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Lense-Thirring Field and the Solar Limb Effect

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Abstract. Solar-Limb Effect is an observational phenomena connected to the solar gravitational red-shift. It shows a variation of the magnitude of the gravitational red-shift from the center to the limb of the solar disc. In the present work an attempt, for interpreting this phenomena using a general relativistic red-shift formula, is given . This formula takes into account the effect of the the Sun's gravitational field, the effect of the solar rotation, the effect of inclination of the line of sight and the motion of the observer. In this study the gravitational field of the Sun is assumed to be given by Lense-Thirring field instead of the Schwarzschild one. The Earth is assumed to move along an elliptic orbit. Comparison with a previous relativistic study and with observation is given.

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1. Introduction

The solar limb effect is an observational phenomena indicating that the value of the gravitational red-shift varies from point to point along the solar disc (cf. Adam (1976, 1979), Peter (1999)). While the theoretical study using orthodox general relativity (GR) gives constant value for gravitational red-shift, observations show that this value increases as we move from the center to the limb of the Sun's disc. Many authors have attempted to find satisfactory interpretation for this effect (cf. Mikhail et al.(2002) and references listed therein).

In a previous attempt (Mikhail et al.(2002)) two of us , in collaboration with Mikhail have tried to find an interpretation using a more general formula for the gravitational red-shift in the context of GR. In that study the assumptions have been:
1) The gravitational field of the Sun is given by the Schwarzschild exterior solution.
2) The observer, on the Earth's surface , moves in a circular orbit about the Sun.

In the present work we are going to use the same general formula for the gravitational red-shift, used in the above mentioned study. The main differences between the present study and above mentioned one are:
(1) The Sun's gravitational field is given by Lense-Thiring solution of GR in free space.
(2) The observer, on the Earth's surface, is assumed to move in an elliptic trajectory about the Sun.

2. A General Formula for Red-Shift Variation

Kermack, McCrea and Whittaker (1933), established and proved two theorems on null-geodesics. As a direct result of application of these theorems, they found the following formula:

$$\underset{0}{\lambda_1} = \frac{[\rho_\mu\ \eta^\mu]_{C_1}}{[\bar{\omega}_\mu\ \eta^\mu]_{C_0}}\ \lambda_1\ , \tag{2.1}$$

where we assume that we have two identical atoms at the points C_1 and C_2 on the solar equator where there are two observers A_1, A_2 respectively, $\lambda_1, \underset{0}{\lambda_1}$ are the wavelengths of a certain spectral line, as measured by an observer A_1 at C_1 and B at the C_0 on the Earth's surface respectively , ρ_μ gives the components of the tangent to geodesic of the observer A_1 at C_1 and η^μ gives the component of the tangent to the null-geodesic connecting A_1 and B and ϖ_μ gives the components of the tangent to the trajectory of the observer B. For the second atom at C_2 we can write a formula similar to (2.1) as:

$$\underset{0}{\lambda_2} = \frac{[\rho_\mu\ \eta^\mu]_{C_2}}{[\bar{\omega}_\mu\ \eta^\mu]_{C_0}}\ \lambda_2\ . \tag{2.2}$$

Now, we can write,

$$\lambda = \lambda_1 = \lambda_2$$

as the two atoms are situated on the same great circle (the solar equator). The quantities between the square brackets in (2.1), (2.2) are evaluated at the points indicated outside the the brackets, respectively.

As the observer B, on the Earth's surface, measures the wavelengths coming from the two atoms at C_1, C_2, he would expect a difference in the gravitational red-shift given by:

$$\Delta\ Z = \frac{\underset{0}{\lambda_2} - \underset{0}{\lambda_1}}{\lambda}. \tag{2.3}$$

Using (2.1), (2.2), we can write (2.3) in the form

$$\Delta\ Z = \frac{[\rho_\mu\ \zeta^\mu]_{C_2}}{[\bar{\omega}_\mu\ \zeta^\mu]_{C_0}} - \frac{[\rho_\mu\ \eta^\mu]_{C_1}}{[\bar{\omega}_\mu\ \eta^\mu]_{C_0}}\ . \tag{2.4}$$

This formula gives variation of the red-shift of spectral lines emitted by two identical atoms situated at two different points, on the equator of the Sun.
In this study we assume that these two atoms are situated in two symmetric positions relative to the the line connecting the observer at B and the center of the Sun.

3. Red-shift Variation in Lense-Thirring Field

In this section, we are going to calculate the quantities necessary to evaluate the variation of the red-shift given by (2.4). For this reason we assume that the exterior gravitational field of the Sun, considered as a slowly rotating object, is given by the Lense-Thirring metric, (cf. Adler et al (1975))

$$dS^2 = \left(1 - \frac{2m}{\varrho}\right) dt^2 - \left(1 - \frac{2m}{\varrho}\right)^{-1} d\varrho^2 - \varrho^2(d\theta^2 + \sin^2\theta\, d\phi^2) - \frac{4am}{\varrho} \sin^2\theta\, dt\, d\phi\ , \tag{3.1}$$

where m is the geometric mass of the Sun and (ma) is its intrinsic angular momentum. Now, we are going to use (3.1) to calculate :
1) The tangent of the geodesic, ρ^μ, representing the trajectory of the two similar atoms,

assumed to be circular motion(the motion is along the solar equator)
2) The tangent of the geodesic , ϖ^{μ} , representing the trajectory of the observer B at the Earth's surface (elliptical motion).
3) The component of the null vectors , η^{μ} , ζ^{μ} , tangent to the null-geodesics Γ_1 , Γ_2 , connecting C_1 *and* C_0 ; C_2 *and* C_0 , respectively. Then substituting the calculated values into (2.4) we obtain,

$$\Delta\ Z = \sqrt{\frac{\frac{\gamma_{\oplus}^2}{N^2} + \frac{\gamma_{\oplus}^2}{r^2}\beta^2 - V_{\oplus}^2 r^2 - \frac{4m(V_{\oplus}^2 a)}{\gamma_{\oplus}^2 r}\Big(V_{\oplus}^{-1} + \beta\Big)}{(\gamma_{\odot} - V_{\odot}^2 R^2) + \frac{4am\gamma_{\odot}}{R^3}\beta}} \times$$
$$\Bigg[\frac{1 + \frac{2am}{R^3}(\beta)}{1 + \frac{2am}{r^3}\beta - \frac{2maV_{\oplus}}{r\gamma_{\oplus}} - \sqrt{1 - \gamma_{\oplus}\Big(\frac{1}{N^2} + \frac{1}{r^2}\beta^2\Big) + \frac{4am}{r^3}\beta}} -$$
$$\frac{\frac{2am}{R^3}\beta + \Big(1 - V_{\odot}\beta\Big)}{\Big(1 + \frac{2am}{r^3}\beta\Big)\Big(1 - V_{\oplus}\beta\Big) + \sqrt{\Big(1 - \frac{\gamma_{\oplus}}{r^2}\beta^2 + \frac{4am}{r^3}(\beta)\Big)^2 - \frac{\gamma_{\oplus}}{N^2}\Big(1 - \frac{\gamma_{\oplus}}{r^2}\beta^2 + \frac{4am}{r^3}\beta\Big)}}\Bigg] \tag{3.2}$$

where $\gamma_{\odot} = 1 - (2m/r)$, $\gamma_{\oplus} = 1 - (2m/R)$,where (R) and (r) are the radius of the Sun and the mean distance from the Earth to the Sun respectively, $V_{\oplus}$ & $V_{\odot}$ are the orbital angular velocities of the Earth and the atoms on the equator of the Sun, respectively and $\beta = \frac{\overline{B}}{N}$ where N & $\overline{B}$ are constants .

It is well known that the angle between any two null geodesics is a right angle. So, let us consider the angle (ε) between the projections of the two null geodesics (a measurable quantity) as defined by Mikhail et al.(2002) as

$$\cos(\varepsilon) \ = a_{ij}\ v^i\ \omega^j. \tag{3.3}$$

where v^i and ω^j are the transport null vectors along the projection of the first and second null geodesics, respectively. It is worth mentioning that the angle ε is a small angle. So, it is more convenient to replace it by the angle ψ, between the projection of the radial null-geodesic and the radius of the Sun passing through the atom. The relation between the two angles is given by ($\sin\varepsilon = \frac{R}{r}\sin\psi$), where (R) and (r) are the radius of the Sun and the mean distance from the Earth to the Sun, respectively. If we neglect terms containing quantities of the orders $(a/r)^2$ or $(a/R)^2$ and higher, where a is the angular momentum per unit mass, we get

$$\Delta\ Z = \sqrt{\frac{(\gamma_\oplus - V_\oplus^2 r^2) - \frac{4amV_\oplus}{r}\left(1 + \frac{V_\oplus R}{\sqrt{\gamma_\oplus}}\sin\psi\right)}{(\gamma_\odot - V_\odot^2 R^2) + \frac{4am\gamma_\odot}{R^2\sqrt{\gamma_\oplus}}\sin\psi}} \times$$
$$\Big[\frac{1 + \frac{2am}{R^2\sqrt{\gamma_\oplus}}\sin\psi}{1 + \frac{2amR}{r^3\sqrt{\gamma_\oplus}}\sin\psi - \frac{2amV_\oplus}{r\gamma_\oplus} - \sqrt{\frac{4amR}{r^3\sqrt{\gamma_\oplus}}\sin\psi}} -$$
$$\frac{1 + \frac{2am}{R^2\sqrt{\gamma_\oplus}}\sin\psi - \frac{2amV_\odot}{r\gamma_\oplus} - \frac{RV_\odot}{\sqrt{\gamma_\oplus}}\sin\psi}{1 - \frac{2amV_\oplus}{r\gamma_\oplus} - \frac{2amV_\oplus R^2}{r^3\gamma_\oplus}\sin^2\psi - \sin\psi\left(\frac{RV_\oplus}{\sqrt{\gamma_\oplus}} - \frac{2amR}{r^3\sqrt{\gamma_\oplus}}\right) - \sqrt{\frac{4amR\sin\psi}{r^3\sqrt{\gamma_\oplus}}\left(1 - \frac{R^2}{r^2}\sin^2\psi\right)}}\Big]. \tag{3.4}$$

Now equation (3.4) represents the difference in the red-shift due to the theoretical treatment using Lense - Thirring gravitational field.

4. Results and Discussion

Now we are going to evaluate the variation in the red-shift as given by (3.4), in order to compare it with the well known observational value. We are going to use the following data for the Sun and the Earth(cf. Arthur (2000)). These data are summarized in the following table in both (M.K.S) units and (relativistic units).

Table (1): Dynamical Quantities in M.K.S Units and Relativistic Units

Dynamical Quantity		**in M.K.S Units**	**in Relativistic Units**
r	Mean distance form Earth to Sun	$1.496 \times 10^{11}\ m$	499.0159779 sec
R	Radius of the Sun	$6.9599 \times 10^{8}\ m$	2.321591781 sec
a	Sun's angular momentum per unit mass	273.28 m	$9.1234 \times 10^{-7} sec$
m	Geometric mass of the Sun	1477 m	$4.9268 \times 10^{-6} sec$
$V_\odot$	The angular velocity of the Sun	-	$2.865 \times 10^{-6} rad.sec^{-1}$
$V_\oplus$	Orbital angular velocity of the Earth	-	$1.991 \times 10^{-7} rad.sec^{-1}$

For the Earth $\gamma_\oplus = 1$, and for the Sun $\gamma_\odot = 0.99999$. By substituting the values of the quantities, tabulated in Table (1), in equation (3.4), we get

$$\Delta\ Z(\psi) = 6.1892 \times 10^{-6} \times \Big[\frac{\sin\psi}{1 - 4.6228 \times 10^{-7}\sin\psi}\Big]. \tag{4.1}$$

It is clear from this relation that the difference in red-shift on Solar disc varies as sin the angle ψ, which means that there is a variation in red-shift from center-to-limb. But this value contains the variation caused by Doppler shift due to the rotation of the Sun. So to eliminate this effect (Doppler shift) we consider two atoms on the equator of the Sun situated at two similar positions ψ *and* $-\psi$ then we take the average value.
The "Center-to-Limb" variation in red-shift (Λ) is given by the following equation,

$$\Lambda = \left(\Delta\ Z\right)_{\psi=90} - \left(\Delta\ Z\right)_{\psi=0}. \tag{4.2}$$

Now by using the numerical values for $\Delta\ Z$, given by equation (4.1) then equation (4.2), will give,

$$\Lambda = 2.861 \times 10^{-12}\sin^2\psi\,. \tag{4.3}$$

To compare this value with those obtained from the previous study (Mikhail et al(2002)) and from observation, let us calculate the maximum value of (4.3) (for $\psi = 90$)in km/sec units, we found that

$$\Lambda_{theo} = 7.58 \times 10^{-7}\ km/sec. \quad (4.4)$$

Although this value is greater than that obtained in the previous study ($\Lambda = 4.8 \times 10^{-7}\ km/sec$), both are still too small compared with the observed value ($\Lambda_{obs} = o.3\ km/sec$). The ratio between the theoretical and observational values of Λ is given by,

$$\frac{\Lambda_{theo}}{\Lambda_{obs}} = 2.5 \times 10^{-6}. \quad (4.5)$$

So, we still have the same conclusion as in the previous work. The ratio given by (4.5) indicate that there is some parameter missing in the theoretical treatment. The order of magnitude of this ratio is the same as that of the square value of the fine structure constant ($\alpha = \frac{1}{137}$). This gives rise to the idea that the spin -torsion interaction is the missing parameter, since the coupling constant of this interaction is the fine structure constant. This interaction is tested by experiment (Wanas et al.(2000))and by using observations (Wanas et al.(2002), Sousa and Maluf (2004)). A theoretical treatment using the parameterized path equation(Wanas (1998), (2000)), in place of the nullgeodesic one, may solve this problem.

References

Adam, M.G. (1976) Mon. Not. Roy. Asrton. Soc. **177**, 687.

Adam, M.G. (1979) Mon. Not. Roy. Asrton. Soc. **188**, 819.

Arthur N.Cox. (2000) "*Allen's Astrophysical Quantities*". 4^{th} ed.

Adler, R., Bazin, M.,and Schiffer, M. (1975)"*Introduction to General Relativity*". 2^{nd} ed., McGraw Hill.

Kermack, W.O., Mc Crea, W.H. and Wittaker, E.T. (1933) Proc. Roy. Soc. Edin. **53**, 31.

Mikhail, F.I., Wanas, M.I. and Morcos, A.B. (2002) Astrophys. Space Sci. **223**, 233; gr-qc/0505132.

Peter, H. (1999) Astrophys. J. **516**, 490.

Sousa, A. A. and Maluf, J. W. (2004) Gen.Rel.Grav. 36, 967-982, gr-qc/0310131 v1

Wanas, M.I.,(1998), Astrophys. Space Sci., **258**, 237; gr-qc/9904019.

Wanas, M.I.,(2000), Turk. J. of Phys., **24**, Issue 3, 473; gr-qc/0010099.

Wanas, M.I., Melek, M. and Kahil, M.E. (2000) Grav.Cosmol. 6. **319**, 322; gr-qc/9812085.

Wanas, M.I., Melek, M. and Kahil, M.E. (2002) Proc. MG IX, Part **B**, 1100; gr-qc/0306086.