Optimization Problems and Algorithms

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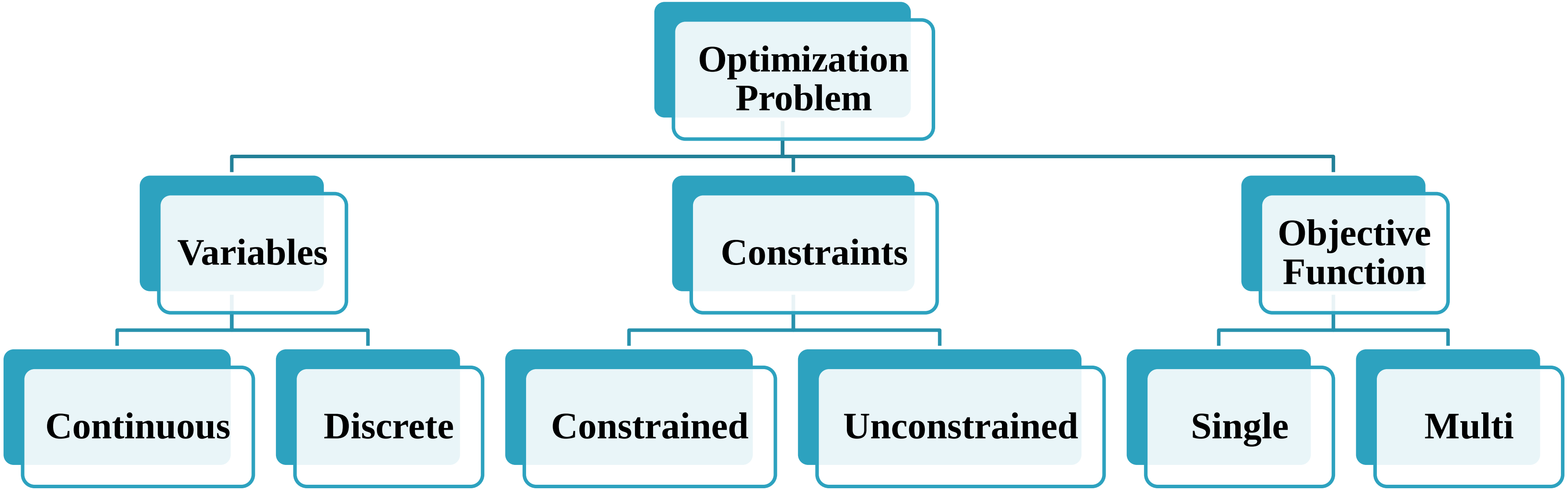
Overview

- Definition of Optimization
- Definition of Optimization Problems
- Types of Optimization Techniques
- Meta-heuristic Algorithms
- An Example : Whale Optimization Algorithm

Definition of Optimization

The process of finding the best values for the variables of a particular problem to minimize or maximize an objective function

Definition of Optimization Problem



Definition of Optimization Problem (cont.)

An example : single objective function

$$f(x_1, x_2) = x_1^2+2x_2^2-0.3\cos(3\pi x_1)(4\pi x_2)+0.3$$

objective function min(f)

variables $\in [10, -10]$

Unconstrained Problem

Definition of Optimization Problem (cont.)

An example : single objective function

$$\text{Min } f(z_1, z_2, z_3) = (-100-(z_1-5)^2 - (z_2-5)^2 + (z_3-5)^2)/100$$

Subject to;

$$h(z_1, z_2, z_3) = (z_1 - 3)^2 + (z_2 - 2)^2 + (z_3 - 5)^2 - 0.0625 \leq 0$$

where;

$$0 \leq z_i \leq 10;$$

Constrained Problem

Definition of Optimization Problem (cont.)

An example : Multi objective function

$$f_1(x_1, x_2) = 0.5(x_1^2 + x_2^2) + \sin(x_1^2 + x_2^2)$$

$$f_2(x_1, x_2) = \frac{(3x_1 - 2x_2 + 4)^2}{8} + \frac{(x_1 - x_2 + 1)^2}{27} + 15$$

$$f_3(x_1, x_2) = \frac{1}{x_1^2 + x_2^2 + 1} - 1.1 \exp(-x_1^2 - x_2^2)$$

objective function **$\min(f_1)$ & $\min(f_2)$ & $\min(f_3)$**

Unconstrained Problem

Definition of Optimization Problem (cont.)

An example : Multi objective function

$$\boldsymbol{min} = \begin{cases} f_1(\vec{x}) = 1.5 - x_1(1 - x_2) \\ f_2(\vec{x}) = 2.25 - x_1(1 - x_2^2) \\ f_3(\vec{x}) = 2.625 - x_1(1 - x_2^3) \end{cases}$$

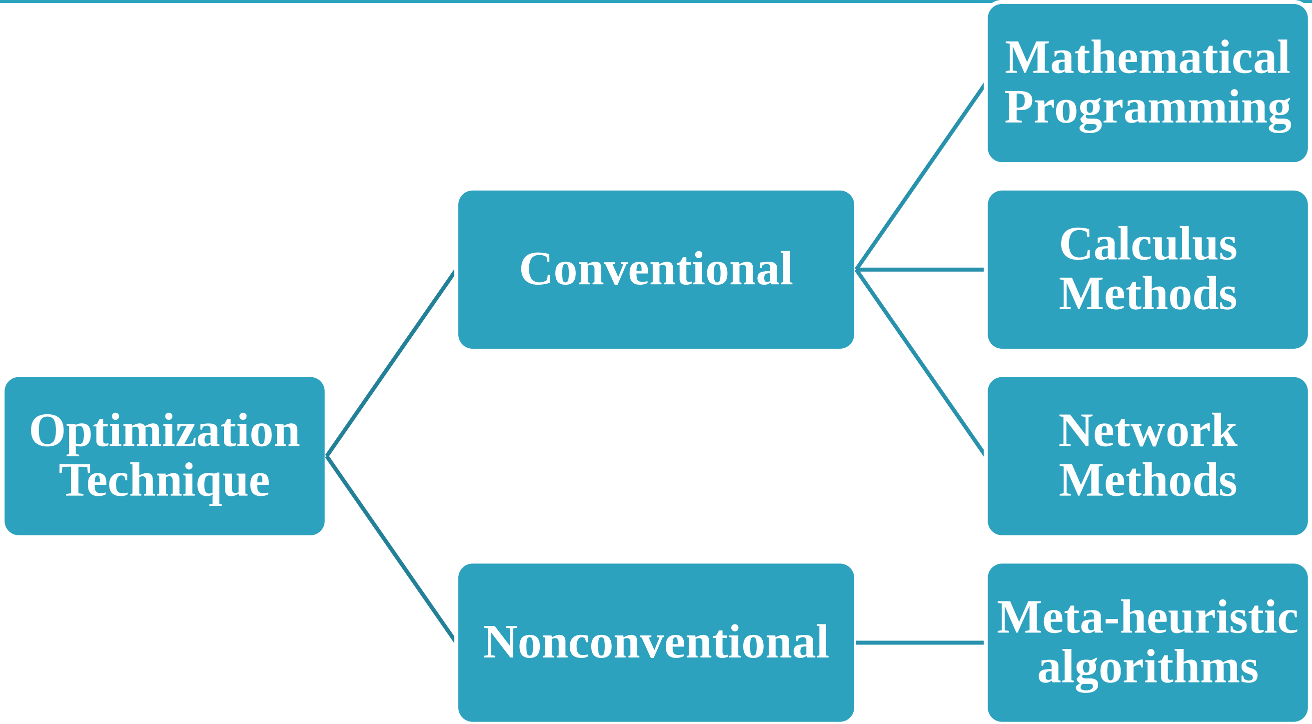
subject to;

$$g_1(\vec{x}) = x_1^2 + x_2^2 - 225 \leq 0$$

$$g_2(\vec{x}) = x_1 - 3x_2 + 10 \leq 0$$

Constrained Problem

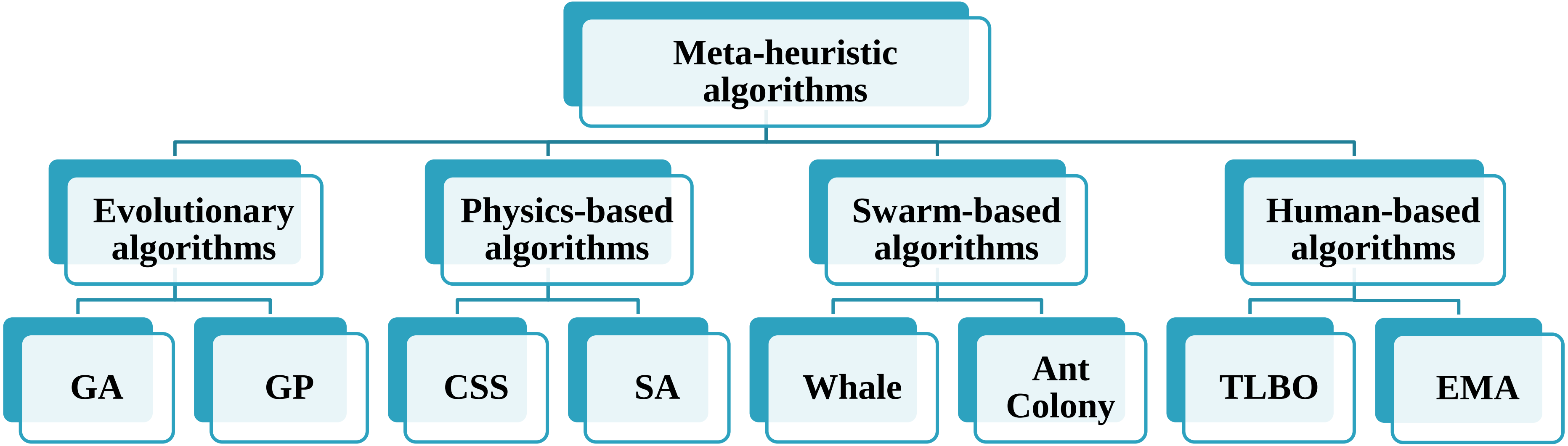
Types of Optimization Techniques



Meta-heuristic Algorithms

Meta-heuristic is a general algorithmic framework which can be applied to different optimization problems with relatively few modifications to make them adapted to a specific problem.

Meta-heuristic Algorithms (cont.)



Genetic Algorithm (GA)
Simulated Annealing (SA)

Genetic Programming (GP)
Teaching Learning Based Optimization(TLBO)

Charged System Search (CSS)
Exchange Market Algorithm (EMA)

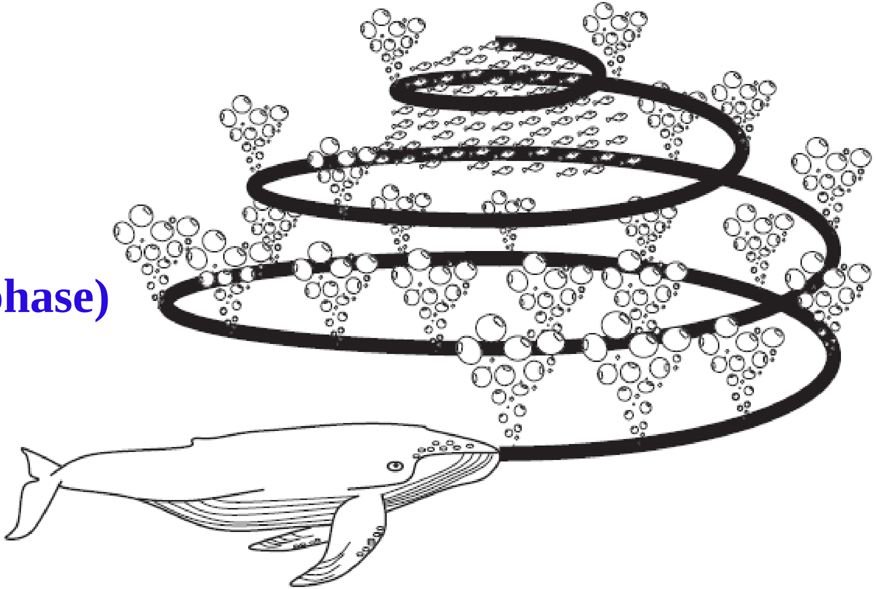
An Example : Whale optimization algorithm

Behavior of Whale

1- Encircling prey

2- Bubble-net attacking method (exploitation phase)

3- Search for prey (exploration phase)



Whale optimization algorithm(cont.)

Mathematical Model

1- Encircling prey

$$D = |C \cdot X^*(t) - X(t)|$$

$$X(t+1) = X^*(t) - A \cdot D$$

Where *t* is the current iteration, A and C are coefficient vectors, X* is the position vector of the best solution, and X indicates the position vector of a solution, | | is the absolute value.

Whale optimization algorithm (cont.)

Mathematical Model (cont.)

The vectors A and C are calculated as follows:

$$A = 2a \cdot r \cdot a$$

$$C = 2 \cdot r$$

Where components of a are linearly decreased from 2 to 0 over the course of iterations and r is random vector in [0; 1]

Whale optimization algorithm (cont.)

Mathematical Model (cont.)

2- Bubble-net mechanism (exploitation phase)

$$X(t+1) = \begin{cases} X^*(t) - A \cdot D & if \;\; p < 0.5 \\ D' \cdot e^{bl} \cdot \cos(2\pi l) + X^*(t) & if \;\; p \geq 0.5 \end{cases}$$

Where the value of A is a random value in interval [-a, a] and the value of a is decreased from 2 to 0 , D' =| X*(t) - X(t) | is the distance between the prey (best solution) and the i^th whale, b is a constant, l is a random number in [-1; 1], and p is a random number in [0; 1]

Whale optimization algorithm (cont.)

Mathematical Model (cont.)

3- search for prey (exploration phase)

In order to force the search agent to move far a way from reference whale, we use the A with values > 1 or < 1

$$D = |C \cdot X_{rand} - X|$$

$$X(t+1) = X_{rand} - A \cdot D$$

Where Xrand is a random position vector chosen from the current population.

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