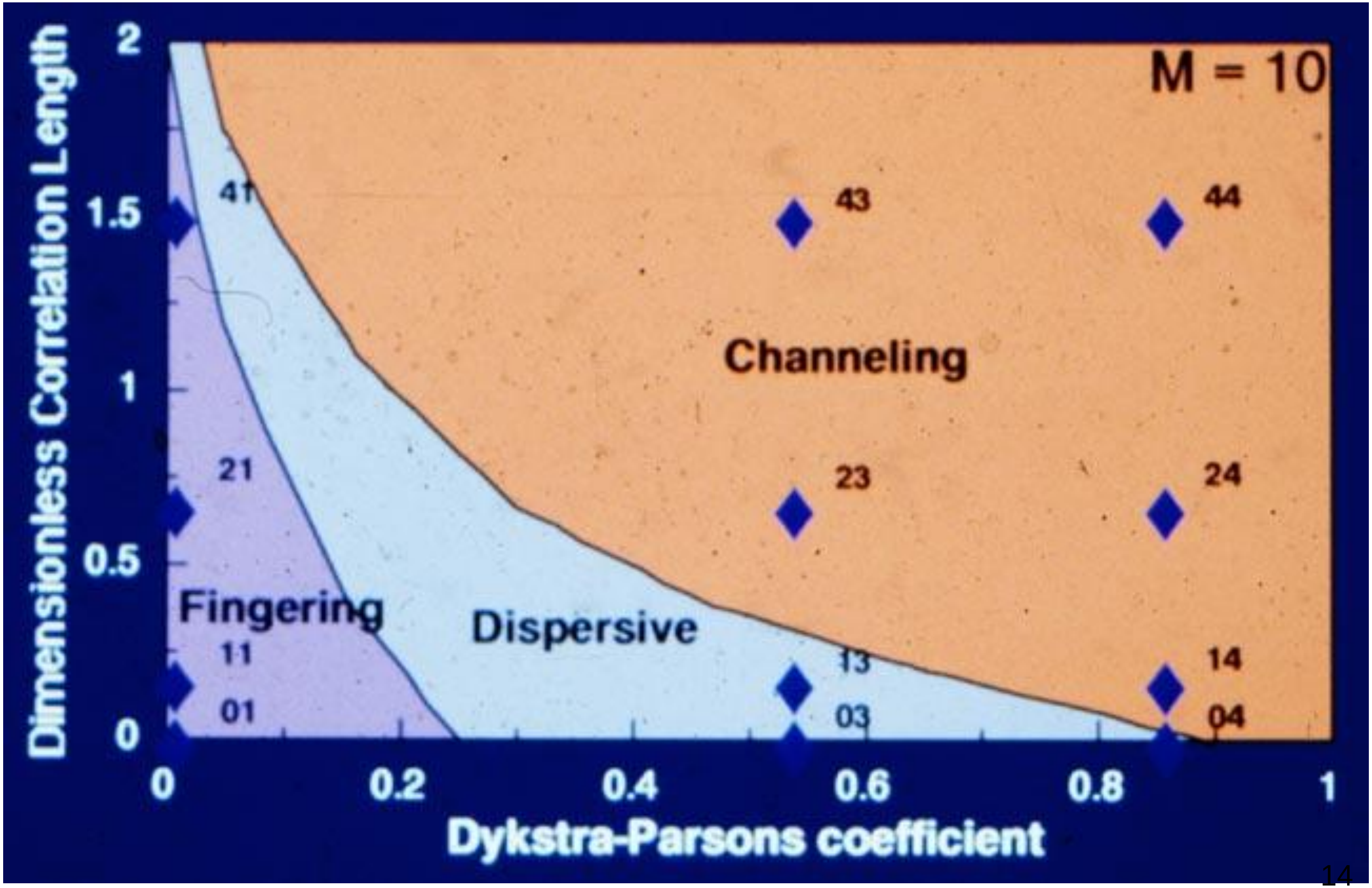
VERTICAL SWEEP EFFICIENCY



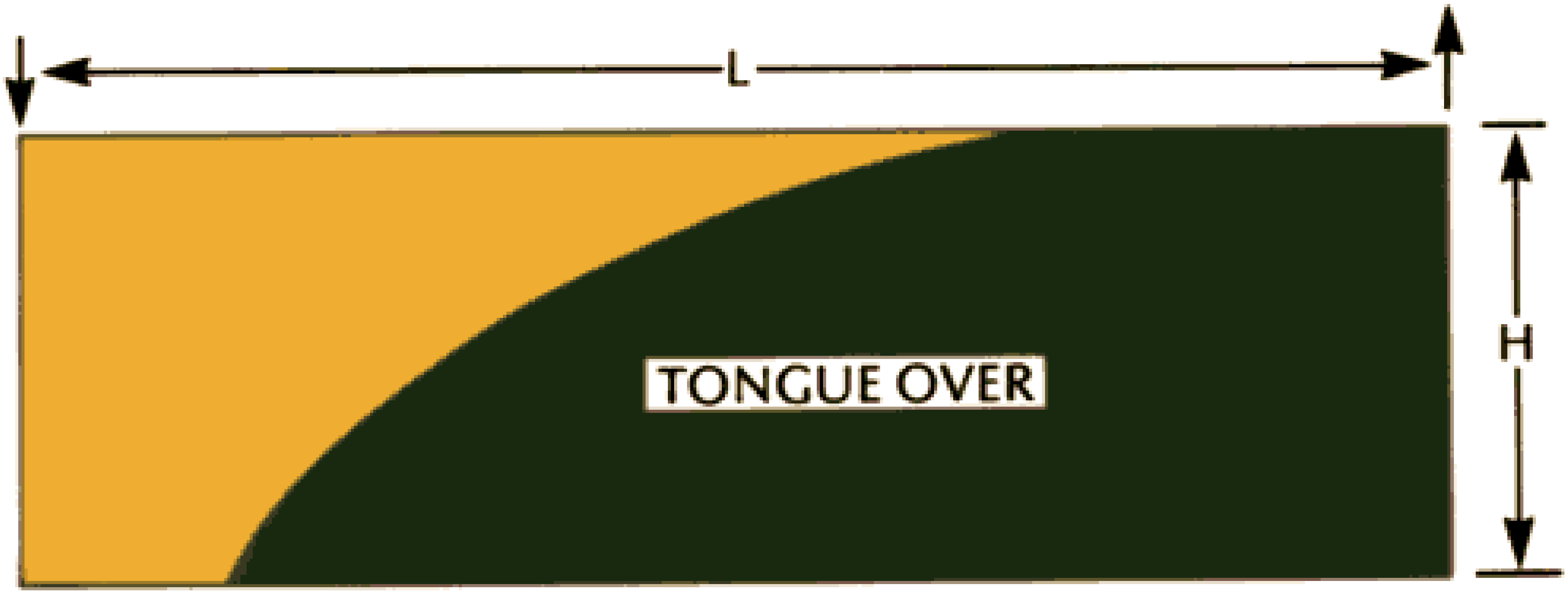
Dykstra Parsons Coefficient Map



Displacement Regime Diagram (M=10)...

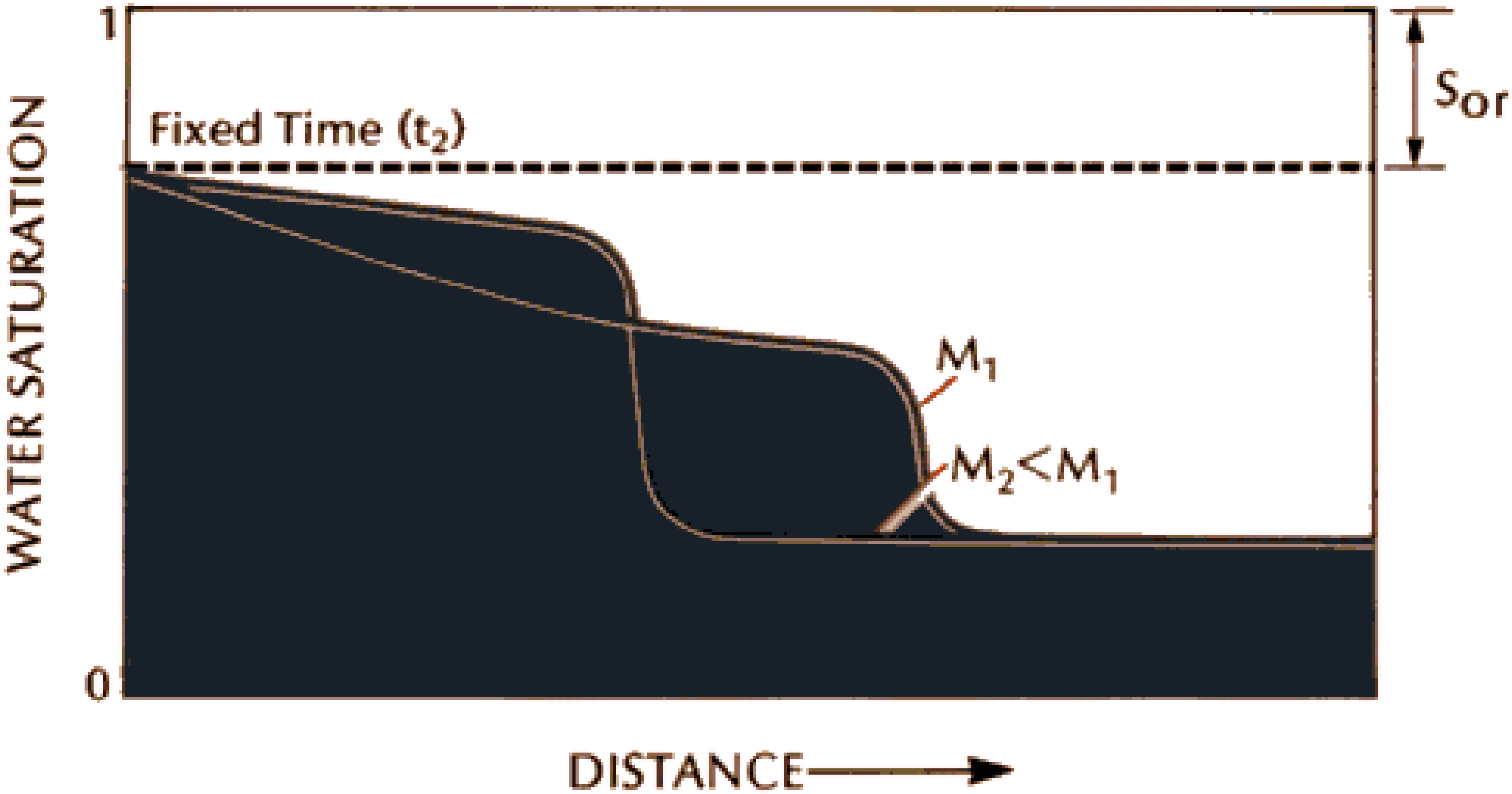


GRAVITY EFFECT



DENSITIES: DISPLACING<DISPLACED
(Gas Displacements)

EFFECT OF MOBILITY RATIO on E_D



Capillary Desaturation Curve...