PHYN001
Cairo University, Faculty of Engineering
Credit Hours System
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Unit 08 The Kinetic Theory of Gases

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Reference

Serway, Raymond A. and Jewett, John W. *Physics for scientists and engineers with modern physics*. 9th Ed.

THE KINETIC THEORY OF GASES

Contents

- Molar specific heat of an ideal gas

- Adiabatic processes for an ideal gas

- Work Done by an Ideal Gas

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MOLAR SPECIFIC HEAT OF AN IDEAL GAS

Due to Kinetic Theory of Gases analysis, the internal energy of an ideal gas:
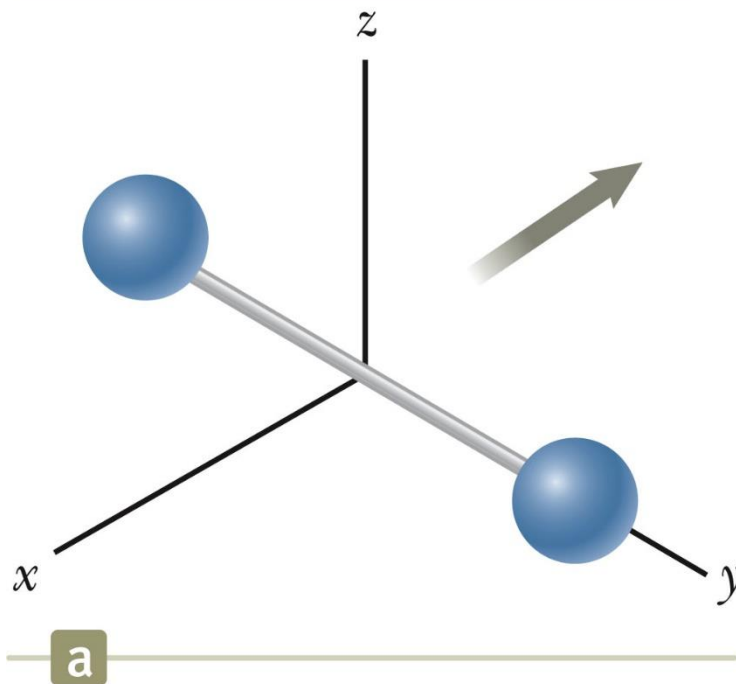
$$E_{int} = Const \times T$$

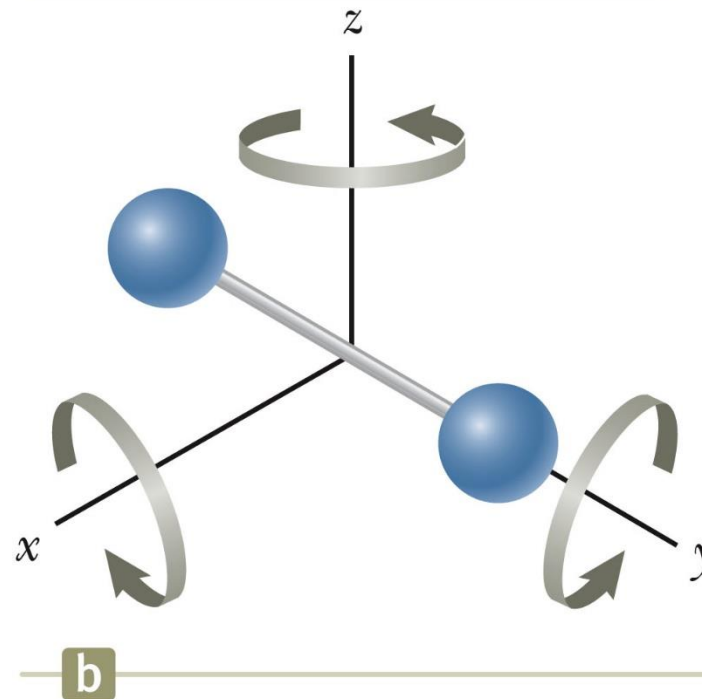$$\therefore \Delta E_{int} \propto \Delta T$$

Examples

- For monoatomic ideal gas: $E_{int} = \frac{3}{2}nRT$

- For diatomic ideal gas: $E_{int} = \frac{5}{2}nRT$

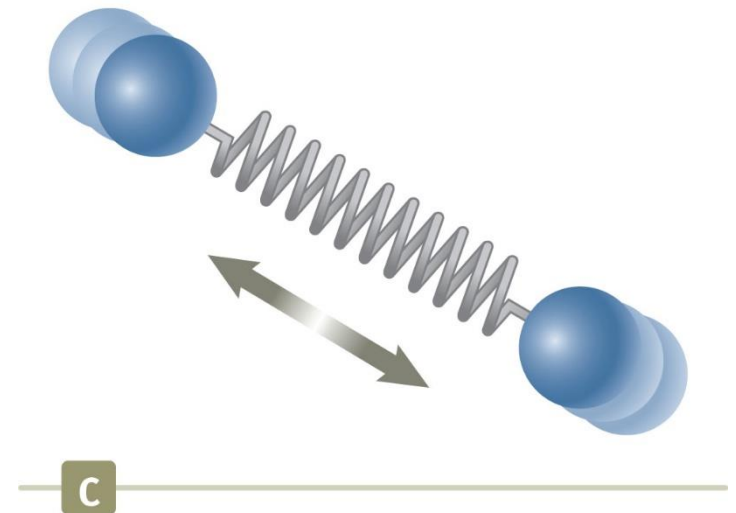# MOLAR SPECIFIC HEAT OF AN IDEAL GAS

Translational motion of the center of mass

Rotational motion about the various axes
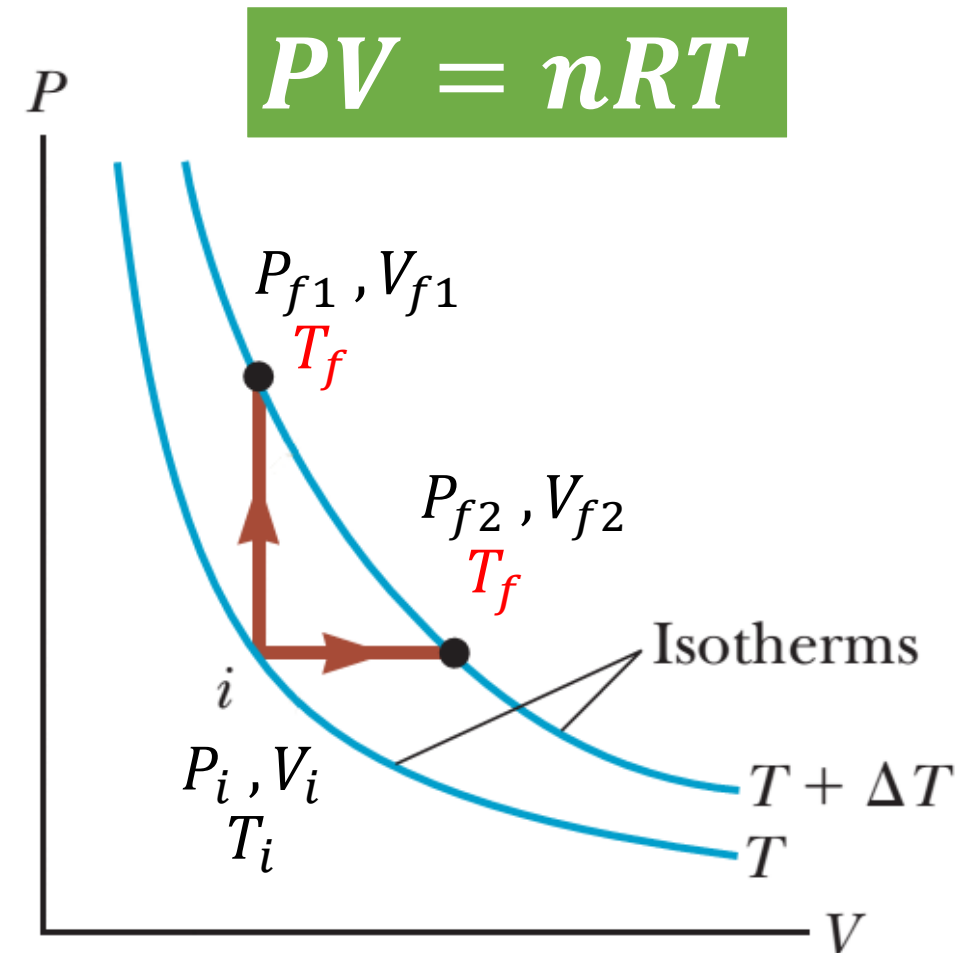
Vibrational motion along the molecular axis

MOLAR SPECIFIC HEAT OF AN IDEAL GAS

Same $\Delta E_{int} = Q - W$ for each process from T_i to T_f.

Each process has unique W

$\therefore$ Each process has a unique Q

We need a flexible definition of specific heat: $Q = m(c)\Delta T$

$$PV = nRT$$

MOLAR SPECIFIC HEAT OF AN IDEAL GAS

Constant volume process

$$Q = nC_V\Delta T$$

C_V, Molar specific heat at const V

n : number of moles

$$\Delta E_{int} = Q - W, \qquad W = 0$$

$$Q = \boxed{\Delta E_{int} = nC_V\Delta T}$$

Constant pressure process

$$Q = nC_P\Delta T$$

C_P, Molar specific heat at const P

n : number of moles

$$\Delta E_{int} = Q - W, \qquad W = P\Delta V$$

$$Q = nC_P\Delta T = \Delta E_{int} + P\Delta V$$

$$nC_P\Delta T = nC_V\Delta T + nR\Delta T$$

$$\boxed{C_P = C_V + R}$$

MOLAR SPECIFIC HEAT OF AN IDEAL GAS

Monoatomic	Diatomic

Monoatomic

Degrees of freedom $f = 3$

$$E_{int} = \frac{3}{2}nRT = nC_VT$$

$$C_V = \frac{3}{2}R = 12.5 \text{ J}/\text{mol}\cdot\text{K}$$

$$C_P = \frac{3}{2}R + R = \frac{5}{2}R = 20.8 \text{ J}/\text{mol}\cdot\text{K}$$

$$\gamma = C_P/C_V = 5/3 = 1.67$$

Diatomic

Degrees of freedom $f = 5$

$$E_{int} = \frac{5}{2}nRT = nC_VT$$

$$C_V = \frac{5}{2}R = 20.8 \text{ J}/\text{mol}\cdot\text{K}$$

$$C_P = \frac{5}{2}R + R = \frac{7}{2}R = 29.1 \text{ J}/\text{mol}\cdot\text{K}$$

$$\gamma = C_P/C_V = 7/5 = 1.4$$

MOLAR SPECIFIC HEAT OF AN IDEAL GAS

	Molar Specific Heat $(J/mol \cdot K)^a$			
Gas	C_P	C_V	$C_P - C_V$	$\gamma = C_P/C_V$
Monatomic gases				
He	20.8	12.5	8.33	1.67
Ar	20.8	12.5	8.33	1.67
Ne	20.8	12.7	8.12	1.64
Kr	20.8	12.3	8.49	1.69
Diatomic gases				
H_2	28.8	20.4	8.33	1.41
N_2	29.1	20.8	8.33	1.40
O_2	29.4	21.1	8.33	1.40
CO	29.3	21.0	8.33	1.40
Cl_2	34.7	25.7	8.96	1.35
Polyatomic gases				
CO_2	37.0	28.5	8.50	1.30
SO_2	40.4	31.4	9.00	1.29
H_2O	35.4	27.0	8.37	1.30
CH_4	35.5	27.1	8.41	1.31

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ADIABATIC PROCESSES FOR AN IDEAL GAS

No heat exchange: $dE_{int} = -dW$

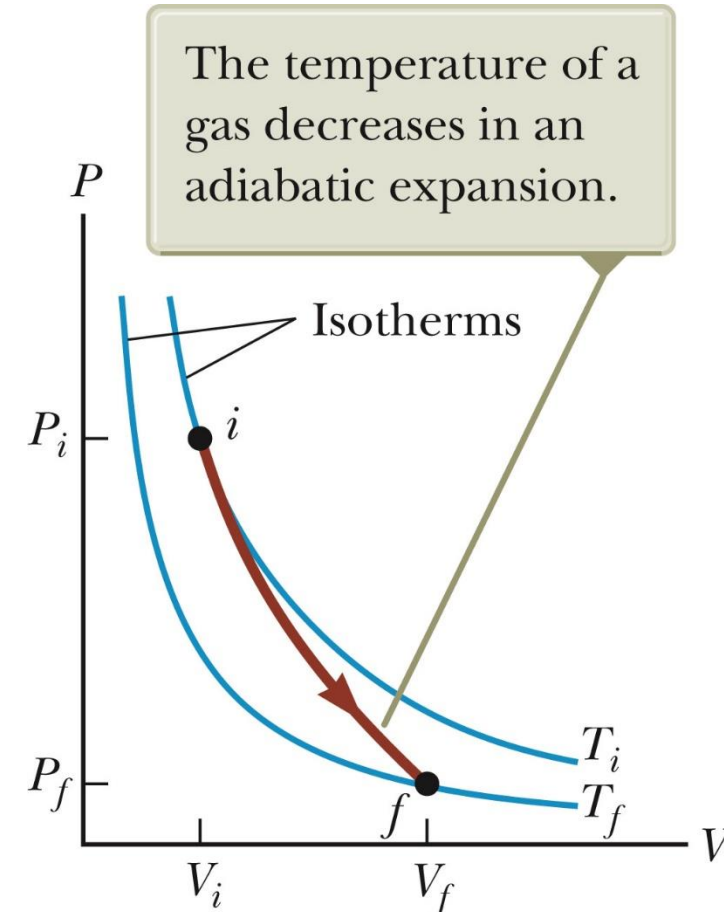$$\therefore nC_V dT = -PdV$$

But $PV = nRT$

$$\therefore PdV + VdP = nRdT$$

$$\therefore ndT = \frac{PdV + VdP}{R}$$

$$\therefore nC_V dT = \frac{C_V(PdV + VdP)}{R} = -PdV$$

$$\therefore C_V(PdV + VdP) = -P(C_P - C_V)dV$$

The temperature of a gas decreases in an adiabatic expansion.

ADIABATIC PROCESSES FOR AN IDEAL GAS

$$\therefore C_V P \, dV + C_V V \, dP = -C_P P \, dV + C_V P \, dV$$

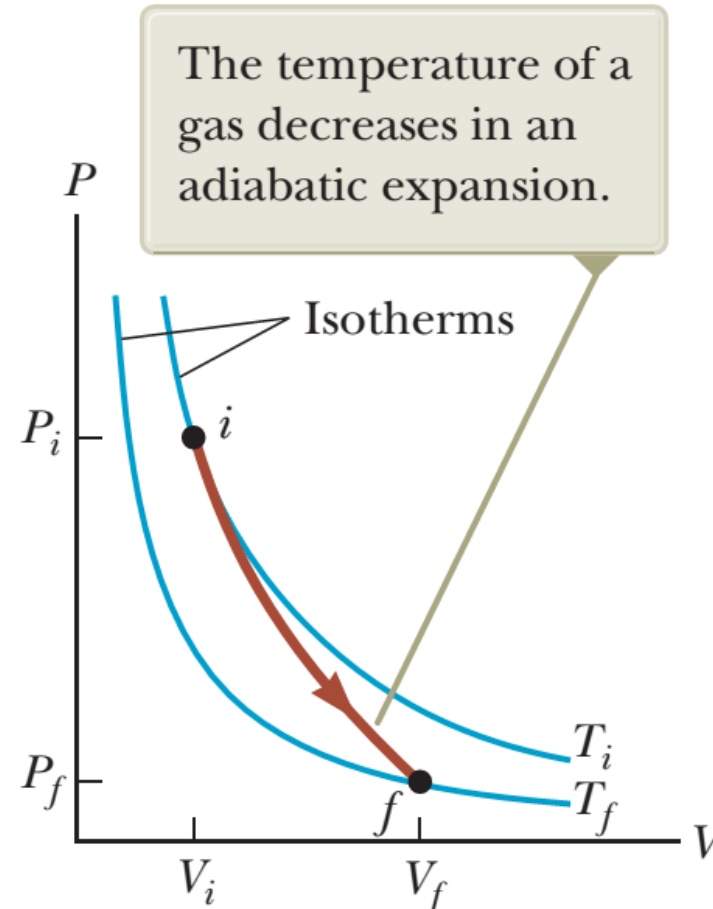$$\therefore C_V V \, dP = -C_P P \, dV$$

$$\frac{dP}{P} = -\frac{C_P}{C_V} \frac{dV}{V} = -\gamma \frac{dV}{V}$$

$$\therefore \ln P = -\gamma \ln V + \text{const}$$

$$\therefore \ln P + \gamma \ln V = \text{const}$$

$$\therefore \ln P V^\gamma = \text{const}$$

$$\boxed{\therefore P V^\gamma = \text{Const}}$$

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WORK DONE BY AN IDEAL GAS

1. Constant P Process (Isobaric)

$$W = P(V_2 - V_1)$$

2. Constant V Process (Isochoric)

$$W = 0$$

WORK DONE BY AN IDEAL GAS

3. Constant T process (Isothermal)

$$PV = nRT$$

$$W = \int_{V_i}^{V_f} P\,dV$$

$$W = \int_{V_i}^{V_f} \frac{nRT}{V}\,dV$$

$$W = nRT \int_{V_i}^{V_f} \frac{dV}{V}$$

$$\therefore \boxed{W = nRT \ln \frac{V_f}{V_i}}$$

WORK DONE BY AN IDEAL GAS

4. Adiabatic process ($Q = 0$)

$$P_i V_i^{\gamma} = P_f V_f^{\gamma} = Const$$

$$W = \int_{V_i}^{V_f} P\, dV$$

$$W = Const \int_{V_i}^{V_f} V^{-\gamma}\, dV = Const\, \frac{V_f^{-\gamma+1} - V_i^{-\gamma+1}}{-\gamma + 1}$$

WORK DONE BY AN IDEAL GAS

4. Adiabatic process (Q = 0)

$$W = \frac{\textcolor{red}{P_f V_f^{\gamma}} V_f^{-\gamma+1} - \textcolor{red}{P_i V_i^{\gamma}} V_i^{-\gamma+1}}{-\gamma + 1}$$

$$\therefore W = \frac{P_f V_f - P_i V_i}{-\gamma + 1}$$

$$\therefore W = nR \frac{T_f - T_i}{-\gamma + 1}$$

$$\boxed{\therefore W = \frac{nR\Delta T}{1 - \gamma}}$$

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Process	Isobaric	Isovolumetric	Isothermal	Adiabatic
Relation	$P = Const$	$V = Const$	$T = Const$	$Q = 0$
$\dfrac{PV}{T} = Const$	$\dfrac{V}{T} = Const$	$\dfrac{P}{T} = Const$	$PV = Const$	$PV^\gamma = Const$ $TV^{\gamma-1} = Const$ $P^{1-\gamma}T^\gamma = Const$
$\Delta E_{int} = Q - W$	$nC_V\Delta T$	$nC_V\Delta T$	$nC_V\Delta T$	$nC_V\Delta T$
W	$P(V_f - V_i)$	0	$nRT\ln\dfrac{V_f}{V_i}$	$-\Delta E_{int}$
Q	$nC_P\Delta T$	ΔE_{int}	$\Delta E_{int} + W$	0