Cairo University
Faculty of Computers and Information
Subject: Algorithms
Subject Code: CS316
Examiner(s): Sherif Khattab

Midterm Exam
Semester: 2nd
Date: 1/4/2017
Duration: 1 hour

Answer as much as you can. Max. points = 40

True/False *Indicate whether the sentence or statement is true or false. (**1 pt. each**)*

____ 1. A subproblem has the same inputs and outputs as the original problem but with a smaller size.

____ 2. The brute-force solution of the closest pair problem has a running-time of $\theta(n^2)$.

____ 3. In the closest pair problem, a pair of points with the minimum difference of x-coordinates are always the closest pair.

____ 4. One can always divide a set of points into two (equal) halves by splitting around the point with the average x-coordinate.

____ 5. If an single edge is added to a tree, a cycle is formed.

____ 6. If the input graph has a negative cycle, then the output of Floyd-Warshall all-pairs shortest paths algorithm is always correct.

____ 7. $\log_a n = O(\log_b n)$ for $a \neq b$.

____ 8. If a single edge is removed from a tree, the tree will be disconnected.

Multiple Choice ***(1 pt. each)***

____ 9. Computing the n^{th} Fibonacci number is __________________.
a. a dynamic programming algorithm b. a greedy algorithm c. a greedy problem d. a problem that can be solved by dynamic programming e. Both (a) and (d) are correct.

____ 10. In Floyd-Warshall all-pairs shortest-paths algorithm, what is *A[i, j, 0]* if (1) $i = j$ (2) $(i, j) \in E$ (3) $i \neq j$ and $(i, j) \notin E$
a. $+\infty$, c_{ij}, $+\infty$ b. 0, 0, $+\infty$ c. 0, c_{ij}, $+\infty$ d. 0, c_{ij}, c_{ij}

____ 11. In Kruskal's MST algorithm, the union-find data structure starts with ________ group(s) and ends with ________ group(s).
a. *1, n* b. *n-1, 1* c. *n, 0* d. *n, 1* e. *n, n*

____ 12. In a union-find data structure with *n* objects, at most _________ union operations can be performed.
a. Exactly *n* b. Any number c. *n log n* d. Two e. Exactly *n-1*

____ 13. Kruskal's algorithm for computing the minimum spanning tree creates no cycles in the resulting tree because ___________________.
a. It selects exactly one crossing edge for each cut in the graph. b. If an edge will create a cycle, it is not added to the tree. c. Because the input graph is always acyclic. d. Both (b) and (c) e. When an edge is added by the algorithm it is the first crossing edge for some cut.

____ 14. In each iteration of the Kruskal's algorithm for computing the minimum spanning tree using the union-find data structure, how many find and union operations are performed?
a. Two find and two union operations. b. At most two find operations and exactly one union.
c. Exactly one find and two union operations. d. Exactly two find operations and one union.
e. Exactly two find operations and at most one union.

ID: ________________________

____ 15. To solve the all-pairs shortest-paths algorithm, one needs ________ invocations of a single-source shortest-paths algorithm.
a. *n-1* b. *n* c. n^2 d. *1* e. *0*

Matching ***(1 pt. each)***

Match each of the following algorithms and recurrences with its corresponding case in the Master method. Each case can be used more than once.

a. Case 1: $a=b^d$ c. Case 2: $a<b^d$

b. Case 3: $a>b^d$

____ 16. Strassen's matrix multiplication
____ 17. Merge Sort
____ 18. $T(n) \le 2T(\frac{n}{2}) + O(n^2)$
____ 19. Gauss's recursive integer multiplication
____ 20. Binary Search

Short Answer ***(1 pt. per space)***

21. Complete the following table that compares different algorithms for solving the all-pairs shortest-paths problem.

Algorithm	Running-time in a sparse graph	Running-time in a dense graph	Limitations
n invocations of Dijkstra's single-source shortest-paths algorithm	______________	______________	________________
n invocations of Bellman-Ford single-source shortest-paths algorithm	______________	______________	negative edges OK no negative cycles
Floyd-Warshall all-pairs shortest-paths algorithm	n^3	______________	________________

22. In the recurrence $T(n) \le aT(\frac{n}{b}) + O(n^d)$ that represents the running-time of a divide-and-conquer algorithm, the constant a represents ______________________, the constant b represents ______________________________, the constant d represents ____, and the number of leaves of the corresponding recursion tree is ______________________.

23. If the minimum spanning tree of a graph *G* is *T*, a contradiction is reached if a different spanning tree *T** can be constructed and T* is __________ than *T*.

24. The number of edges in a tree with *n* nodes is ________________.

ID: ________________________

25. In the union-find data structure with **only** lazy unions, the find operation takes *O(__________)* time and the union operation takes *O(__________)* time.

26. Two optimizations of the union-find data structure are union by _________ and _________ compression.

Problems ***(3 pts. each)***

27. <u>**Correct**</u> the nested loops in the following algorithms. <u>**Explain**</u> the rationale behind your solution.

a. Floyd-Warshall all-pairs shortest-paths algorithm

```
for i = 1 to n
  for j = 1 to n
    for k = 1 to n
      A[i, j, k] = min {A[i, j, k-1], A[i, k, k -1] + A[k, j, k-1]}
```

b. optimal binary search tree algorithm.

```
for i = 1 to n
  for j = 1 to n
```

A[i, j] = $\min_{r=i,\ldots,j}\{\sum_{k=i}^{j} p_k + A[i,r-1] + A[r+1,j]\}$

28. Prove that $a^{\log_b n} = n^{\log_b a}$

ID: ________________________

29. Suppose we have keys x < y < z and we know that: 80% of searches are for x, 10% of searches are for y, 10% of searches are for z.

    What is the average search time (i.e., number of nodes looked at) in the tree