Multi-Phase Flow Equations

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Introduction

- **Multi-phase flow equations are based on the same principles that govern single-phase flow, except that they must account for interactions between simultaneously flowing phases in porous media.**

- **The main parameters that we use to characterize these interactions are relative permeability's, saturations and solution gas-liquid ratios.**

- **Basically, we obtain the flow equation by substituting Darcy’s law into the continuity equation. For multi-phase systems, we write the continuity equation for each of the phases.**

- **Then we use an appropriate form of Darcy’s law, which accounts for the presence of multiple-fluid flux terms. At the same time, we adjust the phase accumulation term using phase saturations.**

Darcy's law for phase "f"

$$v_{fs} = \left(-\alpha\frac{k_s k_{rf}}{\mu_f}\right)\frac{\partial\Phi}{\partial s} \quad (3.26)$$

Continuity equation for phase "f"

$$-\nabla \cdot (\rho_f v_f) = \frac{\partial}{\partial t}(\phi S_f \rho_f) \quad (3.27)$$

Flow equation for phase "f"

$$\frac{\partial}{\partial x}\left(\frac{A_x k_x k_{rf}}{\mu_f B_f}\frac{\partial\Phi_f}{\partial x}\right)\Delta x + \frac{\partial}{\partial y}\left(\frac{A_y k_y k_{rf}}{\mu_f B_f}\frac{\partial\Phi_f}{\partial y}\right)\Delta y + \frac{\partial}{\partial z}\left(\frac{A_z k_z k_{rf}}{\mu_f B_f}\frac{\partial\Phi_f}{\partial z}\right)\Delta z + q_{fsc} = \frac{V_b}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi S_f}{B_f}\right) \quad (3.28)$$

Note that in Equations 3.26 through 3.28, subscript "f" refers to phase "f," subscript "s" refers to flow direction, k_r is relative permeability and S is the phase saturation. The other terms are as defined previously; however, they are defined for the particular phase (e.g. μ_f is the viscosity of phase "f")

Two-phase (oil-water) equations

- **In reservoirs where two-phase flow of oil and water phases dominates, we need to write Equations for the oil and water phases separately.**

- **We can do this easily by setting the subscript**
  f first to o for oil-phase and then to w for water phase.

The complete set of equations for two-phase oil-water transport problems, together with the unknowns to be solved, is summarized below:

- **Oil flow equation**

$$\frac{\partial}{\partial x}\left(\frac{Axkxkro}{\mu oBo}\frac{\partial Po}{\partial x}\right)\Delta x+\frac{\partial}{\partial y}\left(\frac{Aykykro}{\mu oBo}\frac{\partial Po}{\partial y}\right)\Delta y+$$

$$\frac{\partial}{\partial z}\left(\frac{Azkzkro}{\mu oBo}\frac{\partial Po}{\partial z}\right)\Delta z+qosc=\frac{Vb}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi So}{Bo}\right)$$

$$=\frac{\phi Vb}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi So}{Bo}\right)$$

Water flow equation

$$\frac{\partial}{\partial x}\left(\frac{Axkxkrw}{\mu wBw}\frac{\partial Pw}{\partial x}\right)\Delta x + \frac{\partial}{\partial y}\left(\frac{Aykykrw}{\mu wBw}\frac{\partial Pw}{\partial y}\right)\Delta y +$$

$$\left(\frac{\partial}{\partial z}\left(\frac{Azkzkrw}{\mu wBw}\frac{\partial Pw}{\partial z}\right)\Delta z + qwsc = \frac{Vb}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi Sw}{Bw}\right)\right.$$

$$\left. = \frac{Vb}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi Sw}{Bw}\right)\right)$$

- **In these Equations, there are four unknowns: oil-phase pressure, P_o , water-phase pressure, P_w , oil-phase saturation, S_o , and water-phase saturation, S_w.**

- **To solve the system, we need two more equations.**

- **These equations, called the <u>auxiliary equations</u>, are:**
- **Capillary pressure relationship:**
- **$P_{cow}(S_w) = P_o - P_w$**
- **Saturation relationship:**
- **$S_o + S_w = 1$**

Two-phase (oil-gas) equations

- **In a volumetric reservoir, there are three phases present (oil, water and gas); at irreducible water saturation, the dominant flow is oil and gas.**

- **In representing flow in this kind of reservoir, we must account for both the free gas and the gas dissolved in the oil phase.**

Oil Flow Equation

$$\frac{\partial}{\partial x}\left(\frac{Axkxkro}{\mu oBo}\frac{\partial Po}{\partial x}\right)\Delta x+\frac{\partial}{\partial y}\left(\frac{Aykykro}{\mu oBo}\frac{\partial Po}{\partial y}\right)\Delta y+$$

$$\frac{\partial}{\partial z}\left(\frac{Azkzkro}{\mu oBo}\frac{\partial Po}{\partial z}\right)\Delta z+qosc=\frac{Vb}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi So}{Bo}\right)$$

$$=\frac{\phi Vb}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi So}{Bo}\right)$$

Gas Flow Equation

$$\frac{\partial}{\partial x}\left(\frac{A_x k_x k_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial x} + R_{so}\frac{A_x k_x k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)\Delta x$$

$$+\frac{\partial}{\partial y}\left(\frac{A_y k_y k_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial y} + R_{so}\frac{A_y k_y k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial y}\right)\Delta y +$$

$$\frac{\partial}{\partial z}\left(\frac{A_z k_z k_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial z} + R_{so}\frac{A_z k_z k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial z}\right)\Delta z$$

$$+\left[q_{gsc} + R_{so} q_{osc}\right] = \frac{V_b}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g} + R_{so}\frac{\phi S_o}{B_o}\right)$$

- **In the flow terms of gas Equation, the second term in each bracket represents the contribution from the gas dissolved in oil.**
- **Similarly, qo represents the free gas produced (injected), while the product Rsqo represents the dissolved gas produced along with oil. The second term under the temporal derivative represents the accumulation (depletion) of gas dissolved in oil.**
- **The auxiliary equations necessary to complete this formulation are the capillary pressure and saturation relationships.**
- **Pcog (So) = Po - Pg**
- **So + Sg = 1**

Three-phase (oil-water-gas) equations

The widest application of reservoir simulation is for three-phase oil-water-gas systems, in which all three phases are active in the production process. For this class of problems, we need to write Equation for each of the three phases

Oil flow equation

$$\frac{\partial}{\partial x}\left(\frac{A_x k_x k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)\Delta x+\frac{\partial}{\partial y}\left(\frac{A_y k_y k_{ro}}{\mu_o B_o}\frac{\partial Po}{\partial y}\right)\Delta y+$$

$$\frac{\partial}{\partial z}\left(\frac{A_z k_z k_{ro}}{\mu_o B_o}\frac{\partial Po}{\partial z}\right)\Delta z+q_{osc}=\frac{V_b}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right)$$

Water flow equation

$$\frac{\partial}{\partial x}\left(\frac{A_x k_x k_{rw}}{\mu_w B_w}\frac{\partial P_w}{\partial x}\right)\Delta x+\frac{\partial}{\partial y}\left(\frac{A_y k_y k_{rw}}{\mu_w B_w}\frac{\partial P_w}{\partial y}\right)\Delta y+$$

$$\frac{\partial}{\partial z}\left(\frac{A_z k_z k_{rw}}{\mu_w B_w}\frac{\partial P_w}{\partial z}\right)\Delta z+q_{wsc}=\frac{V_b}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right)$$

Gas flow equation (ignoring the solubility of gas in water)

$$\frac{\partial}{\partial x}\left(\frac{A_x k_x k_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial x}+R_{so}\frac{A_x k_x k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial x}\right)\Delta x$$

$$+\frac{\partial}{\partial y}\left(\frac{A_y k_y k_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial y}+R_{so}\frac{A_y k_y k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial y}\right)\Delta y +$$

$$\frac{\partial}{\partial z}\left(\frac{A_z k_z k_{rg}}{\mu_g B_g}\frac{\partial P_g}{\partial z}+R_{so}\frac{A_z k_z k_{ro}}{\mu_o B_o}\frac{\partial P_o}{\partial z}\right)\Delta z$$

$$+\left[q_{gsc}+R_{so}q_{osc}\right]=\frac{V_b}{\alpha}\frac{\partial}{\partial t}\left(\frac{\phi S_g}{B_g}+R_{so}\frac{\phi S_o}{B_o}\right)$$

- **Auxiliary equations:**
- Pcow (Sw) = Po – Pw
- Pcgo (So) = Pg – Po
- So + Sw + Sg = 1

Isothermal compositional formulation

- **The strategy for formulating the governing equations of isothermal compositional systems is the same as for the black-oil model, except the focus is on each component within the control volume rather than the bulk phase.**

- **In modern computations, we often obtain these PVT data by using an appropriate equation of state.**
- **The common ones used in the petroleum industry include the Peng-Robinson equation of state (or any of its modifications).**

- **Isothermal flow is a special case where existing temperature gradients within a given reservoir are negligible.**

- **Therefore, in isothermal compositional formulation, temperature distribution does not come into play.**

- **Hence, all the phase behavior and transport calculations are performed under isothermal conditions.**

- **For an N-component system, we can write N equations and will have N equations in (3N+6) unknowns.**

- **The following Table summarizes the auxiliary equations (2N+6) needed.**

Source of Equations		Number of Unknowns
$\sum x_{io} = 1$	Conservation of each component	3
$\sum x_{ia} = 1$		
$\sum x_{iv} = 1$		
$S_0 + S_a + S_v = 1$	Saturation relationship	1
$P_{coa}(S_a) = P_o - P_a$	Capillary pressure relationships	2
$P_{cvo}(S_a + S_v) = P_v - P_o$		
$\frac{x_{iv}}{x_{io}} = k_{ivo}; i = 1, 2, \ldots, N$	Equilibrium relationships	2N
$\frac{x_{iv}}{x_{ia}} = k_{iva}; i = 1, 2, \ldots, N$		
Total Number of Auxiliary Equations		2N+6

The above 2N+6 equations, when combined with the N equations arising from molar balances (Equation 3.47) provide the 3N+6 equations needed to close the system.