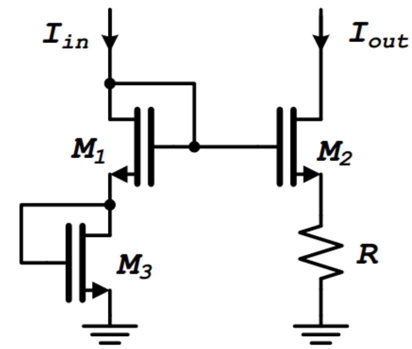
Faculty of Engineering – Cairo University
Electronics and Electrical Communications Department
EECE2020 – Electronics II
Problem Set # 2

$\boldsymbol{\lambda = 0.1, K'_n = \mu_n C_{ox} = 0.138 mA/V^2}$

Problem 1: Using the circuit on the right, answer the following parts keeping all answers in terms of the device parameters μn, Cox, (W/L), Vth. You may ignore channel length modulation.



a) Derive a large signal expression relating Iout to Iin in terms of device parameters and bias voltages (Vgs).

b) Now assume that the (W/L) ratio of M3 is very large so that $\sqrt{2I_{D3}/\mu_n C_{ox}\left(\frac{W}{L}\right)_3} \approx 0$, also assume that nominally $V_{GS1} \approx V_{GS2}$. What is the approximate value of Iout In terms of R?

Solution:

a)

Assuming all devices are in SAT region

$$I_{in} = \frac{1}{2}K_{n1}(V_{GS1} - V_{TH1})^2 = \frac{1}{2}K_{n3}(V_{GS3} - V_{TH3})^2$$

$$I_{out} = \frac{1}{2}K_{n2}(V_{GS2} - V_{TH2})^2$$

$$V_{GS2} + I_{out}R = V_{GS1} + V_{GS3}$$

$$V_{GS2} + I_{out}R = V_{TH1} + \sqrt{\frac{2I_{in}}{K_{n1}}} + V_{TH3} + \sqrt{\frac{2I_{in}}{K_{n3}}}$$

$$I_{out} = \frac{\sqrt{\frac{2I_{in}}{K_{n1}}} + \sqrt{\frac{2I_{in}}{K_{n3}}} + V_{TH1} + V_{TH3} - V_{GS2}}{R}$$

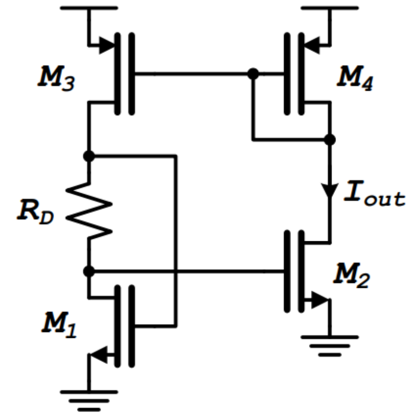
b)

If M3 is large, $\sqrt{\frac{2I_{in}}{K_{n3}}} \approx 0$

Assuming $V_{GS1} = V_{GS2} = V_{TH1} + \sqrt{\frac{2I_{in}}{K_{n1}}}$

$$I_{out} = \frac{V_{TH3}}{R}$$

Problem 2: Use the circuit shown on the right. Neglect L modulation and assume that (W/L)3 = (W/L)4. Derive a large-signal expression for the current Iout in terms of the (W/L)1, (W/L)2, RD, and device parameters. Assume Vth1 = Vth2.



Solution:

M3 and M4 → forms a current mirror.

$I_{D3} = I_{D4} = I_{out}$

Assume $V_{TH1} = V_{TH2} = V_{TH}$

$$I_{out} = \frac{1}{2}K_{n2}(V_{GS2} - V_{TH})^2 = \frac{1}{2}K_{n1}(V_{GS1} - V_{TH})^2$$

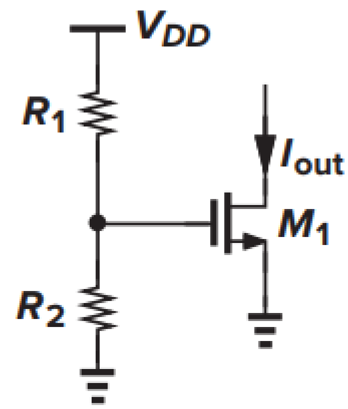
$V_{GS1} = I_{out}R_D + V_{GS2}$

$$I_{out}R_D = V_{GS1} - V_{GS2} = \sqrt{\frac{2I_{out}}{K_{n1}}} + V_{TH} - \sqrt{\frac{2I_{out}}{K_{n2}}} - V_{TH} = \sqrt{\frac{2I_{out}}{\mu_n C_{ox}}}\left[\sqrt{\frac{1}{\left(\frac{W}{L}\right)_1}} - \sqrt{\frac{1}{\left(\frac{W}{L}\right)_1}}\right]$$

$$I_{out}^2 R_D^2 = \frac{2I_{out}}{\mu_n C_{ox}}\left[\frac{1}{\left(\frac{W}{L}\right)_1} + \frac{1}{\left(\frac{W}{L}\right)_2} - 2\sqrt{\frac{1}{\left(\frac{W}{L}\right)_1\left(\frac{W}{L}\right)_2}}\right]$$

$$I_{out} = \frac{2}{\mu_n C_{ox} R_D^2}\left[\frac{1}{\left(\frac{W}{L}\right)_1} + \frac{1}{\left(\frac{W}{L}\right)_2} - 2\sqrt{\frac{1}{\left(\frac{W}{L}\right)_1\left(\frac{W}{L}\right)_2}}\right]$$

Razavi - Problem 5.1: In the circuit shown on the right, assume that $(W/L)_1 = 50/0.5$, $\lambda = 0$, Iout = 0.5 mA, and M1 is saturated.



a) Determine R2/R1.
b) Calculate the sensitivity of Iout to VDD, defined as ∂ Iout/∂VDD and normalized to Iout.
c) How much does Iout change if VTH changes by 50 mV?
d) If the temperature dependence of μn is expressed as $\mu n \propto T^{-3/2}$ but VTH is independent of temperature, how much does Iout vary if T changes from 300°K to 370°K?
e) What is the worst-case change in Iout if VDD changes by 10%, VTH changes by 50 mV, and T changes from 300°K to 370°K?

Solution:

a)

$$V_{GS1} = V_{DD}\frac{R_2}{R_1 + R_2} = V_{TH} + \sqrt{\frac{2I}{K_n'\frac{W}{L}}}$$

$$\frac{R_2}{R_1} = \frac{\sqrt{\frac{2I}{K_n'\frac{W}{L}}} + V_{TH}}{V_{DD} - \left(\sqrt{\frac{2I}{K_n'\frac{W}{L}}} + V_{TH}\right)} = 0.44$$

b)

$$I_D = \frac{1}{2}K_n'\left(\frac{W}{L}\right)\left(V_{DD}\frac{R_2}{R_1 + R_2} - V_{TH}\right)^2$$

$$Sensitivty = \frac{\frac{\partial I_D}{\partial V_{DD}}}{I_D} = \frac{K_n'\left(\frac{W}{L}\right)\left(V_{DD}\frac{R_2}{R_1 + R_2} - V_{TH}\right)\left(\frac{R_2}{R_1 + R_2}\right)}{\frac{1}{2}K_n'\left(\frac{W}{L}\right)\left(V_{DD}\frac{R_2}{R_1 + R_2} - V_{TH}\right)^2} = \frac{2}{V_{DD} - V_{TH}\left(1 + \frac{R_1}{R_2}\right)} = 2.84$$

c)

$$I_D = \frac{1}{2}K_n'\left(\frac{W}{L}\right)\left(V_{DD}\frac{R_2}{R_1 + R_2} - V_{TH}\right)^2$$

$$\frac{\partial I_D}{\partial V_{TH}} = -K_n'\left(\frac{W}{L}\right)\left(V_{DD}\frac{R_2}{R_1 + R_2} - V_{TH}\right)$$

$$\Delta I_D \approx -K_n'\left(\frac{W}{L}\right)\left(V_{DD}\frac{R_2}{R_1 + R_2} - V_{TH}\right)\Delta V_{TH} = -233\mu A$$

$$\Delta I_D = I_D(V_{TH} = 0.75V) - I_D(V_{TH} = 0.7) = -205\mu A$$

d)

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{DD}\frac{R_2}{R_1+R_2} - V_{TH}\right)^2$$

$$\mu_n = \mu_{no}\left(\frac{T}{T_o}\right)^{-\frac{3}{2}}$$

$$\frac{\partial I_D}{\partial T} = -\frac{3}{2}\left(\frac{T}{T_o}\right)^{-\frac{3}{2}}\left(\frac{1}{T}\right) I_{Do}$$

$$\Delta I_D \approx -\frac{3}{2}\left(\frac{T}{T_o}\right)^{-\frac{3}{2}}\left(\frac{1}{T}\right) I_{Do}\Delta T = -103\mu A$$

$$I_D = I_D(T = 370^o K) - I_D(T = 300^o K) = -135\mu A$$

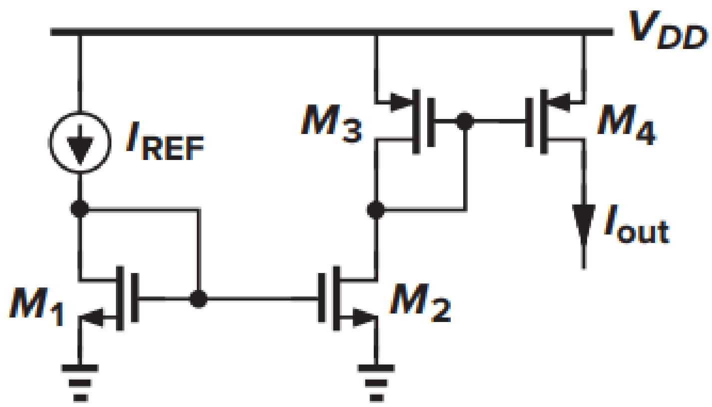
e)

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)\left(V_{DD}\frac{R_2}{R_1+R_2} - V_{TH}\right)^2$$

$$I_{D,W.C.} = \frac{1}{2}\mu_{n_o}\left(\frac{T_o + \Delta T}{T_o}\right)^{-\frac{3}{2}} C_{ox}\left(\frac{W}{L}\right)\left((V_{DD} - \Delta V_{DD})\frac{R_2}{R_1+R_2} - (V_{TH} + \Delta V_{TH})\right)^2 = 43\mu A$$

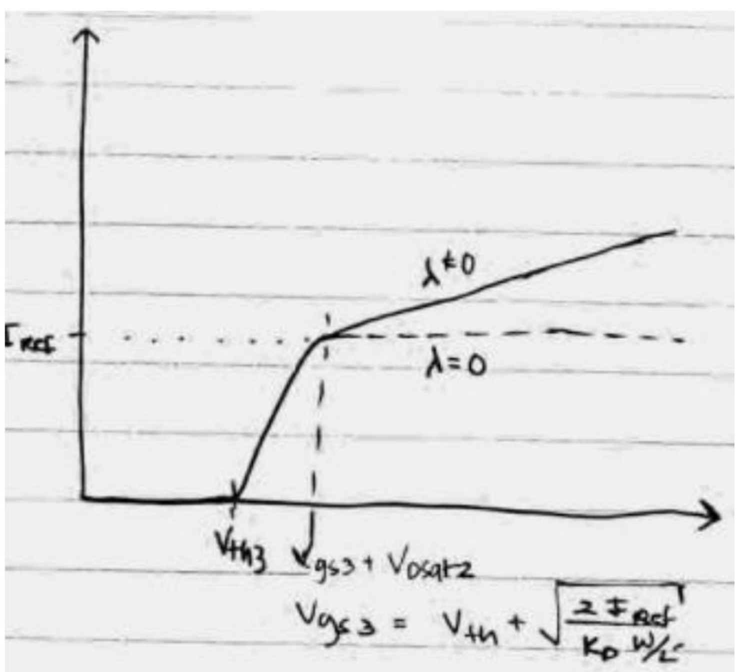
$$\Delta I_{W.C.} = I_{D,W.C.} - I_{Do} = -457\mu A$$

Razavi - Problem 5.2: Consider the circuit shown below. Assuming I_{REF} is ideal, sketch Iout versus VDD as VDD varies from 0 to 3 V.

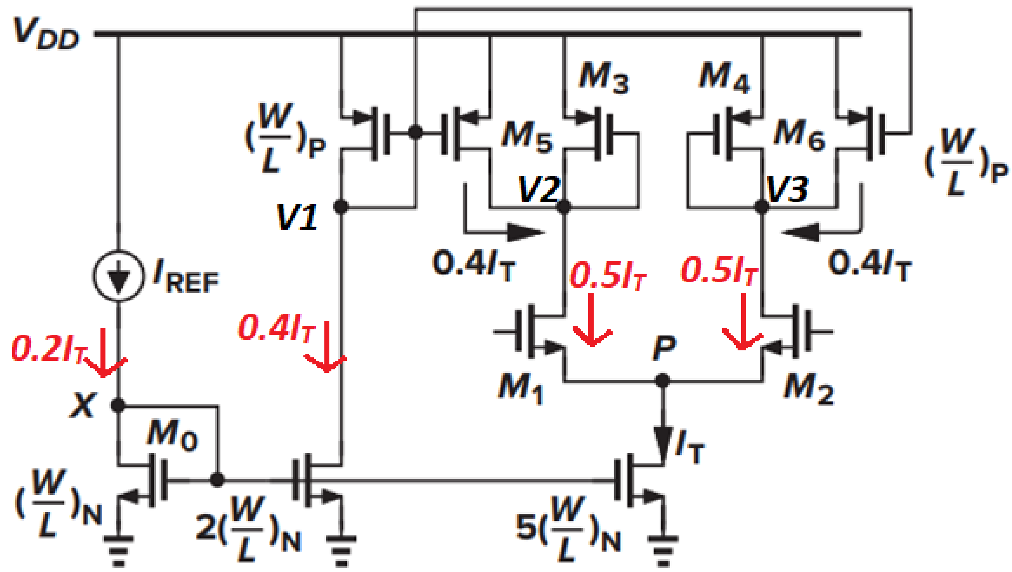


Solution:

- If $V_{DD} < V_{THP}$
  - M3 and M4 are off
  - Iout=0
- At $V_{THP} \leq V_{DD} < V_{SG3,4} + V_{ov2}$
  - M3 and M4 are on and SAT.
  - M3 and M4 act as a current mirror.
  - M2 is in Triode as acts as the Iref for the M3/M4 current mirror.
  - Iout current will increase linearly with VDD.
- As $V_{DD} \geq V_{SG3,4} + V_{ov2}$
  - M2 goes into SAT region
  - Two current mirrors in cascade.
  - Iout=Iref
  - Considering channel length modulation, Iout will increase as VDD increases.



Razavi - Problem 5.3: In the circuit shown below, $(W/L)_N = 10/0.5$, $(W/L)_P = 10/0.5$, and IREF = 100 μA. The input CM level applied to the gates of M1 and M2 is equal to 1.3 V. Assuming $\lambda = 0$, calculate VP and the drain voltage of the PMOS diode-connected transistors.



Solution:

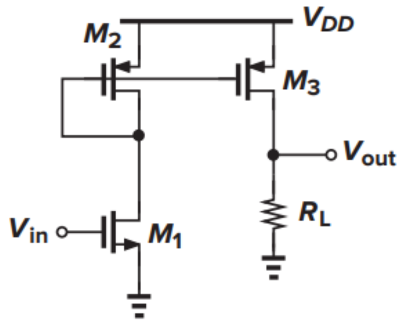
$$I_D = \frac{1}{2} K_n (V_{GS} - V_{TH})^2$$

$$V_1 = V_{DD} - V_{SGP} = V_{DD} - |V_{THP}| - \sqrt{\frac{2 \times 2I_{REF}}{K'_p \left(\frac{W}{L}\right)_p}} = 3 - 0.8 - \sqrt{\frac{0.4}{0.038 \times \left(\frac{10}{0.5}\right)}} = 1.475V$$

$$V_2 = V_3 = V_{DD} - V_{SG2,3} = V_{DD} - |V_{THP}| - \sqrt{\frac{2 \times 0.5I_{REF}}{K'_p \left(\frac{W}{L}\right)_{2,3}}} = 3 - 0.8 - \sqrt{\frac{0.1}{0.038 \times \left(\frac{10}{0.5}\right)}} = 1.837V$$

$$V_P = V_{CM} - V_{GS1} = V_{CM} - V_{THN} - \sqrt{\frac{2 \times 2.5I_{REF}}{K'_p \left(\frac{W}{L}\right)_1}} = 1.3 - 0.7 - \sqrt{\frac{0.5}{0.139 \times \left(\frac{10}{0.5}\right)}} = 0.176V$$

Razavi - Problem 5.4: In the circuit shown below, sketch Vout versus VDD as VDD varies from 0 to 3V.



Solution:

- If $V_{DD} < V_{THP}$
  - M2 is M3 are off
  - I3=0
  - Vout=0
- As $V_{DD} \geq V_{THP}$ but less than $(V_{SG2} + V_{ov1})$
  - M1 is in deep triode region, linearly approaching saturation.
  - M2 and M3 are on and are in SAT region and acting as current mirror.
  - Vout increasing linearly with VDD.

$$V_{out} = I_{D3} \times R_L$$

$$I_{D3} = I_{D2} = I_{D1}$$

$$I_{D1} = K_{n1}(V_{GS1} - V_{THN})V_{DS1} = K_{n1}(V_{in} - V_{THN})(V_{DD} - V_{SG2})$$

$$V_{out} = K_{n1}(V_{in} - V_{THN})(V_{DD} - V_{SG2})R_L$$

$$\frac{\partial V_{out}}{\partial V_{DD}} = K_{n1}(V_{in} - V_{THN})R_L = \frac{R_L}{R_{on}}$$

- When $V_{DD} \geq V_{SG2} + V_{ov1}$
  - All transistors are on and in the SAT region.
  - As V_{DD} increases, V_{DS1} increases and hence I_{D1} increases.
  - Vout increases linearly with V_{DD} with a slope of λ.

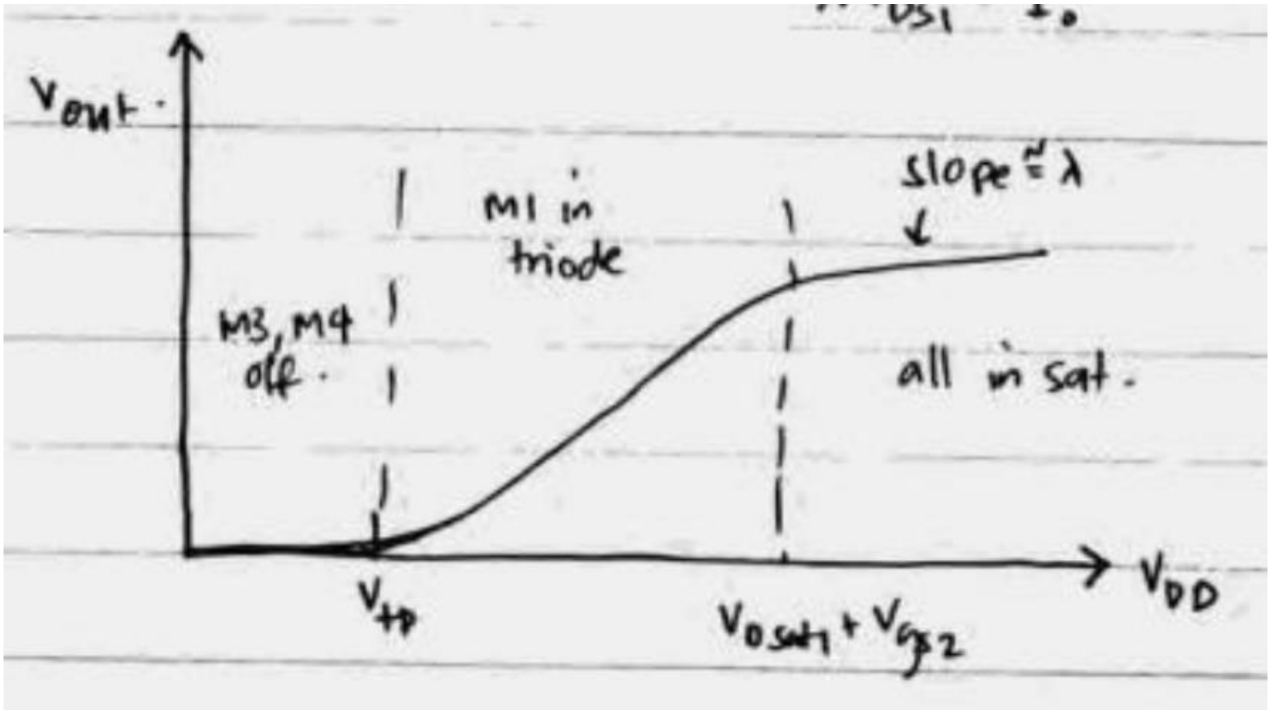
$$V_{out} = I_{D3} \times R_L$$

$$I_{D3} = I_{D2} = I_{D1}$$

$$I_{D1} = \frac{1}{2}K_{n1}(V_{GS1} - V_{THN})^2(1 + \lambda V_{DS1}) = \frac{1}{2}K_{n1}(V_{in} - V_{THN})^2(1 + \lambda(V_{DD} - V_{SG2}))$$

$$V_{out} = \frac{1}{2}K_{n1}(V_{in} - V_{THN})^2(1 + \lambda(V_{DD} - V_{SG2}))R_L$$

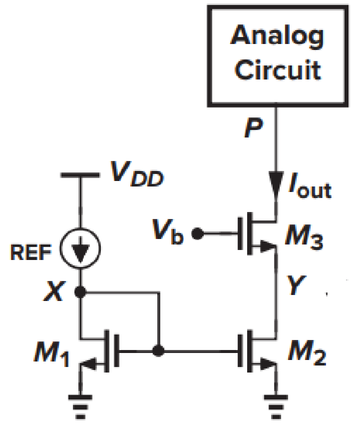
$$\frac{\partial V_{out}}{\partial V_{DD}} = \frac{1}{2}K_{n1}(V_{in} - V_{THN})^2\lambda R_L$$



Vout.
M1 in triode
slope ≈ λ
M3, M4 off.
all in sat.
Vtp
Vdsat1 + Vgs2
VDD

Razavi - Problem 5.5: Consider the circuit shown below, assume (W/L)1–3 = 40/0.5, and IREF=0.3 mA,.

a) Determine Vb such that VX = VY .
b) If Vb deviates from the value calculated in part (a) by 100 mV, what is the mismatch between Iout and IREF?
c) If the circuit fed by the cascode current source changes VP by 1V, how much does VY change?



Solution:

a)

$$V_{DS2} = V_{DS1} = V_{GS1}$$

$$V_b = V_{GS3} + V_{DS2} = 2V_{GS1}$$

$$I_{REF} = \frac{1}{2}K_{n1}(V_{GS1} - V_{TH})^2(1 + \lambda V_{DS1})$$

$$0.3 = \frac{1}{2} \times 0.138 \times \frac{40}{0.5} \times (V_{GS1} - 0.7)^2(1 + 0.1V_{GS1})$$

$$\frac{5}{92} = (V_{GS1} - 0.7)^2(1 + 0.1V_{GS1})$$

$$V_{GS1} = 0.9231V$$

$$V_b = 2V_{GS1} = 1.8462V$$

b)

$$I_{out} = I_{REF} \times \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}} = I_{REF} \times \frac{1 + \lambda(V_{GS1} + \Delta V_y)}{1 + \lambda V_{GS1}} = I_{REF} + I_{REF} \times \frac{\lambda \Delta V_y}{1 + \lambda V_{GS1}} = I_{REF} + \Delta I_{REF}$$

If V_b increases by $\Delta V_b = 100mV$ → V_y will increase by $\Delta V_y < \Delta V_b$ → We need to get ΔV_y?

$$I_{D2} = \frac{1}{2}K_{n2}(V_{GS2} - V_{TH})^2(1 + \lambda V_{DS2}) = \frac{1}{2}K_{n2}(V_{GS1} - V_{TH})^2\left(1 + \lambda(V_{GS1} + \Delta V_y)\right)$$

$$I_{D3} = \frac{1}{2}K_{n3}(V_{GS3} - V_{TH})^2(1 + \lambda V_{DS3}) = \frac{1}{2}K_{n3}\left(V_{GS1} + \Delta V_b - \Delta V_y - V_{TH}\right)^2\left(1 + \lambda(V_{GS1} - \Delta V_y)\right)$$

$$I_{D2} = I_{D3}$$

$$(V_{GS1} - V_{TH})^2 \left(1 + \lambda(V_{GS1} + \Delta V_y)\right) = (V_{GS1} + \Delta V_b - \Delta V_y - V_{TH})^2 \left(1 + \lambda(V_{GS1} - \Delta V_y)\right)$$

$$0.05(1.09231 + 0.1\Delta V_y) = (0.3231 - \Delta V_y)^2 (1.09231 - 0.1\Delta V_y)$$

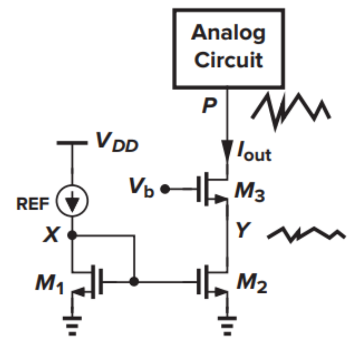
$$\Delta V_y = 97.49mV$$

$$\text{Mismatch} = \Delta I_{REF} = I_{REF} \times \frac{\lambda \Delta V_y}{1 + \lambda V_{GS1}} = 0.3mA \times \frac{0.1 \times 0.09749}{1 + 0.1 \times 0.9231} = 2.678\mu A$$

c)

If V_P increases by 1V → V_y will increase by ΔV_y

We need to get ΔV_y?



$$I_{out} = I_{D2} = I_{REF} \times \frac{1 + \lambda V_{DS2}}{1 + \lambda V_{DS1}} = I_{REF} \times \frac{1 + \lambda(V_{GS1} + \Delta V_y)}{1 + \lambda V_{GS1}} \qquad (I)$$

$$I_{out} = I_{D3} = \frac{1}{2} K_{n3} (V_{GS3} - V_{TH})^2 (1 + \lambda V_{DS3}) = \frac{1}{2} K_{n3} (V_{GS1} - \Delta V_y - V_{TH})^2 \left(1 + \lambda(V_{GS1} + 1 - \Delta V_y)\right) \quad (II)$$

Equating (*I*) *and* (*II*)

$$I_{REF} \times \frac{1 + \lambda(V_{GS1} + \Delta V_y)}{1 + \lambda V_{GS1}} = \frac{1}{2} K_{n3} (V_{GS1} - \Delta V_y - V_{TH})^2 \left(1 + \lambda(V_{GS1} + 1 - \Delta V_y)\right)$$

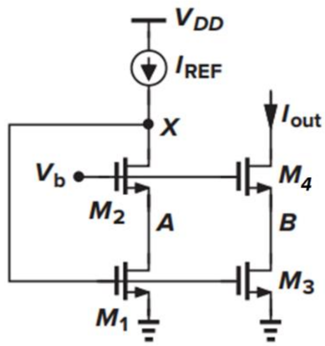
$$0.3m \times \frac{1 + 0.1(0.9231 + \Delta V_y)}{1 + 0.1 \times 0.9231} = \frac{1}{2} \times 0.138m \times \frac{40}{0.5} \times (0.9231 - \Delta V_y - 0.7)^2 \left(1 + 0.1(0.9231 + 1 - \Delta V_y)\right)$$

$$(1.09231 + 0.1\Delta V_y) = 20.1 \times (0.2231 - \Delta V_y)^2 (1.19231 - 0.1\Delta V_y)$$

$$\Delta V_y = 9.43mV$$

Razavi - Problem 5.6: The circuit shown below is designed with $(W/L)_{1,2} = 20/0.5$, $(W/L)_{3,4} = 60/0.5$, and IREF = 100 µA.

a) Determine VX and the acceptable range of Vb.
b) Estimate the deviation of Iout from 300 µA if the drain voltage of M4 is higher than VX by 1 V.



Solution:

a)

$$V_X = V_{GS1}$$

$$I_{REF} = I_{D1} = \frac{1}{2}K_{n1}(V_{GS1} - V_{TH})^2(1 + \lambda V_{DS1})$$

$$0.1 = \frac{1}{2} \times 0.138 \times \frac{20}{0.5} \times (V_{GS1} - 0.7)^2\big(1 + 0.1(V_{GS1} - V_{TH})\big)$$

$$\frac{5}{138} = (V_{GS1} - 0.7)^2(0.93 + 0.1V_{GS1})$$

$$V_X = V_{GS1} = 0.889V$$

Acceptable range of V_b:

$$V_b \geq V_{GS2} + V_{ov1}$$

$$I_{REF} = I_{D2} = \frac{1}{2}K_{n2}(V_{GS2} - V_{TH})^2(1 + \lambda V_{DS2})$$

$$0.1 = \frac{1}{2} \times 0.138 \times \frac{20}{0.5} \times (V_{GS2} - 0.7)^2(1 + 0.1 \times 0.7)$$

$$V_{GS2} = 0.884V$$

$$V_b \geq 0.884 + (0.889 - 0.7)$$

$$V_b \geq 1.073V$$

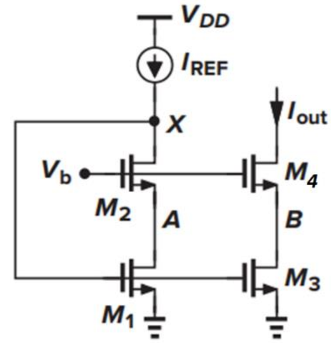
If V_b increases above $V_{GS1,2} + V_{TH}$:

- M2 and M4 go into triode.
- $V_A = V_X$
- $V_B = V_{D4}(Drain\ of\ M4)$
- As long as V_{D4} doesn't drop below V_{ov3}, I_{out} will reasonably track I_{REF}.

b)

Like previous problem, if $V_{drain,M4}$ increases by 1V → V_B will increase by ΔV_B

We need to get ΔV_B?



$$I_{out} = I_{D3} = 3I_{REF} \times \frac{1 + \lambda V_{DS3}}{1 + \lambda V_{DS1}} = 3I_{REF} \times \frac{1 + \lambda(V_{ov1} + \Delta V_B)}{1 + \lambda V_{ov1}} \qquad (I)$$

$$I_{out} = I_{D4} = \frac{1}{2} K_{n4}(V_{GS4} - V_{TH})^2(1 + \lambda V_{DS4}) = \frac{1}{2} K_{n4}(V_{GS2} - \Delta V_B - V_{TH})^2\big(1 + \lambda(V_{TH} + 1 - \Delta V_B)\big) \quad (II)$$

Equating $(\boldsymbol{I})$ $\boldsymbol{and}$ $(\boldsymbol{II})$

$$3I_{REF} \times \frac{1 + \lambda(V_{ov1} + \Delta V_B)}{1 + \lambda V_{ov1}} = \frac{1}{2} K_{n4}(V_{GS2} - \Delta V_B - V_{TH})^2\big(1 + \lambda(V_{TH} + 1 - \Delta V_B)\big)$$

$$V_{ov1} = V_{GS1} - V_{TH} = 0.889 - 0.7 = 0.189V$$

$$0.3m \times \frac{1 + 0.1(0.189 + \Delta V_B)}{1 + 0.1 \times 0.189} = \frac{1}{2} \times 0.138m \times \frac{60}{0.5} \times (0.884 - \Delta V_B - 0.7)^2\big(1 + 0.1(0.7 + 1 - \Delta V_B)\big)$$

$$(1.0189 + 0.1\Delta V_B) = 27.6 \times (0.184 - \Delta V_B)^2(1.17 - 0.1\Delta V_B)$$

$$\Delta V_B = 6.27mV$$

$$I_{out} = 3I_{REF} \times \frac{1 + \lambda(V_{ov1} + \Delta V_B)}{1 + \lambda V_{ov1}} = 300.2\mu A$$