X – Rays

1. OBJECTIVE

1. Record X-Ray Spectrum for Copper Anode, using: (i) Lithium Fluoride (LiF) Monocrystal, and (ii) NACL Monocrystal.
2. Verification of Duane-Hunt Displacement Law.
3. Determination of Planck's Constant.

2. APPARATUS

X-Ray Unit (Built-in Counter Tube -Pulse Rate Meter) - Mounted LiF and NaCL Monocrystals

3. EXPERIMENTAL SETUP

The X-ray unit shown in Figure (1) consists of three separate chambers: (a) The X-ray tube, shown in Figure (2), is located to the left of the unit. (b) The base of the housing, which contains a microprocessor-controlled electronic circuit, connected to all the controls on the front panel. (c) The experimental section, which is the largest chamber, closed with a transparent sliding door.

Figure (1): The X-ray unit.

The X-ray tube, with copper anode, is supplied with a high tension DC voltage that can be increased in 100 V steps, up to a maximum value of 35 KV, and is read on the 3-figure digital display (1) on the front panel. The heating current of the cathode is adjusted at a constant value of 1.0 mA.

The X-ray radiation enters the experimental section as a diverging beam from a focal spot on the copper anode of the tube and fills the fluorescent screen (16) at the far end through an entry aperture of 21 mm diameter. To collimate the X-ray beam (i.e. to make the beam parallel and narrow), two screens with 2 mm slits are inserted into the aperture of the X-ray tube.

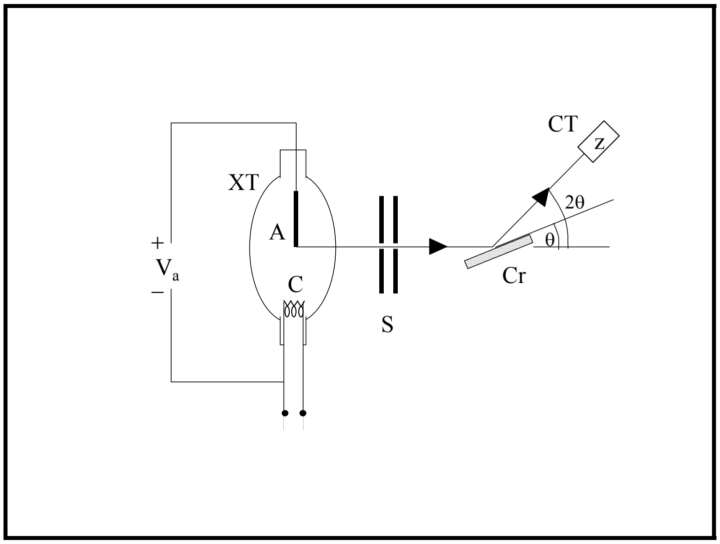

Figure (2): Diagrammatic representation of the arrangement used for Bragg reflection (V_a = anode voltage, XT = X-ray tube, C = cathode, A = copper anticathode, S = screen system, Cr = crystal, CT = Counting tube, θ = Bragg angle).

In the experimental section, the rotatable Crystal Holder (15) is situated immediately in front of the entry aperture of the X-ray tube (14). The counter tube (21) is rotated about the crystal axis of rotation. Two pointers are fixed to the rotating Crystal Holder (20) and the Counter tube (22) ending at the demonstration angular scale (19). These pointers indicate the two angles between the direction of the incident X-ray beam and (i) the irradiated face of the crystal (Bragg's angle) and (ii) The axis of the counter tube (counter angle).

Safety Notes: Any risk to the user due to X-ray unit is eliminated by two micro-switches that turn off the X-ray tube, if the sliding door is not properly closed. The sliding door (18) in the front of the experimental section, the light entry window on its top, and the cover of the fluorescent screen (16) at its far end, are made of special leaded plastic (crystal glass). This type of absorbing glass ensures that the radiation level outside the X-ray unit is many times below the limits given in the X-ray regulations for permissible radiation. Yet, students should always comply with the following precautions:

1. This experiment must not be started except under the supervision of the laboratory staff.
2. The X-ray unit must not be switched on for longer than is necessary.

3. Persons do not remain in the immediate vicinity of the X-ray unit when it is switched ON.

The counter tube (type A) is a self-quenching counter for detecting α, β and γ radiation as well as X-rays.

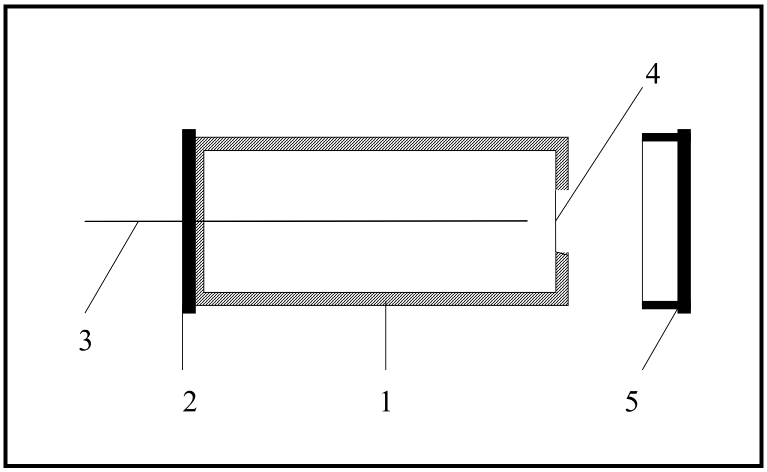

Figure (3): the Counter tube.

It consists as shown in Figure (3), of a thin metallic cylindrical casing (1) of about 40 mm length, 15 mm diameter, and 0.3 mm thickness, provided with a metallic wire (3) extending along its axis and insulated at one end (2) of the tube. The other end is sealed with a sheet of Mica (4), a few μm thick, which represents the counter tube window protected by a plastic cap (5) because of its sensitivity to mechanical stress. It is provided with a BNC socket, such that the counter wire is connected to its central conductor through a 10 $M\Omega$ resistor and the casing is connected to its outer conductor.

To operate the counter tube, it is connected to a DC power supply of 500 V with its wire being positive with respect to the casing. Every ionizing particle (α or β) or photon (γ or X-ray) entering the tube, regardless of the primary ionizing level, causes a discharge pulse. Subsequently, the voltage across the counter tube is slightly reduced. This generates a negative voltage pulse across the working resistance (10 M) that can be amplified. The Halogen component of the filling gas ensures that the discharge is automatically cut off, and hence the counter tube is called self-quenching. For operation, the counter tube is connected by a B.N.C. cable to the input socket of the digital counter.

It is very important to know the characteristics of the counter tube; i.e. the relation between the pulse rate (number of pulses measured per unit time), versus the tube voltage. Figure (4) shows that the tube starts counting, at a threshold of 275 V. The pulse rate is practically independent of the applied voltage over the range from 350 V to 650 V; called the "Plateau" of the tube. Choosing the operating voltage of the tube at the center of the Plateau (e.g. 500 V) avoids any possible effects on the counted pulse rate due to voltage variations. The tube must not be operated at voltages beyond the upper end of the Plateau (650 V). Beyond the Plateau, the pulse rate increases steeply ending in a continuous discharge or a "Break Down" of the tube, hence, it becomes useless for measurement purposes.

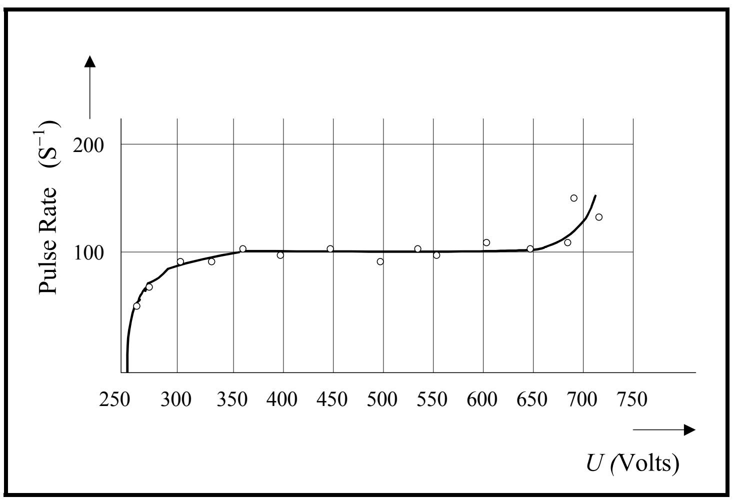

Figure (4): The plateau of the Geiger counter.

The digital counter shown in Figure (5) is used to measure the pulse rate of the counter tube. It has a six-figure digital display (25), for recording the pulse rate, frequency or period, with additional three-figure matrix for displaying the units.

4. THEORY

4.1. X-RAY PRODUCTION

When electrons of high energy impinge on the metallic anode of the X-ray tube (copper in this case), X-rays are produced. Figure (2) shows the simple structure of the X-ray tube. There are two types of X-ray spectra namely:

1. The continuous spectrum.
2. The characteristic line spectrum.

4.1.1. CONTINUOUS SPECTRUM

It depends on the accelerating potential *V* of the electrons. The continuous energy distribution of the X-rays is due to the elastic collisions between the incident electrons and the anode of the X-ray tube. In such collisions each electron loses its kinetic energy (K.E.) in one or more impinges. This loss of energy appears as one or more photons of the generated X-rays, according to the following formula:

$$\text{K.E.} = h\nu = hc/\lambda \qquad (1)$$

where h is Planck's Constant $= 6.625 \times 10^{-34}$ Joule.sec., ν is the frequency of the X-ray quantum or photon, λ is its wavelength, and c is the velocity of light in vacuum $= 2.9979 \times 10^{8}$ m/sec. If the electron loses its energy in one collision, the produced X-ray photon should have a maximum frequency ν_{max} or a minimum wavelength λ_{min}. This quantity characterizes the short wavelength end of the continuous spectrum. Furthermore it is related directly to the potential difference *V* across the X-ray tube, according to the following formula:

$$\text{K.E.}_{max} = eV = h\nu_{max} = hc/\lambda_{min} \qquad (2)$$

or

$$\lambda_{min} = hc/eV \qquad (3)$$

where e is the electron charge $= 1.6021\times10^{-19}$ Coulomb. Duan and Hunt (1915) empirically found that the product of the anode voltage V and the corresponded shortest wavelength $\lambda_{\min}$ of the X-ray continuous spectrum is constant, viz.

$$V\lambda_{\min} = 1.25\times10^{-6}\ \text{Vm}. \tag{4}$$

This is called Duane and Hunt Displacement Law. Equation (4) can be theoretically derived from Equation (3); specifically:

$$\begin{aligned} V\lambda_{\min} &= hc/e = 6.6256\times10^{-34}\times2.9979\times10^{8}/1.6021\times10^{-19} \\ &= 1.2398\times10^{-6}\ \text{Vm}. \end{aligned} \tag{5}$$

4.1.2. Characteristic Line Spectrum

The X-ray line spectrum is characteristic of the material of the target anode of the X-ray tube (copper in this case). It is due to the nonelastic collision between the energetic electrons and the anode atoms. Some of these atoms can be ionized through electron impact in their K-shells. Electrons from higher energy levels (L or M shells) could then fill the vacancy. This process is shown in Figure (6). The energy released by such de-energization process appears as characteristic X-ray spectral lines called K_α and K_β lines. $M_1 \rightarrow K$ and $L_1 \rightarrow K$ transitions do not usually take place, also the spectral lines $K_{\alpha1}$ and $K_{\alpha2}$ are inseparable, hence the Mean Value $K_{\overline{\alpha}}$ is used.

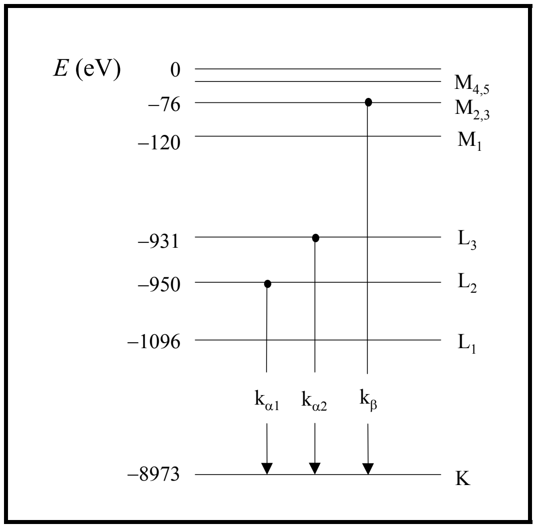

Figure (6): Energy level diagram of copper atom.

The characteristic X-Ray Spectral Lines for Copper, are given by:

$$E_{K_{\bar{\alpha}}} = \frac{1}{2}(E_{L_2} + E_{L_3}) - E_K = hc/\lambda_{K_{\bar{\alpha}}} \tag{6}$$

$$\therefore \; \lambda_{K_{\bar{\alpha}}} = hc/E_{K_{\bar{\alpha}}} = 6.6256\times10^{-34}\times2.9979\times10^{8}\big/8038\times1.6021\times10^{-19}$$

$$\therefore \; \lambda_{K_{\bar{\alpha}}} = hc/E_{K_{\bar{\alpha}}} = 1.55\times10^{-10}\text{ m} = 1.55\,\mathring{A} \tag{7}$$

Similarly:

$$E_{K_\beta} = E_M - E_K = (-74)-(-8979) = 8905\text{ eV}$$

$$\lambda_{K_\beta} = hc\big/E_{K_\beta} = 1.39\,\mathring{A} \tag{8}$$

4.2. X-RAY DIFFRACTION

Monocrystals as lithium fluoride (LiF) and Sodium Cholride (NaCL) are used for spectral analysis of X-rays. Their crystal lattice represents a natural three dimensional diffraction grating for which Bragg's Law of diffraction of X-rays can be applied. In 1912, Bragg showed that the equidistant parallel atomic planes of a crystal lattice act like plane mirrors in reflecting X-rays. Consider a parallel beam of X-rays of wavelength λ incident at a glancing angle θ on two adjacent parallel atomic planes as shown in Figure (7). The diffracted X-rays can interfere constructively or destructively depending on the angle of incidence. Consider the case of X-rays diffracted from two adjacent atomic planes of a crystal. The condition for constructive interference is that the optical path difference between the diffracted rays equals an integer multiple of the wavelength λ; specifically:

$$2d\sin\theta = n\lambda \tag{9}$$

where d is the interplanar crystal spacing, and n is the order of diffraction. The energy E of the X-ray quantum is given by:

$$E = h\nu = hc/\lambda\,. \tag{10}$$

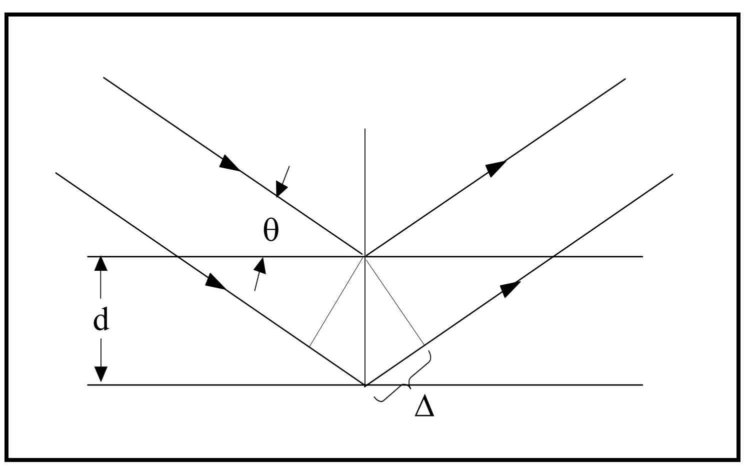

Figure (7): Bragg’s diffraction.

Substituting equation (9) into equation (10):

$$E = nhc/2d\sin\theta\,. \qquad (11)$$

Knowing the spacing d of the crystal, the wavelength λ and the quantum energy E of the diffracted rays, the glancing angle θ can be obtained. Figure (8) shows the spectral energy distribution of X-rays (both the continuous and line spectra), as a function of the glancing angle θ on LiF monocrystal, for anode voltage $V = 25$ KV. It shows $K_{\bar{\alpha}}$ and K_{β} Spectral Lines in the first order, and K_{β} line only in the second order. Figure (9) shows the same on NaCl monocrystal. It shows both $K_{\bar{\alpha}}$ and K_{β} spectral lines in the first, second and third orders.

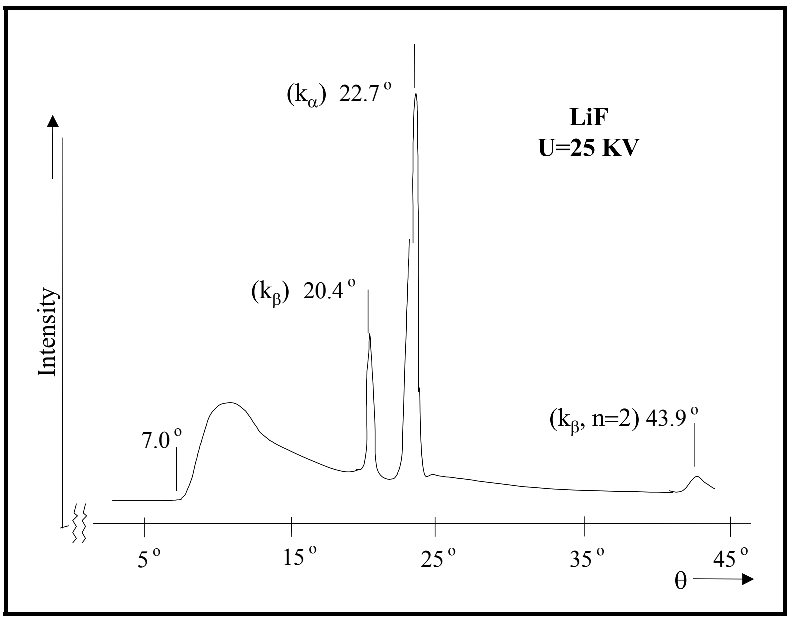

Figure (8): X-ray spectrum scattered from LiF monocrystal.

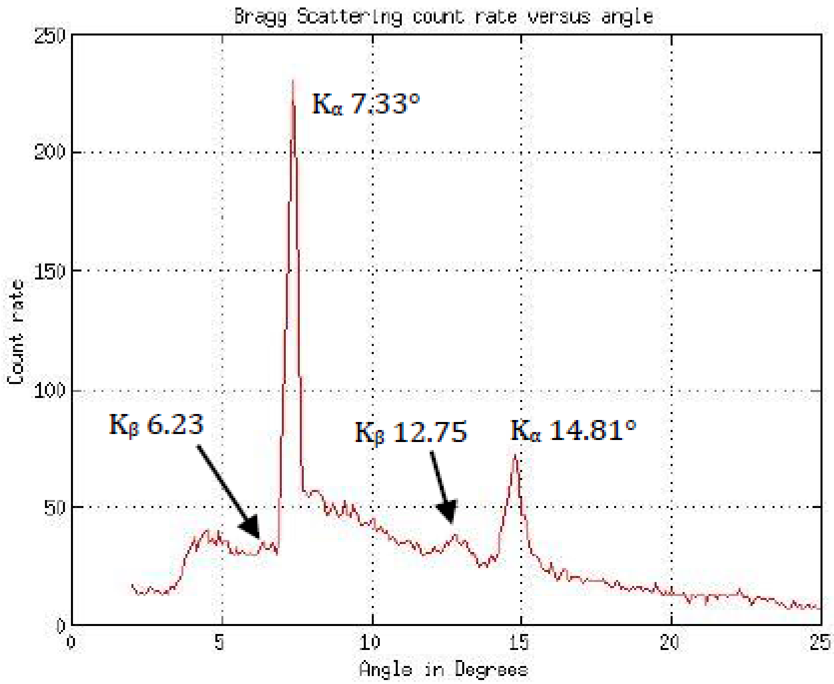

Figure (9): X-ray spectrum scattered from NaCl monocrystal.

5. PROCEDURE

5.1. RECORDING THE X-RAY SPECTRUM FOR COPPER ANODE USING LITHIUM FLUORIDE (LIF) MONOCRYSTAL (d = 2.014 A)

5.1.1. AUTOMATIC RECORDING BY PC USING AVAILABLE SOFTWARE

5.1.2. MANUAL RECORDING

Carry out the following steps for manual recording.

1. To adjust the crystal and counter tube angles, push the appropriate buttons on the front panel of the X-ray unit for increasing and decreasing these angles. Each brief pressing of these buttons changes these angles by a step of 0.2° and 0.4° respectively. Holding these buttons down, the angular speeds of the drives of the crystal and counter tubes (e.g. 0.25°/second and 0.50°/second) selected in step (11) are set.
2. Measure the increasing values of the crystal angle (i.e., the Bragg angle θ) as shown by the "Crystal Pointer" on the angular scale of the X-ray unit. In table 1 record the angle θ and the corresponding values of pulse rate (intensity of X-rays) that is measured by the digital counter.
3. Plot the spectral energy distribution curve of X-rays as intensity I versus Bragg's angle θ (as a measure of the wavelength $\lambda = 2d\sin\theta/n$), as shown in Figure (8). Be careful to determine the start of the curve and its peaks accurately.

5.2. RECORDING THE X-RAY SPECTRUM FOR COPPER ANODE USING NACL MONOCRYSTAL (d = 3.295 A)

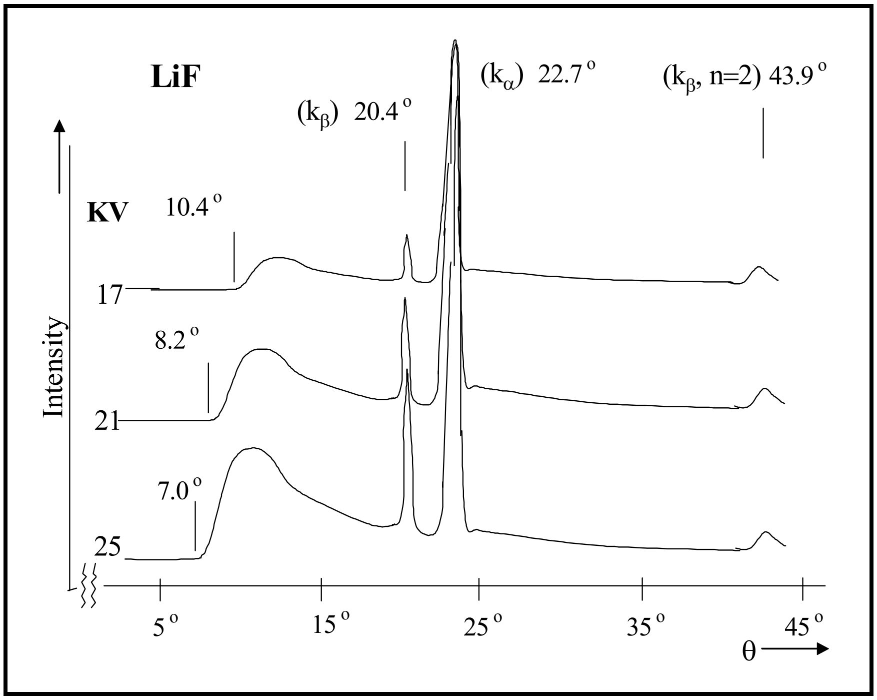

Figure (11): X-ray spectra produced at different high tension voltages.

Carry out the same procedures described above except that the (LiF) monocrystal on the holder of the X-ray unit is replaced by the (NaCL) monocrystal. In this case the first, second and third orders of diffraction appear in the spectral energy distribution as shown in Figure (9).

5.3. Verification of Duane-Hunt Displacement Law and Determination of Planck's Constant

1. Carry out the same procedure for the (LiF) monocrystal at different values of the anode voltage of the X-ray tube using the "Voltage" push buttons for setting the High Tension to 10, 15, 20 and 25 KV.
2. Plot the spectral energy distribution curves of X-rays at these values of the high-tension voltage as shown in Figure (11).
3. Determine the minimum Bragg angle θ_{min} corresponding to the short wavelength limit of the continuous spectrum in each case.
4. Find the corresponding values of minimum wavelengths (λ_{min}) by applying Bragg's equation for the first order of diffraction: $\lambda_{min} = 2d \sin\theta_{min}$, where $d = 2.014\,\mathring{A}$.

5. Record your results in table (3).
6. Plot a graph showing the relation between λ_{min} versus $1/V$. It should be a straight line passing through the origin whose slope $= 1.25 \times 10^{-6}$ Vm. If it is so, it verifies experimentally Duane-Hunt displacement law. (cf. Figure (12)).
7. The slope of the straight line $= \lambda_{min}/(1/V) = V\lambda_{min} = hc/e$. Planck's constant $= h = (e/c) \times \text{Slope}$, where $e = 1.6021 \times 10^{-19}$ Coulomb is the electron charge, and $c = 2.9979 \times 10^{8}$ m/sec is the velocity of light in vacuum.

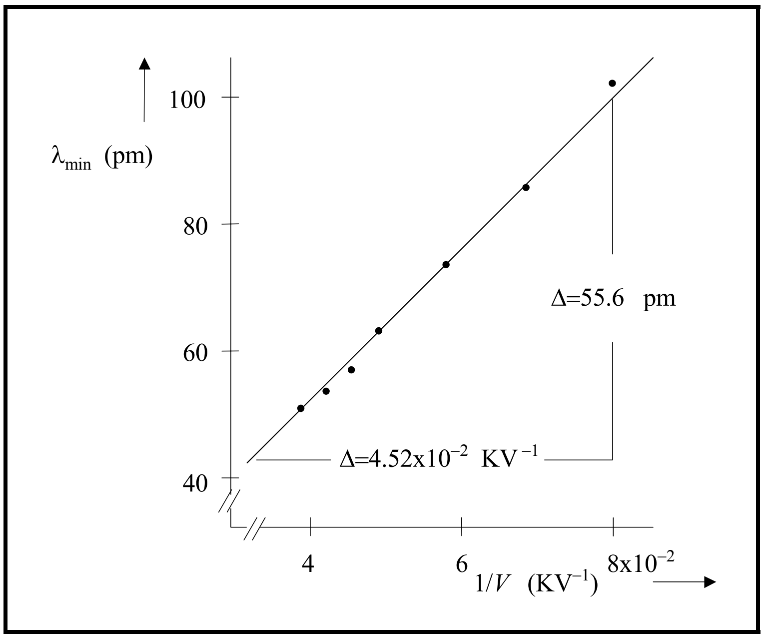

Figure (12): Duane-Hunt displacement law and Planck's "quantum of action".

8.6 RESULTS

Table 1: For (LiF)

θ																		
I																		

Table 2: For (NACLr)

θ																		
I																		

Table 3

V (KV)	25	20	15	10
θ_{min}				
$\sin\theta_{min}$				
$\lambda_{min} = 2d\sin\theta_{min}$