Student name: _____ Student ID: _____

Cairo University
Faculty of Computers and Information
Subject: Algorithms
Subject Code: **NCS316**
Examiner(s): **Sherif Khattab**

Mid-term exam

Semester: 1st
Date: 26/11/**2015**
Duration: 1 **hour**

Answer as much as you can. Max. grade is 20.

Question 1 [5 marks] Answer the following questions:

1. The asymptotic worst-case running time of Karatsuba's integer multiplication algorithm, as a function of the number of digits *n*, is …… because ……..
   ……
   ……
   ……

2. The running time of depth-first search, as a function of n and m, if the input graph is G=(V,E), where as usual n=|V| and m=|E| is …….. .

3. The asymptotic running time of Randomized Selection on arrays of length n, in expectation (over the choice of random pivots) is ……. .

4. Suppose that the running time of an algorithm is governed by the recurrence $T(n)=7*T(n/2)+n^2$.
   What's the overall asymptotic running time (i.e., the value of T(n))? ……..... .
   Why? …….
   ……
   ……
   ……

Question 2 [4 marks] True or False? correct the wrong statements. ***[1 mark each]***

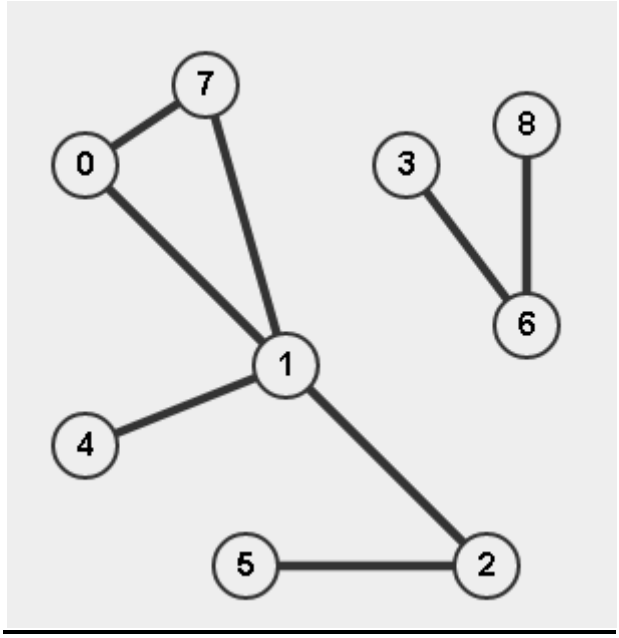
1. Depth-first search can be used to compute a topological ordering of a directed acyclic graph in $O(m^2)$ time.

2. Breadth-first search can be used to compute the strongly connected components of a directed graph in O(m+n) time.

3. Breadth-first search can be used to compute the connected components of an undirected graph in O(m+n) time.

4. Depth-first search can be used to compute shortest paths in O(m+n) time (when every edge has unit length).

Question 3 [11 marks] Consider the graph below and answer the following questions about it.



a. ***[3 marks]*** Draw the adjacency matrix **<u>and</u>** adjacency and edge lists of the graph.

b. ***[2 marks]*** What is the value of the minimum cut of each connected component of the graph? List a minimum cut in each component.

c. ***[6 marks]*** **Write** pseudo-code for an efficient algorithm to compute the connected components of an undirected graph. **Trace** your algorithm on the graph.

Question 4 [5 marks] **<u>Write</u>** pseudo-code for an efficient algorithm to compute a topological sort of a directed graph. **<u>Trace</u>** your algorithm on the graph below