Cairo University
Faculty of Computers and Information
Postgraduate Final Exam (Pre M.Sc.)
Department : Computer Science
Lecturer : Sherif Khattab
Course : Advanced mathematics and algorithms
Course Code : CS612 **Marks** : 60
Date: 21/5/2016 **Time** : 2 hours

Answer as much as you can. Max. grade is 60.

Q1. [19 marks; 1 mark each] Indicate **True or False** and **explain why for both true and false statements**.

a. Gauss's recursive integer multiplication algorithm is $O(n^2)$.
b. Gauss's recursive integer multiplication algorithm is $\theta(n^2)$.
c. Merge sort is an important example of divide and conquer problems.
d. The counting inversions problem reduces to the sorting problem.
e. The input size of the matrix multiplication problem is *n*, where each matrix is *n* x *n*.
f. The in-place implementation of the partition function in quick sort needs no extra memory.
g. $2^{5n} = O(2^n)$.
h. The little-oh is roughly equivalent to strictly less than (<).
i. Linearity of expectation holds only when the random variables are independent.
j. A graph with *n* vertices has exactly 2^n cuts.
k. The number of edges of any graph without parallel edges is $\Omega(n)$ and $O(n^2)$.
l. In Dijkstra's shortest-path algorithm implemented using heaps, the key of each node *u* in the heap is the smallest greedy score of an edge *(u, v)* with *v* not visited.
m. Computing the shortest cycle-free paths in a graph with negative cycles is NP-hard.
n. In a graph without negative cycles, a shortest path has at most *n-1* edges.
o. The fractional knapsack problem is in P.
p. The 0/1 knapsack problem is in NP.
q. The 0/1 knapsack problem is NP-complete.
r. If an edge is the only edge crossing a cut of a graph then the edge can be part of a cycle.
s. The running time of Kruskal's minimum spanning tree algorithm improves by using an optimized implementation of the union-find data type that performs *n* operations in less than *nlogn*.

Q2. [9 marks; 1 mark per space] **Complete** the following statements.

a. The merge function to merge sort is like the _____ function to quick sort. Both have a running time of θ (_____).
b. In asymptotic analysis we ignore _____, _____ because we focus on large values of _____.
c. The knapsack problem can be solved using two dynamic programming algorithms, one runs in *O(_____)* and the other in *O(_____)*.
d. The running time of Kruskal's minimum spanning tree algorithm can be improved using the _____ data type,whereas that of Prim's algorithm can be improved using the _____ data structure.

Q3. [20 marks] **Prove** the following statements.
a. Randomized selection has an average running time of *O(n).*
b. Gauss's recursive algorithm for integer multiplication has a running time of $O(n^{1.59})$.
c. Kosaraju's two-pass algorithm for computing strongly connected components is correct.
*d.*The number of minimum cuts in a graph is at most $\binom{n}{2}$.
e. The 3-step greedy heuristic for knapsack is an ½-approximation algorithm.

Q4. [15 marks] **Write an algorithm** for each of the following. **State** the running time of your algorithms.
a. The *most efficient* algorithm to compute all-pairs shortest paths for a graph with equal edge costs.
b. The Floyd-Warshall all-pairs shortest-paths algorithm.
c. An efficient algorithm to compute the minimum spanning tree of a graph.

Q5. [20 marks] **Explain how** to achieve each of the following.
a. Use Dijkstra's shortest path algorithm to compute single-source longest-paths of a graph.
b. Use Bellman-Ford shortest-paths algorithm to detect cycles (positive and negative).
c. Reconstruct the actual shortest paths by keeping a single one-dimensional array of predecessor pointers in Bellman-Ford shortest paths algorithm.
d. Solve the shortest-path problem using an appropriate decision problem.

Best Wishes

2-2

Examiners	Name	Signature
Examiner 1	Sherif Khattab	
Examiner 2		