Numerical Models

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Two techniques for solving mathematical reservoir models:

analytical* and *numerical

Each of these has certain strengths and limitations.

- **Analytical techniques offer the advantage of providing *exact* solution**

- **Analytical methods fall short when we start dealing with varying formation thickness, non-uniform porosity and permeability, and changing fluid properties, and other such conditions that describe most real reservoirs.**

The net result is

exact solution to an approximate problem.

- **Numerical solutions** involve discretizing, or approximating the mathematical model.

- **Using a numerical tool such that continuous forms of the partial differential equations are written in a discrete form.**

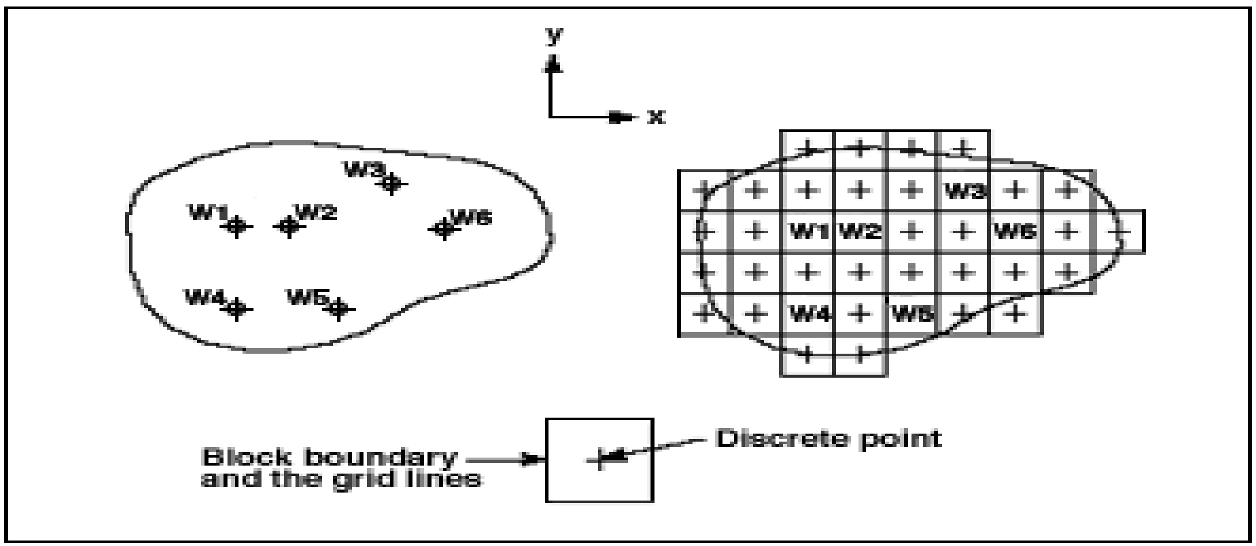
- **The clear advantage of the numerical approach is that it allows us to assign representative properties to as many parts of a system as we have information for.**

- **We must not forget that we inevitably lose some measure of accuracy in discretizing the partial differential equations.**

- **The net result is that: *approximate solution to an exact problem.***

- In *body-centered* (or block-centered) grids, the discrete points are located at the center of each cell.

- There are no discrete points located on the reservoir boundaries



Multi-well reservoire and the imposed body-centered grid (38 active blocks)

- **In setting up the grid system, we must be especially careful in placing the grid with respect to the wells.**

- **First, the well should be located at the center of its host grid block.**

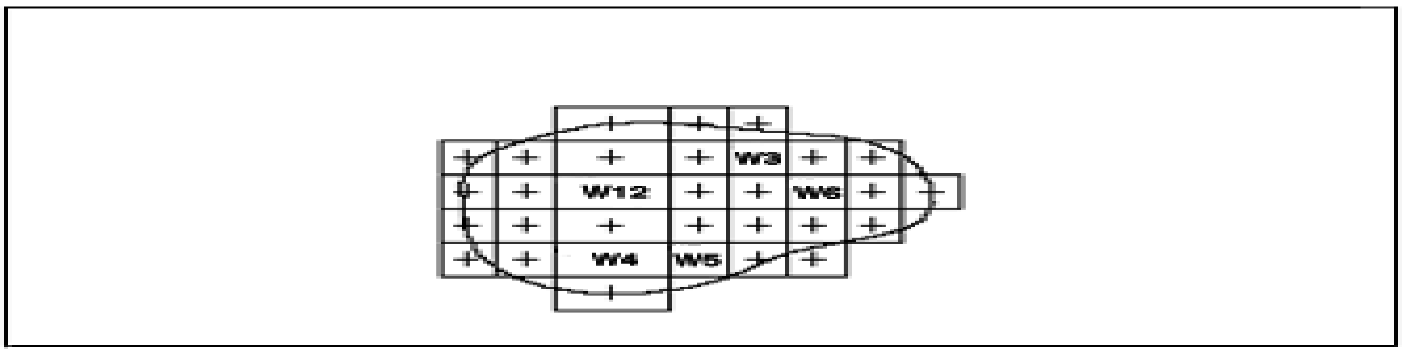
- **Second, each grid block should have no more than one well within the same grid block.**

- **If it is difficult to locate the wells such that they coincide with the grid center, we can shift them slightly to satisfy this condition.**

- **If we cannot avoid having more than one well in the same grid block, we can mathematically replace them with one well of equivalent strength.**

- Two wells are combined into one equivalent well (wells W1 and W2) located in the center of a new block.

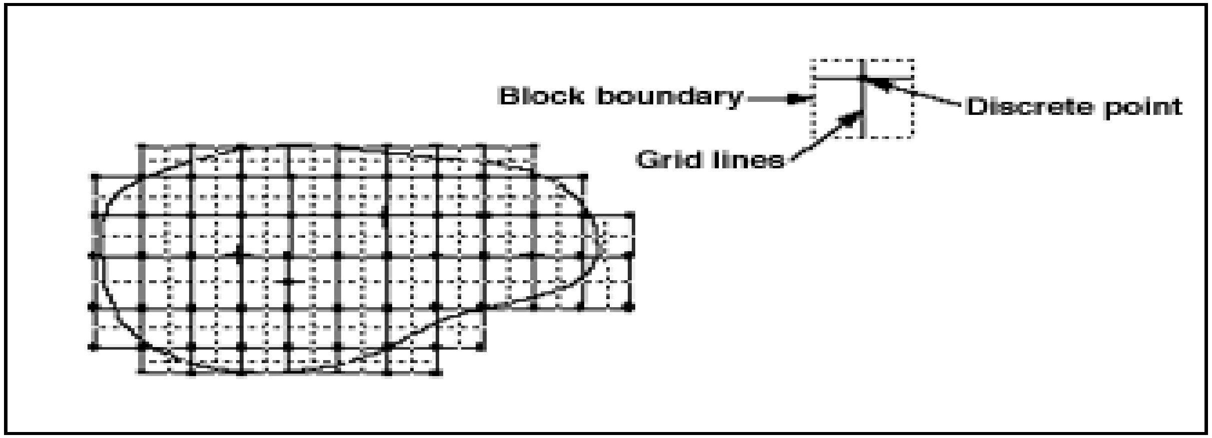
- The number of blocks along the x- and y-directions is decreased by one, which results in an overall decrease of 6 blocks.



Combining wells in a body-centered grid (32 active blocks)

- **In *mesh-centered* (or point-distributed) grids, the discrete points are located at the grid line *intersections* .**

- **In contrast to body-centered grids, there are discrete points located on the reservoir boundaries**



Multi-well reservoir and the imposed mesh-centered grid, showing a typical block

- **We must always be aware that the quality of output depends on the quality of input.**

- **Each block added to the system demands information in terms of accurate property representation.**

- **In some reservoir studies, the quality and detail of the information we need may require us to use much finer grid blocks. A case in point is the simulation of an *EOR process*, where we need to track a fluid front in order to better control the outcome.**

- **In *steam flooding or miscible flooding*, where changes are abrupt and the interactions between the phases are strong, a fine grid system will help capture the details of the changes taking place.**

- **Representing *geological complexity* with a grid system sometimes requires the use of a large number of grid blocks.**
- **It is not possible to construct a grid system for a lenticular sand with several pinch-outs without imposing finer grids that capture peculiar features of this system.**

- **The number of wells in the reservoir plays a dominant role in grid size selection.**

- **The presence of wells in the reservoir further complicates an already complex system, since most of the significant activities and rapid changes that take place in the reservoir occur in the near wellbore region.**

- **As a rule, the more wells we have in a reservoir, the more grid blocks we need.**

Approximating a partial differential equation using its finite-difference analogue introduces certain errors.

The magnitude of these errors and the stability of the numerical algorithm depend on the grid size.

Finer grids **produce more accurate results**

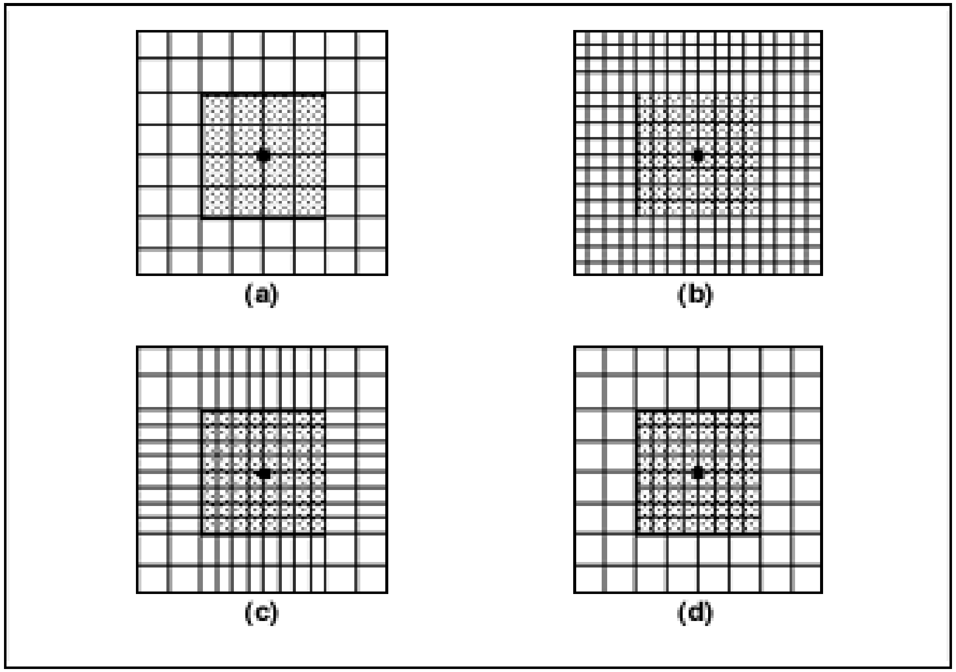
Local grid refinement

- **A good compromise—and a way to avoid placing blocks where they are not needed—is to refine the grids around the locality where we require more detailed information, and/or where rapid changes occur.**

- *local grid refinement.*

Part (b) shows *the global grid refinement.*

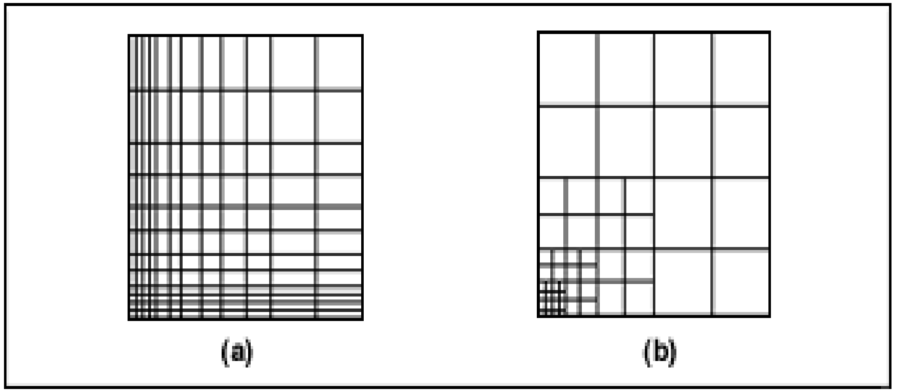
- Part (a) shows a coarse grid composed of 81mesh-centered grid points.
- The shaded area around the well is the section from which we need detailed information.



Part (c) depicts *the conventional approach.*

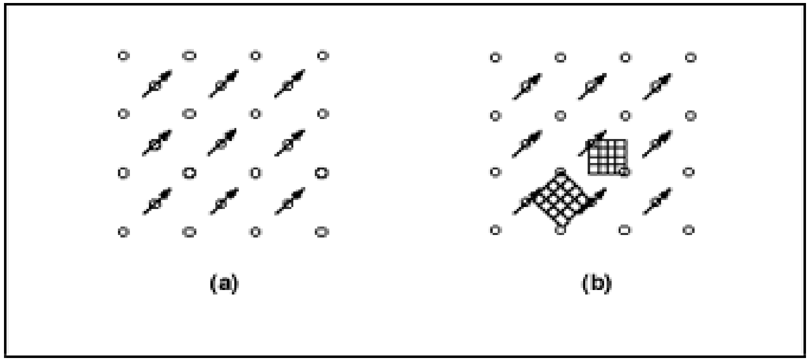
Part (d) illustrates ***the best solution.***

- <u>In this scheme</u>, the level of refinement increases as we move toward the well, where the most rapid changes take place.
- A total of four levels of refinement are shown in conventional local refinement and true local refinement schemes -Parts (a) and (b)



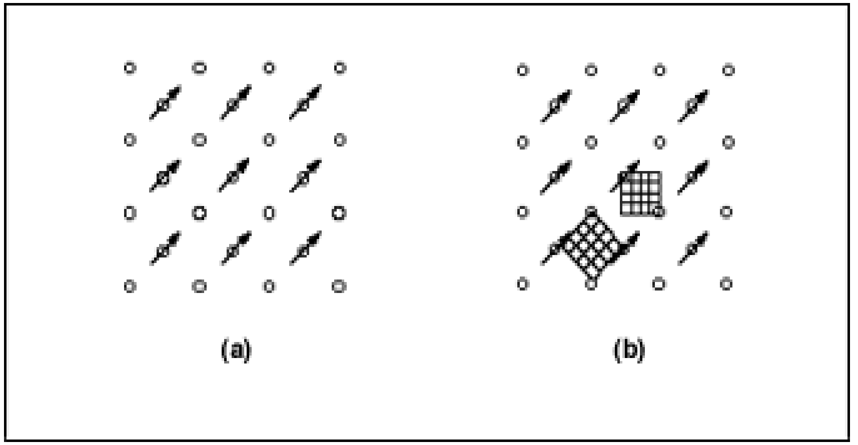
Grid orientation

- **The placement of the axes.**
- **In orienting the grid, we have to consider two factors:**
- **Permeability anisotropy, if any**
- **Co-ordinate orthogonality**



- **Align the coordinate axes with the principal flow directions.**

- **We must ensure that we preserve the coordinate system's orthogonality.**



Time step selection

- **We can use uniform time steps in numerical modeling, but it is neither necessary nor warranted by the nature of the reservoir problem being solved.**

- **More often than not, the changes are more rapid initially, thus requiring us to use finer time step sizes in order to capture the essence of these changes.**

- **Most modern simulators implement automatic time control strategies, which are generally known as *automatic time step size selectors*.**

- **These selectors operate based on changes in pressure and saturation over the previous time step.**

- **As time step size progressively increases, it is common for material balance errors to appear.**

- **Errors in material balance reflect the loss of accuracy that results from using coarse time step sizes to calculate pressure and saturation distributions.**

- **The material balance checks help the engineer to determine the maximum time step size that is admissible by the particular problem.**