CHAPTER 10

Determining
How Costs Behave

CHAPTER 10 LEARNING OBJECTIVES

1. Describe linear cost functions and three common ways in which they behave
2. Explain the importance of causality in estimating cost functions
3. Understand various methods of cost estimation
4. Outline six steps in estimating a cost function using quantitative analysis
5. Describe three criteria used to evaluate and choose cost drivers

CHAPTER 10 LEARNING OBJECTIVES, CONCLUDED

6. Explain nonlinear cost functions, in particular those arising from learning curve effects
7. Be aware of data problems encountered in estimating cost functions

COST FUNCTION, DEFINED

- A cost function is a mathematical description of how a cost changes with changes in the level of an activity relating to that cost.
- Managers often estimate cost functions based on two assumptions:
  - Variations in the level of a single activity (the cost driver) explain the variations in the related total costs, and
  - Cost behavior is approximated by a linear cost function within the relevant range.

COST TERMINOLOGY

From prior chapters, we are familiar with the distinction between variable and fixed costs and in this chapter introduce mixed costs.

- Variable costs—costs that change in total in relation to some chosen activity or output.
- Fixed costs—costs that do not change in total in relation to some chosen activity or output.
- Mixed costs—costs that have both fixed and variable components; also called semivariable costs.

LINEAR COST FUNCTION

$$y = a + bX$$

The dependent variable: the cost that is being predicted

The intercept: fixed costs

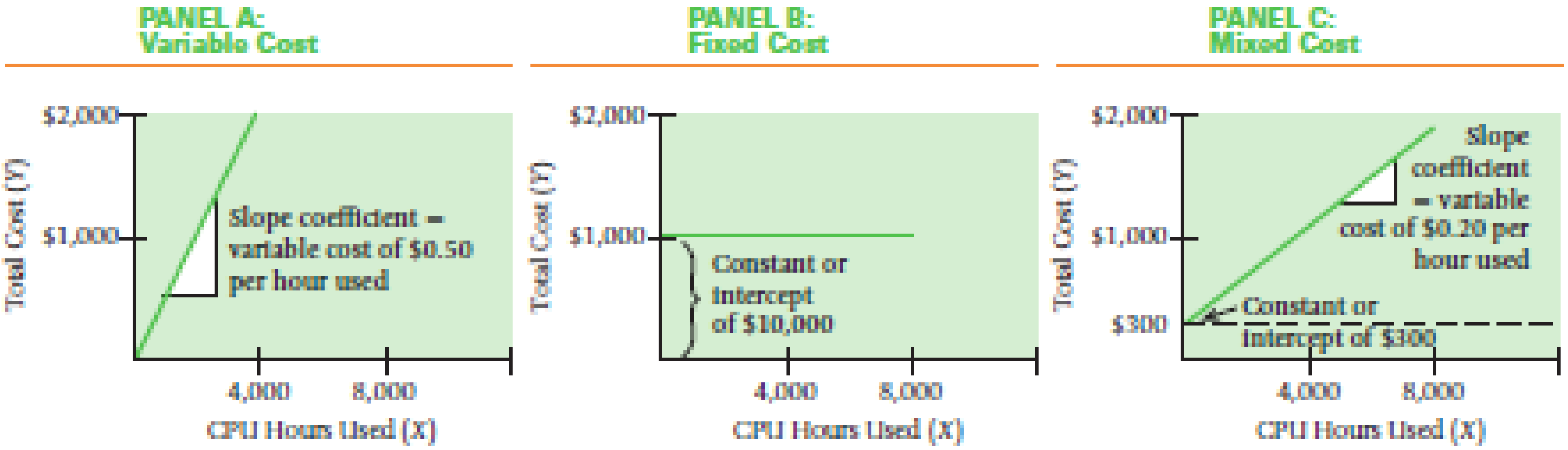
The slope of the line: variable cost per unit

The independent variable: the cost driver

BRIDGING ACCOUNTING AND STATISTICAL TERMINOLOGY

Accounting	Statistics
Variable Cost	Slope
Fixed Cost	Intercept
Mixed Cost	Linear Cost Function

LINEAR COST FUNCTIONS ILLUSTRATED



CRITERIA FOR CLASSIFYING VARIABLE AND FIXED COMPONENTS OF A COST

1. Choice of cost object—different objects may result in different classification of the same cost.
2. Time horizon—the longer the period, the more likely the cost will be variable.
3. Relevant range—behavior is predictable only within this band of activity.

CAUSE AND EFFECT AS IT RELATES TO COST DRIVERS

- The most important issue in estimating a cost function is determining whether a cause-and-effect relationship exists between the level of an activity and the costs related to that level of activity.
- Without a cause-and-effect relationship, managers will be less confident about their ability to estimate or predict costs.

CAUSE AND EFFECT AS IT RELATES TO COST DRIVERS, CONT'D

- A cause-and-effect relationship might arise as a result of:
  - A physical relationship between the level of activity and the costs
  - A contractual agreement
  - Knowledge of operations
- Note: A high correlation (connection) between activities and costs does not necessarily mean causality.
- Only a cause-and-effect relationship – not merely correlation – establishes an economically plausible relationship between the level of an activity and its costs.

COST DRIVERS & THE DECISION MAKING PROCESS

To correctly identify cost drivers in order to make decisions, managers should always use a long time horizon. Managers should follow the five-step decision-making process outlined in Chapter 1 to evaluate how changes can affect costs and product decisions.

COST ESTIMATION METHODS

1. Industrial engineering method
2. Conference method
3. Account analysis method
4. Quantitative analysis methods
    1. High-low method
    2. Regression analysis

These method are not mutually exclusive and often more than one is used.

INDUSTRIAL ENGINEERING METHOD

- Estimates cost functions by analyzing the relationship between inputs and outputs in physical terms.
- Includes time-and-motion studies.
- Very thorough and detailed when there is a physical relationship between inputs and outputs, but also costly and time-consuming.
- Also called the work-measurement method.

CONFERENCE METHOD

- Estimates cost functions on the basis of analysis and opinions about costs and their drivers gathered from various departments of a company.
- Pools expert knowledge, increasing credibility.
- Reliance on opinions makes this method subjective, though often quicker and less expensive.

ACCOUNT ANALYSIS METHOD

- Estimates cost functions by classifying various cost accounts as variable, fixed, or mixed with respect to the identified level of activity.
- Typically, managers use qualitative rather than quantitative analysis when making these cost-classification decisions.
- Widely used because it is reasonably accurate, cost-effective, and easy to use, but is subjective.

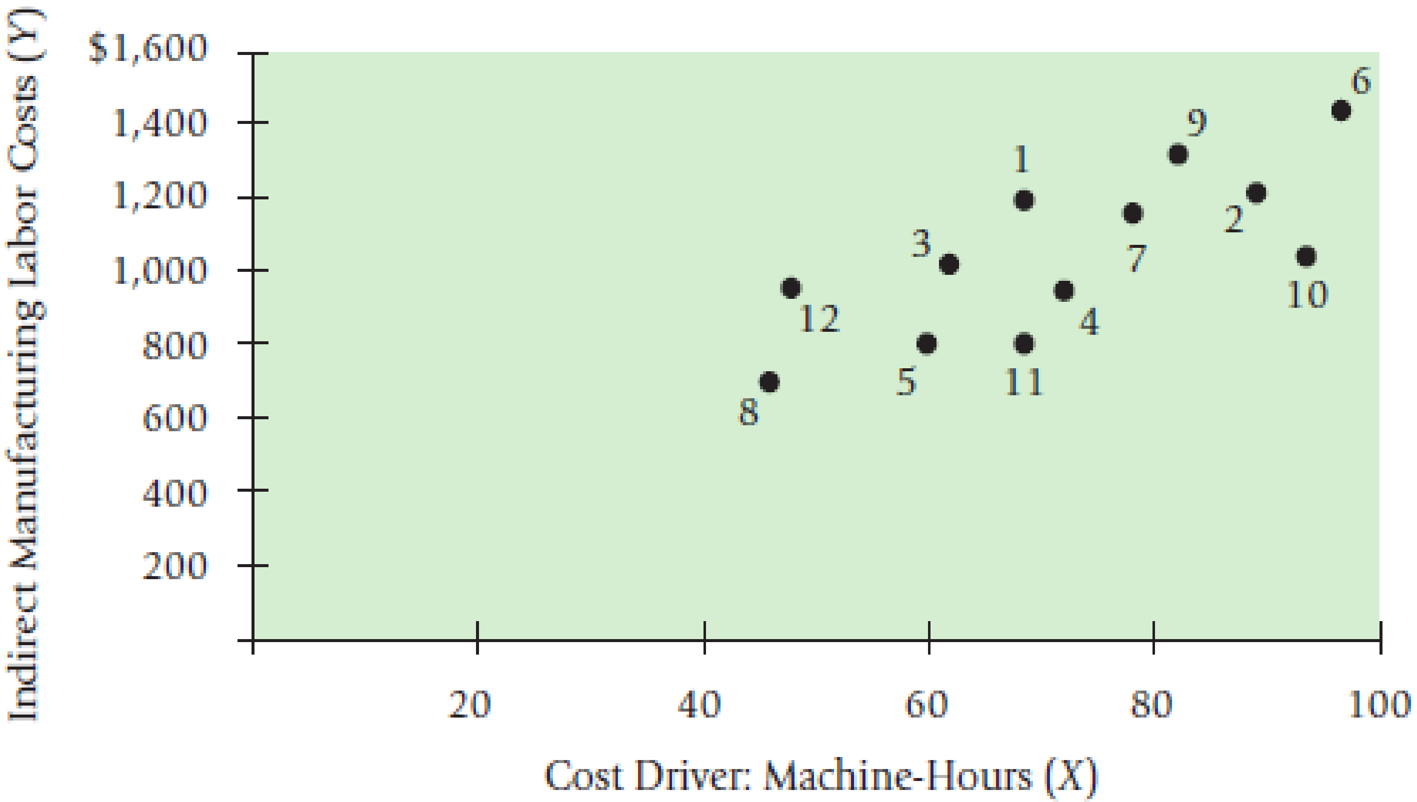
QUANTITATIVE ANALYSIS

- Uses a formal mathematical method to fit cost functions to past data observations.
- Advantage: results are objective.
- Advantage: most rigorous approach to estimate costs.
- Challenge: requires more detailed information about costs, cost drivers, and cost functions and is therefore more time-consuming.

STEPS IN ESTIMATING A COST FUNCTION USING QUANTITATIVE ANALYSIS

1. Choose the dependent variable. (the cost to be predicted and managed)
2. Identify the independent variable. (the level of activity or cost driver)
3. Collect data on the dependent variable and the cost driver.
4. Plot the data to observe the general relationship.
5. Estimate the cost function using two common forms of quantitative analysis: the high-low method or regression analysis.
6. Evaluate the cost driver of the estimated cost function.

SAMPLE COST—ACTIVITY PLOT



HIGH-LOW METHOD

- Simplest method of quantitative analysis.
- Uses only the highest and lowest observed values.
- "Fits" a line to data points which can be used to predict costs.
- Three steps in the high-low method to obtain the estimate of the cost function.

STEPS IN THE HIGH-LOW METHOD

1. Calculate variable cost per unit of activity.

$$\text{Variable Cost per Unit of Activity} = \frac{\left\{\text{Cost associated with highest activity level} - \text{Cost associated with lowest activity level}\right\}}{\text{Highest activity level} - \text{Lowest activity level}}$$

If we had high activity of 100 at cost of $2,500 and low activity of 80 at cost of $2,100, our formula for variable cost per unit of activity would be:

(2500-2100) / (100-80) or 400 / 20 or $20.00

STEPS IN THE HIGH-LOW METHOD

2. Calculate total fixed costs.

Total Cost from either the highest or lowest activity level
- (Variable Cost per unit of activity X Activity associated with above total cost) =
Fixed Costs

Continuing our example, let's calculate fixed costs using both the high and low levels of activity:

High- $2500 – ($20 x 100) = $500 (fixed costs)

Low- $2100 – ($20 x 80) = $500 (fixed costs)

STEPS IN THE HIGH-LOW METHOD

3. Summarize by writing a linear equation.

Y = Fixed Costs + (Variable cost per unit of Activity * Activity)

$$Y = FC + (VC_u * X)$$

Finally, we can summarize our calculations into the equation:

Y = \$500 + (\$20 x X)

If we wondered what costs would be at a 120 level of activity, we'll simply plug that number for X in our equation: Y = \$500 + (\$20 x 120) or Y = \$2,900

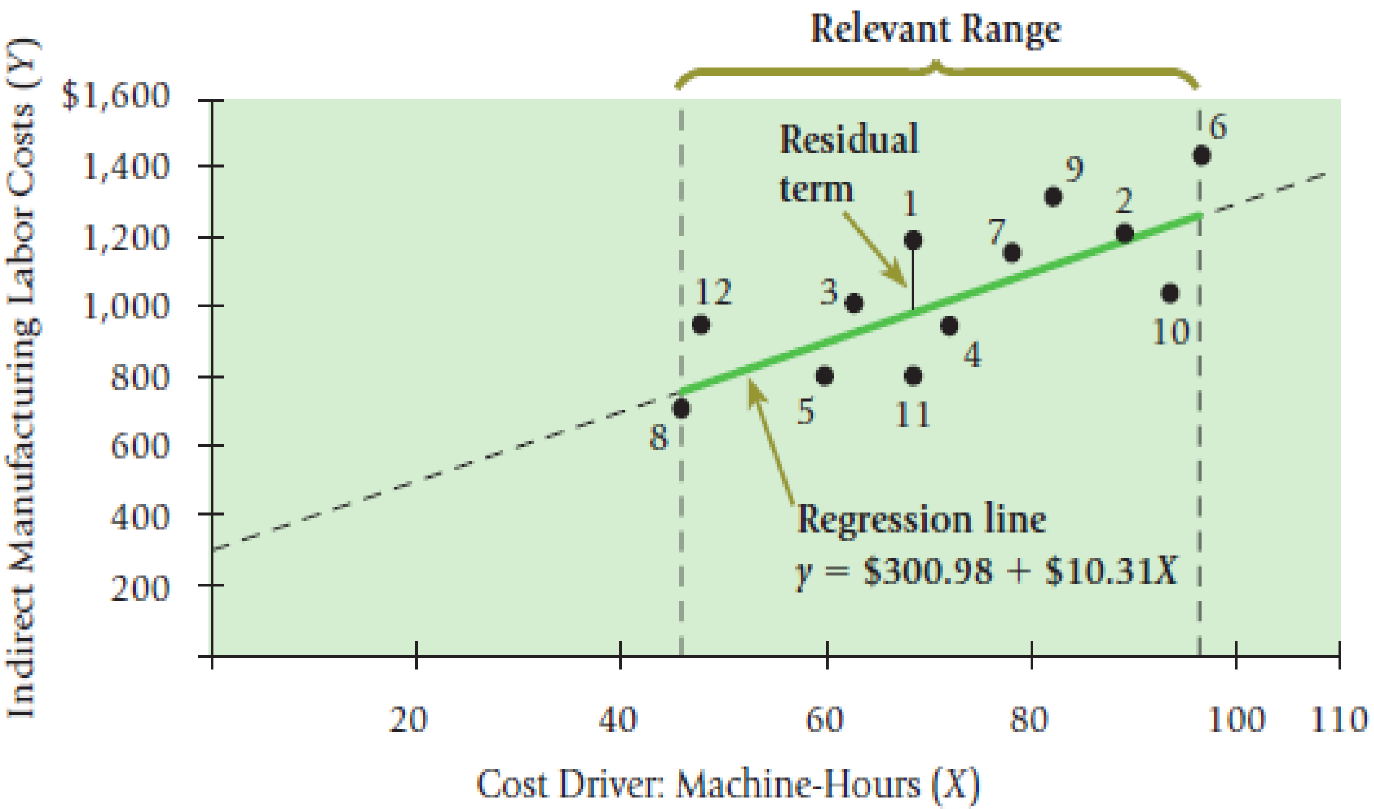
REGRESSION ANALYSIS

- Regression analysis is a statistical method that measures the average amount of change in the dependent variable associated with a unit change in one or more independent variables.
- Is more accurate than the high-low method because the regression equation estimates costs using information from *all* observations whereas the high-low method uses only *two* observations.

TYPES OF REGRESSION

- Simple—estimates the relationship between the dependent variable and *one* independent variable.
- Multiple—estimates the relationship between the dependent variable and *two or more* independent variables.
- Regression analysis is widely used because it helps managers understand why costs behave as they do and what managers can do to influence them.

SAMPLE REGRESSION MODEL PLOT



REGRESSION ANALYSIS: TERMINOLOGY

- Goodness of fit—indicates the strength of the relationship between the cost driver and costs.
- Residual term—measures the distance between actual cost and estimated cost for each observation. The smaller the residual term, the better is the fit between the actual cost observations and estimated costs.

EVALUATING AND CHOOSING COST DRIVERS

How does a company determine the best cost driver when estimating a cost function? An understanding of both operations and cost accounting is helpful. Here are the three criteria used:

1. Economic plausibility
2. Goodness of fit
3. Significance of the independent variable.

EVALUATING AND CHOOSING COST DRIVERS – SIGNIFICANCE OF

QUESTION: Why is choosing the correct cost driver to estimate costs important?

ANSWER: Identifying the wrong drivers or misestimating cost functions can lead management to incorrect and costly decisions along a variety of dimensions.

COST DRIVERS AND ACTIVITY-BASED COSTING

Estimating cost drivers in an activity-based costing system doesn’t differ in general from what’s been discussed.

However, since ABC systems have a great number and variety of cost drivers and cost pools, managers must estimate many cost relationships.

They will do so using the same methods, taking special care with the cost hierarchy. If a cost is batch-level, for example, only batch-level cost drivers can be used.

NONLINEAR COST FUNCTIONS, DEFINED

- Cost functions are not always linear.
- A nonlinear cost function is a cost function for which the graph of total costs is not a straight line within the relevant range.
- Some examples of nonlinear cost functions follow.

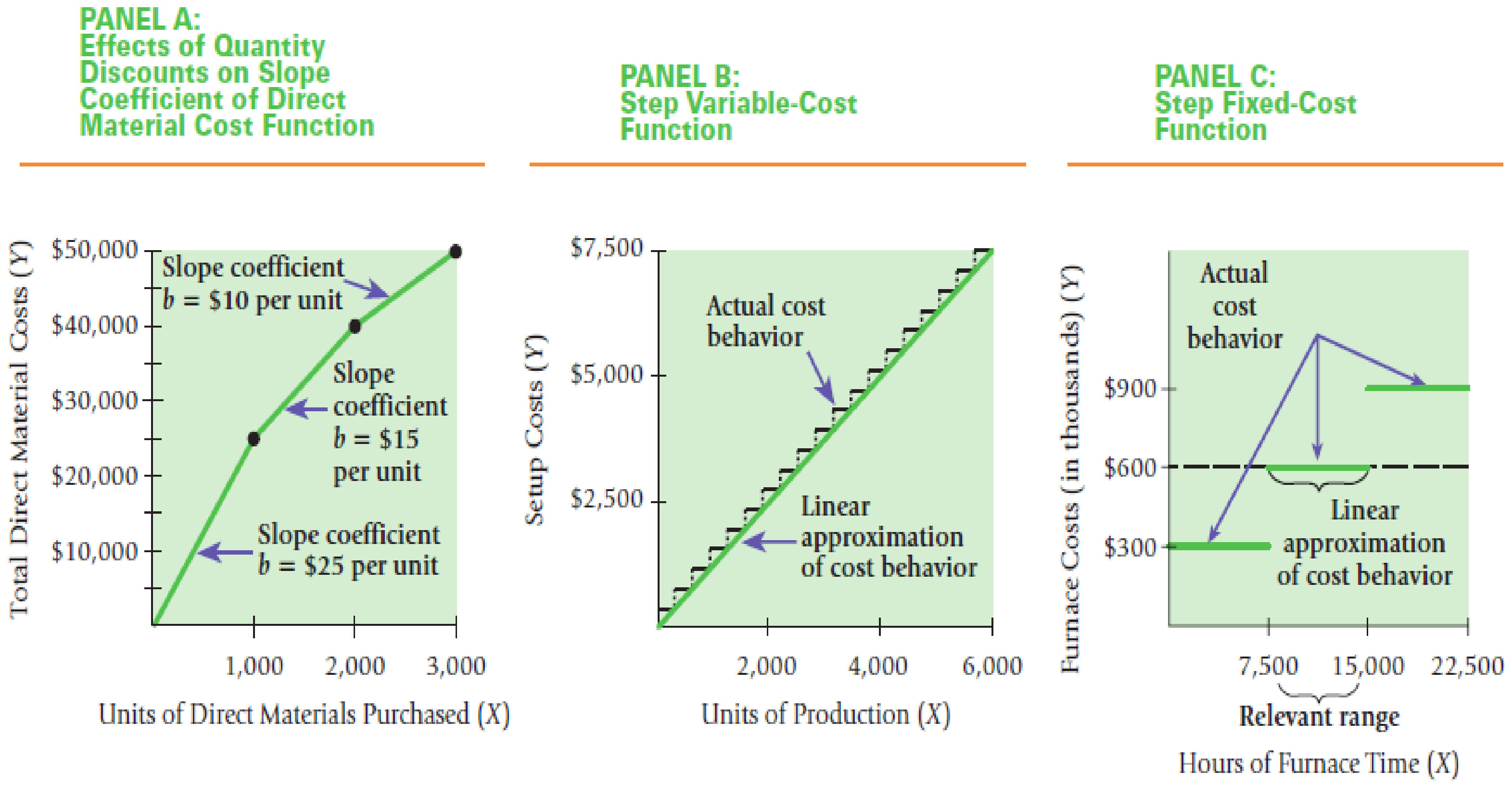
NONLINEAR COST FUNCTIONS, EXAMPLES

1. Economies of scale (produce double the number of advertisements for less than double the cost).
2. Quantity discounts (direct material costs rise but not in direct proportion to increases in quantity due to the nonlinear relationship caused by the quantity discounts).
3. Step cost functions—resources increase in “lot-sizes”, not individual units.

NONLINEAR COST FUNCTIONS, EXAMPLES CONT'D

4. Learning curve—a function that measures how labor-hours per unit decline as units of production increase because workers are learning and becoming better at their jobs.
5. Experience curve —measures the decline in the cost per unit of various business functions as the amount of these activities increases. It is a broader application of the learning curve that extends to other business functions in the value chain such as marketing, distribution and customer service.

NONLINEAR COST FUNCTIONS ILLUSTRATED



TYPES OF LEARNING CURVES

- Cumulative average-time learning model—cumulative *average time per unit* declines by a constant percentage each time the cumulative quantity of units produced doubles.
- Incremental unit-time learning model—incremental *time needed to produce the last unit declines* by a constant percentage each time the cumulative quantity of units produced doubles.

SAMPLE CUMULATIVE AVERAGE-TIME MODEL

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	A	B	C	D	E
1	Cumulative Average-Time Learning Model for Rayburn Corporation				
2					
3		80% Learning Curve			
4					
5	Cumulative	Cumulative		Cumulative	Individual Unit
6	Number	Average Time		Total Time:	Time for *X*th
7	of Units (*X*)	per Unit (*y*)*: Labor Hours		Labor-Hours	Unit: Labor Hours
8					
9				D = Col A x Col B	
10					
11	1	100.00		100.00	100.00
12	2	80.00	=(100x0.8)	160.00	60.00
13	3	70.21		210.63	50.63
14	4	64.00	=(80x0.8)	256.00	45.37
15	5	59.56		297.82	41.82
16	6	56.17		337.01	39.19
17	7	53.45		374.14	37.13
18	8	51.20	=(64x0.8)	409.60	35.46
19	9	49.29		443.65	34.05
20	10	47.65		476.51	32.86
21	11	46.21		508.32	31.81
22	12	44.93		539.22	30.89
23	13	43.79		569.29	30.07
24	14	42.76		598.63	29.34
25	15	41.82		627.30	28.67
26	16	40.96	=(51.2x0.8)	655.36	28.06
27					

E13 = D13 - D12
= 210.63 - 160.00

*The mathematical relationship underlying the cumulative average-time learning model is:

$$y=aX^b$$

where y = Cumulative average time (labor-hours) per unit
X = Cumulative number of units produced
a = Time (labor-hours) required to produce the first unit
b = Factor used to calculate cumulative average time to produce units

The value of b is calculated as

$$\frac{\text{ln (learning-curve \% in decimal form)}}{\text{ln2}}$$

For an 80% learning curve, $b = \ln 0.8/\ln 2 = -0.2231/0.6931 = -0.3219$
when $X = 3, a = 100, b = -0.3219,$

$$y = 100 \times 3^{-0.3219} = 70.21 \text{ labor hours}$$

Numbers in table may not be exact because of rounding.

SAMPLE INCREMENTAL UNIT-TIME LEARNING MODEL

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	A	B	C	D	E
1	Incremental Unit-Time Learning Model for Rayburn Corporation				
2					
3		80% Learning Curve			
4					
5	Cumulative	Individual Unit Time		Cumulative	Cumulative
6	Number	for Xth Unit (y)*:		Total Time:	Average Time
7	of Units (X)	Labor Hours		Labor-Hours	per Unit:
8					Labor-Hours
9					
10					E = Col D ÷ Col A
11					
12	1	100.00		100.00	100.00
13	2	80.00	=(100x0.8)	180.00	90.00
14	3	70.21		250.21	83.40
15	4	64.00	=(80x0.8)	314.21	78.55
16	5	59.56		373.77	74.75
17	6	56.17		429.94	71.66
18	7	53.45		483.39	69.06
19	8	51.20	=(64x0.8)	534.59	66.82
20	9	49.29		583.89	64.88
21	10	47.65		631.54	63.15
22	11	46.21		677.75	61.61
23	12	44.93		722.68	60.22
24	13	43.79		766.47	58.96
25	14	42.76		809.23	57.80
26	15	41.82		851.05	56.74
27	16	40.96	=(51.2x0.8)	892.01	55.75
28					

D14 = D13 + B14
= 180.00 + 70.21

*The mathematical relationship underlying the incremental unit-time learning model is:

$$y = aX^b$$

where y = Time (labor-hours) taken to produce the last single unit
X = Cumulative number of units produced
a = Time (labor-hours) required to produce the first unit
b = Factor used to calculate incremental unit time to produce units

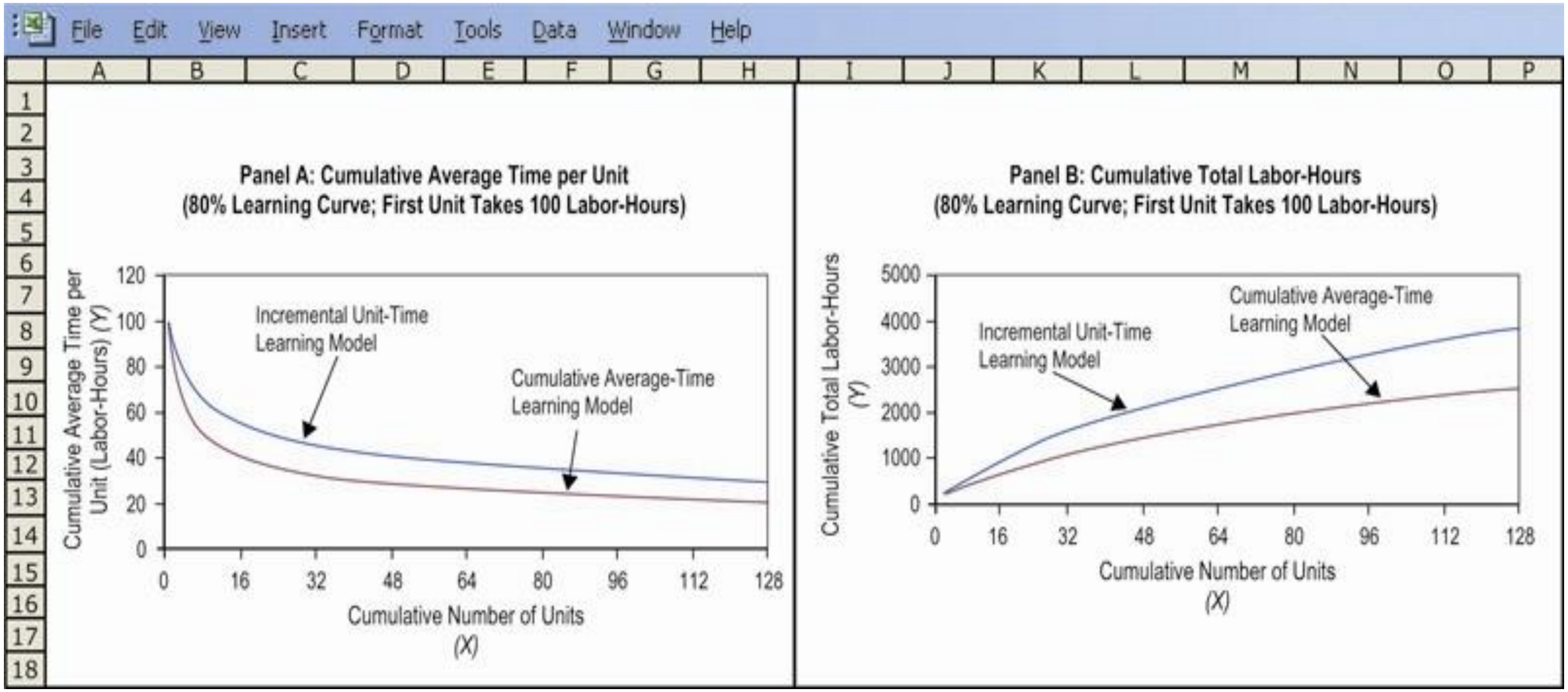
$$= \frac{\ln (\text{learning-curve \% in decimal form})}{\ln 2}$$

For an 80% learning curve, $b = \ln 0.8 \div \ln 2 = -0.2231 \div 0.6931 = -0.3219$
Where $X = 3$, $a = 100$, $b = -0.3219$,

$$y = 100 \times 3^{-0.3219} = 70.21 \text{ labor hours}$$

The cumulative total time when $X = 3$ is 100+80+70.21=250.21 labor-hours.
Numbers in the table may not be exact because of rounding.

TIME LEARNING MODEL COMPARATIVE PLOTS



PREDICTING COSTS USING ALTERNATIVE TIME LEARNING MODELS

	A	B	C	D	E	F
1		Cumulative				
2	Cumulative	Average Time	Cumulative	Cumulative Costs		Additions to
3	Number of	per Unit:	Total Time:	at $50 per		Cumulative
4	Units	Labor-Hours[a]	Labor-Hours[a]	Labor-Hour		Costs
5	1	100.00	100.00	$ 5,000	(100.00 x $50)	$ 5,000
6	2	80.00	160.00	8,000	(160.00 x $50)	3,000
7	4	64.00	256.00	12,800	(256.00 x $50)	4,800
8	8	51.20	409.60	20,480	(409.60 x $50)	7,680
9	16	40.96	655.36	32,768	(655.36 x $50)	12,288
10						
11	[a]Based on the cumulative average-time learning model. See Exhibit 10-10 for the computations					
12	of these amounts.					

DATA COLLECTION AND ADJUSTMENT ISSUES

The ideal database has two characteristics:

1. The database should contain numerous reliably measured observations of the cost driver and the related costs.
2. The database should consider many values spanning a wide range for the cost driver.

DATA PROBLEMS

1. The time period for measuring the dependent variable does not properly match the period for measuring the cost driver.
2. Fixed costs are allocated as if they are variable.
3. Data are either not available for all observations or are not uniformly reliable.

DATA PROBLEMS, CONT'D

4. Extreme values of observations occur.
5. There is no homogeneous relationship between the cost driver and the individual cost items in the dependent variable-cost pool. (A homogeneous relationship exists when each activity whose costs are included in the dependent variable has the same cost driver.)

DATA PROBLEMS, CONCLUDED

6. The relationship between the cost driver and the cost is not stationary. This can occur when the underlying process that generated the observations has not remained stable over time.
7. Inflation has affected the costs, the cost driver, or both.

TERMS TO LEARN

TERMS TO LEARN	PAGE NUMBER REFERENCE
Account analysis method	Page 377
Coefficient of determination	Page 399
Conference method	Page 377
Constant	Page 372
Cost estimation	Page 374
Cost function	Page 371
Cost predictions	Page 374
Cumulative average-time learning model	Page 390
Dependent variable	Page 379
Experience curve	Page 389

TERMS TO LEARN

TERMS TO LEARN	PAGE NUMBER REFERENCE
High-low method	Page 381
Incremental unit-time learning model	Page 391
Independent variable	Page 379
Industrial engineering method	Page 376
Intercept	Page 372
Learning curve	Page 389
Linear cost function	Page 371
Mixed cost	Page 372
Multicollinearity	Page 406
Multiple regression	Page 383

TERMS TO LEARN

TERMS TO LEARN	PAGE NUMBER REFERENCE
Nonlinear cost function	Page 388
Regression analysis	Page 383
Residual term	Page 383
Semivariable cost	Page 372
Simple regression	Page 383
Slope coefficient	Page 372
Specification analysis	Page 401
Standard error of the estimated coefficient	Page 400
Standard error of the regression	Page 399
Step cost function	Page 388
Work-measurement method	Page 376