

## The Accuracy of Neglecting The S-dependence and its Effect on The Exchange Parts of $\alpha$ - $\alpha$ Interaction Potential.

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**Abstract:** The study show the effect of the local density and the accuracy of neglecting s-dependence on the exchange part of  $\alpha$ - $\alpha$  interaction potential. This effect is about 30% for the force BDM3Y2-Reid while it is less than 7% for BDM3Y1-Reid. For BDM3Y3 force which corresponds to large value of compressibility coefficient, the corresponding error is too large at separation distance  $R=0$ .

**Key word:**  $\alpha$ - $\alpha$  interaction potential - local density- exchange part.

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### INTRODUCTION

The most famous models used for deriving the interaction potential are the two-center shell model (Gupta *et al.*, 1974; Greiner and Scheid 1971), the energy density formalism (Holm and Greiner 1972; Ismail *et al.*, 1996), the proximity method (Blucki *et al.*, 1977) and the double folding model (Satchler and Love 1979). Recently, the double folding model was developed in two ways. The first is the calculation of its exchange part using finite range NN force. This development produces some computational difficulties if the non diagonal densities, appearing in this part, were not approximated using the density matrix expansion method of Negele and Vautherin (Negele and Vautherin 1972). The other development is the use of realistic effective NN force which is usually density dependent (Dao khoa 1988; Dao T.Khoa *et al.*, 1997). Again density dependence of NN force produces difficulties in numerical calculations. To overcome this difficulty the nucleon-nucleon separation distance in the density dependent NN force is considered when calculating the direct HI potential part and is neglected for the exchange part. This approximation in considering the density dependence of NN interaction needs to be tested.

The double-folding model (Sinha 1975; Chaudhuri *et al.*, 1985) proved to be quite successful in obtaining the correct values of the real part of the optical model potential needed to fit elastic scattering data. In this model the real potential  $U_n(R)$  is the sum of the direct part  $U_D(R)$  and the exchange part  $U_{ex}(R)$ . Earlier studies led to the calculation of the real potential on the assumption that the exchange part of the nucleon-nucleon interaction is a zero range (Misicu and Greiner 2002; Azab Farid and Satchler 1985). Such approximation produces correct ion-ion potential in the tail region only.

The inner and surface regions can not be calculated with good accuracy using  $\delta$ -force assumption. However, it is well established now that in certain cases of nuclear scattering observed first in  $\alpha$ -particle (Goldberg *et al.*, 1973) and later for other light HI systems (Bohlen *et al.*, 1982; Stiliais *et al.*, 1989), where the data are sensitive to the HI optical potential over a wider radial domain, the simple double folding model (Sinha 1975; Chaudhuri, Basu and Sinha 1985) failed to give a good description to the data. Therefore, some further developments of the folding model have been made to obtain a more realistic shape of the folding potential. One of these approaches is to impose on the widely used M3Y-NN interaction (Bertsch *et al.*, 1977) explicit density dependence, the DDM3Y1 interaction (Kobos *et al.*, 1982). Another is to treat correctly the simple knock out effect arising from the Pauli principle. In the later approach the exchange part of the real HI optical potential is derived from the first principle instead of using a zero-range pseudo potential (Brandon and Satchler 1997; 1991), adopted in the calculation with M3Y and DDM3Y1 interactions (Sartor *et al.*, 1981; Kobos *et al.*, 1982; Krappe, Nix, and Sierk 1979). In ref. (Dao khoa 1988; Dao T.Khoa *et al.*, 1997; Osman, Balkan 1998), a simple microscopic approach has been developed based on generalization of the double folding and the density matrix (Ismail, M.M. Osman and F. Salah 1996; Sinha, and Moszkowski 1979). In the framework of this model, the real part of the HI optical potential at a separation distance  $R$  between the centers of two ions is obtained by:

$$U_n(R) = U_D(R) + U_{ex}(R) = \sum_{\substack{i \in A1 \\ j \in A2}} \{ \langle ij | V_D | ij \rangle + \langle ij | V_{ex} | ji \rangle \} \quad (1)$$

where  $|i\rangle$  and  $|j\rangle$  refer to the single-particle wave functions of nucleons in the two colliding nuclei  $A1$  and  $A2$ , respectively;  $V_D$  and  $V_{ex}$  are the direct and exchange parts of the effective NN interaction.

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In this work we calculate  $U_{ex}(R)$  for  $\alpha$ - $\alpha$  nuclear pairs in the frame work of the generalized double folding model (Dao *et al.*, 1195; Bertsch *et al.*, 1977) using the effective BDM3Y (Dao *et al.*, 1995; Khoa *et al.*, 1995; Khoa *et al.*, 1997; Bertsch *et al.*, 1977; Faessler, 1984; Saloner and Toepffer, 1977) nucleon-nucleon force. This work shows the accuracy of neglecting the S-dependence in the density-dependent NN interaction on the exchange parts of  $\alpha$ - $\alpha$  interaction potential.

**Calculation of The Exchange HI Potential Using Density Dependent NN Forces:**

The exchange part of the real heavy ion potential must be non-local (Sinha and Moszkowski 1979). Since the exact treatment of the non-local exchange term is too complicated and needs a lot of numerical calculations, one usually obtains the equivalent local potential by representing the relative-motion wave function of the two interacting nucleons as plane wave(Sinha and Moszkowski 1979). Under such an assumption, the exchange part of the HI potential (Dao T.khoa 1988: Chaudhuri,D.N.Basu and B.Sinha 1985). is represented by,

$$U_{ex}(R) = \sum_{\substack{i \in A_1 \\ j \in A_2}} \{ \langle ij | v_{ex}(s) | ji \rangle \} \tag{2}$$

By introducing the one -body density matrix (Dao T.khoa 1988: Dao Khoa, Satchler and Von Oertzen 1997), one can explicitly write equation (2) as:

$$U_{ex}(R) = \int \rho_1(\vec{r}_1, \vec{r}_1 + \vec{s}) \rho_2(\vec{r}_2, \vec{r}_2 - \vec{s}) v_{ex}(s) \exp\left(\frac{i\vec{k}(R) \cdot \vec{s}}{M}\right) d\vec{r}_1 d\vec{r}_2 \tag{3}$$

where  $\vec{k}(R)$  is the relative -motion momentum given by (Dao T.khoa 1988: Dao Khoa, Satchler and Von Oertzen 1997).

$$k^2(R) = \frac{2mM}{\hbar^2} [E_{c.m} - U_n(R) - V_c(R)] \tag{4}$$

where M and  $E_{c.m}$  are the reduced mass,  $M = \frac{A_1 A_2}{A_1 + A_2}$  and relative energy in the center of mass system and m is the nucleon mass. Here  $U_n(R)$  and  $V_c(R)$  are total nuclear potential ( $U_n(R) = U_D(R) + U_{ex}(R)$ ), and coulomb potential, respectively. For density dependent NN force equation (3) can be written as,

$$U_{ex}(R) = \int_0^\infty d\vec{s} e^{\frac{i\vec{k} \cdot \vec{s}}{M}} v_{ex}(s) G^{ex}(\vec{R}, \vec{s}) \tag{5}$$

where  $G^{ex}(\vec{R}, \vec{s})$  is given by (Misicu and W.Greiner 2002: Golin, petrovich and Robson 1976).

$$G^{ex}(\vec{R}, \vec{s}) = \int d\vec{y} \rho_T(\vec{y}) \hat{j}_1(k_{eff}(\vec{y})) \rho_P\left(\vec{y} - \vec{R} + \frac{1}{2}\vec{s}, \vec{y} - \vec{R} - \frac{1}{2}\vec{s}\right) F\left(\rho_T\left(\vec{y} - \frac{1}{2}\vec{s}\right), \rho_P\left(\vec{y} - \vec{R} + \frac{1}{2}\vec{s}\right)\right) \tag{6}$$

where density matrix expansion was used to simplify the non-diagonal density of the target nucleus. In most calculations (Dao T.khoa 1988: Dao Khoa, Satchler and Von Oertzen 1997) the s-dependence in the function F is neglected. In this case the direct and exchange parts of HI potential are not computed in consistent way regarding to the variables of density dependence in NN force. The function F has the following expression for BDM3Yn (n=1,2,3) type of forces

$$F(\rho_P + \rho_T) = c(1 - \alpha(\rho_P + \rho_T)^\beta) \tag{7}$$

$\beta$  have the values =1,2 and 3. The projectile non-diagonal density matrix can be expanded using DME(Campi, and Bonnyssy 1978) or it can be calculated exactly using oscillator model if the projectile is double

closed shell light nucleus (as O16 and He4 ) (Ismail Osman and Salah 1996). If one neglects the s-dependence in  $\rho$  appearing in the function F, it becomes

$$F(\rho) = F(\rho_p \left( \left| \vec{y} - \vec{R} \right| \right) + \rho_T(y)) \tag{8}$$

In general  $G^{ex}$  depends on both the magnitude and direction of  $\vec{S}$ . In this case, it is very difficult to calculate  $U_{ex}(R)$ . So, we should use approximate methods to take into account the s-dependence in density dependent part of NN force. We use the approximations (Phys 1976: Goldfarb Nagel 1980).

$$\rho \left( \vec{R} \pm \frac{1}{2} \vec{S} \right) = \rho(\vec{R}) + \frac{35}{2sk_f^3} j_3(sk_f) \left( \frac{1}{4} \nabla^2 \rho(\vec{R}) \right) \tag{9}$$

$$\rho \left( \vec{R} \pm \frac{1}{2} \vec{S} \right) = \rho(\vec{R}) \hat{j}_1(k_{eff}(R)s) \tag{10}$$

And

$$\rho \left( \vec{R} \pm \frac{1}{2} \vec{S} \right) = \left[ \frac{3j_1(sk_f)}{sk_f} + \frac{35}{2sk_f^3} j_3(sk_f) \left( \frac{1}{4} \nabla^2 + \frac{3}{5} k_f^2 \right) \right] \rho(\vec{R}) \tag{11}$$

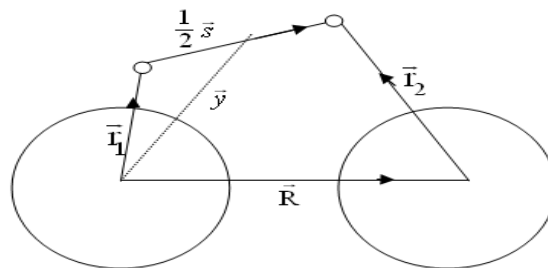
Taking  $\vec{R}$  in Z-direction and integrating over the variable  $\phi_y$  to get,

$$U_{ex}(R) = 4\pi \int s^2 ds v_{ex}(s) j_0 \left( \frac{ks}{M} \right) G^{ex}(R, s) \tag{12}$$

where  $G^{ex}(R, s)$  is given by (Dao Khoa *et al.*, 1997; Ismail and Ellithi 2002).

$$G^{ex}(R, s) = c \int y^2 dy d\cos\theta_y \rho(y, s) \rho(d, s) [1 - \alpha[\rho_T(y) + \rho_p(d)]]^\beta \tag{13}$$

where  $d = \sqrt{y^2 - 2Ry\cos\theta + R^2}$ . Since the nucleon relative momentum depends on the total real potential, the calculation of Uex need UD as input.



**Fig. 1:** Coordinates used in the exchange part of the nucleus-nucleus interaction:

### RESULTS AND DISCUSSION

Since the exact treatment of s-dependence in calculating nucleus-nucleus is too difficult. We use the approximations in equations (9-11). For  $\alpha$  - particle,

$$\rho(r) = \frac{4\alpha^3}{\pi\sqrt{\pi}} e^{-\alpha^2 r^2}, \rho(r, s) = \rho(r) e^{-\frac{1}{4}\alpha^2 s^2} \tag{14}$$

where  $\alpha = \frac{1}{b}$  and b is the oscillator parameter and it has the value b= (1.33) fm.

The effect of taking s-dependence in density appearing in NN force is to produce more attractive  $\alpha$ -  $\alpha$  interaction potential at small separation distances. The tables show that the error in neglecting s-dependence affects strongly the values of U<sub>ex</sub> and U<sub>tot</sub> .

Table (1) contains the results for the separation distance variation of the  $\alpha$ -  $\alpha$  interacting pair calculated using BDM3Yn (n=1,2,3)- Reid force. Table (2) is the same as Table (1) expect Paris version of NN force was used. Part (a), (b) and (c) denote calculations using BDM3Yn with n=1,2 and 3 respectively. Both the tables were calculated for projectile energy per nucleon EL/AP= 5.2 MeV/N. The first column of each table shows the value of the nucleus-nucleus separation distance (R), the second column shows both U<sub>ex</sub> and UD+U<sub>ex</sub> (denoted by U<sub>tot</sub>) . The second column was calculated using harmonic oscillator wave function to construct the diagonal and non diagonal densities. The third, forth and fiftieth column are our calculations by using the three approximations. The error on the tables are calculated as

$$\text{Error} = \left( \frac{V_s(R) - V(R)}{V_s(R)} \right) \times 100$$

Tables (1) and (2) show the results of  $\alpha$ -  $\alpha$  calculated using a value b=1.33. The tables show that the effect of neglecting s-dependence increases as the NN force becomes corresponding to higher value of incompressibility coefficient. This effect is about 30%(23%) in U<sub>ex</sub> (U<sub>tot</sub>) for the force BDM3Y2-Reid type while it is less than 7% for BDM3Y1-Ried. It decreases as the  $\alpha$ -  $\alpha$  separation distance increases. For BDM3Y3 type of force which corresponds to large value of compressibility coefficient, the corresponding error is too large at separation distance R=0, as Tables (1c) and (2c) indicate .This error becomes reasonable as the separation distance between the two  $\alpha$  particles increases. The huge error obtained is due to the small radius of  $\alpha$ -particle (about 1.58 fm.) and its large central density. Since the force BDM3Y3 corresponds to nuclear matter saturation curve in which  $\frac{E}{A}$  varies strongly when  $\rho$  becomes greater than the saturation density

$\rho_0 (\cong 0.17 \text{ fm}^{-3})$  ,we expect large change in potential when the density changes slightly .In other words the force BDM3Y3 is too sensitive to the value of  $\rho$  when the latter has value more than  $0.17 \text{ fm}^{-3}$  . In order to decrease this error,  $\alpha$  -particle is artificially made less localised. For this purpose we increased  $\langle r^2 \rangle^{\frac{1}{2}}$  by about 13% to becomes (1.83 fm.) which corresponds to b=1.5 fm. Tables (3a) and (3b) are the same as (1c) and (2c) respectively except they are calculated using b=1.5fm. These tables show that when the  $\alpha$ -particle becomes less localised, the error becomes reasonable.

**Table(1a):** Calculation of the exchange and total  $\alpha$ -  $\alpha$  potential using density dependent (BDM3Y1-Reid) NN interaction. Harmonic oscillator wave functions used in the calculations to construct the densities. The second column covers r and s to calculations

using  $\rho = \rho_T \left( \bar{r}_1 + \frac{1}{2} \bar{s} \right) + \rho_P \left( \bar{r}_2 - \frac{1}{2} \bar{s} \right)$  which the other columns are calculated by approximating the expression

$$\rho = \rho_T \left( \bar{r}_1 \right) + \rho_P \left( \bar{r}_2 \right) . b=1.33 \text{ fm.}$$

R	HO	Approximation1	Error%	Approximation2	Error%	Approximation3	Error%
	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>
0	-70.35	-74.44	-5.81	-74.9	-6.47	-75.07	-6.71
1	-60.47	-63.43	-4.89	-63.81	-5.52	-63.94	-5.74
2	-35.14	-36.18	-2.96	-36.36	-3.47	-36.41	-3.61
3	-11.7	-11.82	-1.03	-11.86	-1.37	-11.86	-1.37
4	-1.98	-1.98	0.00	-1.98	0.00	-1.98	0.00
5	-0.17	-0.17	0.00	-0.17	0.00	-0.17	0.00
6	-0.01	-0.01	0.00	-0.01	0.00	-0.01	0.00

**Table(1b):** Is the same as Table (1a) but using density dependent (BDM3Y2) force.

R	HO	Approximation1	Error%	Approximation2	Error%	Approximation3	Error%
	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>
0	-42.93	-54.66	-27.32	-55.83	-30.05	-56.34	-31.24
1	-46.5	-53.67	-15.42	-54.49	-17.18	-54.82	-17.89
2	-33.54	-35.26	-5.13	-35.53	-5.93	-35.61	-6.17
3	-11.49	-11.65	-1.39	-11.69	-1.74	-11.69	-1.74
4	-1.87	-1.87	0.00	-1.88	-0.53	-1.88	-0.53
5	-0.15	-0.15	0.00	-0.15	0.00	-0.15	0.00
6	-0.01	-0.01	0.00	-0.01	0.00	-0.01	0.00

**Table(1c):** Is the same as Table (1a) but using density dependent ( BDM3Y3) force.

R	HO	Approximation1	Error%	Approximation2	Error%	Approximation3	Error%
	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>
0	33.06	-12.85	138.87	-16.01	148.43	-17.3	152.33
1	-14.48	-34.84	-140.61	-36.78	-154.01	-37.52	-159.12
2	-31.38	-34.35	-9.46	-34.76	-10.77	-34.88	-11.15
3	-11.38	-11.58	-1.76	-11.62	-2.11	-11.63	-2.20
4	-1.81	-1.82	-0.55	-1.82	-0.55	-1.82	-0.55
5	-0.15	-0.15	0.00	-0.15	0.00	-0.15	0.00
6	-0.01	-0.01	0.00	-0.01	0.00	-0.01	0.00

**Table(2a):** Is the same as Table (1a) but for M3Y Paris

R	HO	Approximation1	Error%	Approximation2	Error%	Approximation3	Error%
	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>
0	-132.3	-136.91	-3.48	-137.28	-3.76	-137.4	-3.85
1	-113.96	-117.19	-2.83	-117.49	-3.10	-117.59	-3.19
2	-65.07	-66.12	-1.61	-66.26	-1.83	-66.3	-1.89
3	-20.81	-20.93	-0.58	-20.96	-0.72	-20.96	-0.72
4	-3.44	-3.44	0.00	-3.44	0.00	-3.44	0.00
5	-0.3	-0.3	0.00	-0.3	0.00	-0.3	0.00
6	-0.01	-0.01	0.00	-0.01	0.00	-0.01	0.00

**Table(2b):** Is the same as Table (2a) but using density dependent ( BDM3Y2) force.

R	HO	Approximation1	Error%	Approximation2	Error%	Approximation3	Error%
	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>
0	-57.25	-71.13	-24.24	-72.22	-26.15	-72.7	-26.99
1	-75.19	-83.22	-10.68	-83.93	-11.62	-84.22	-12.01
2	-60.17	-61.93	-2.93	-62.14	-3.27	-62.21	-3.39
3	-20.29	-20.44	-0.74	-20.47	-0.89	-20.47	-0.89
4	-3.21	-3.22	-0.31	-3.22	-0.31	-3.22	-0.31
5	-0.26	-0.26	0.00	-0.26	0.00	-0.26	0.00
6	-0.01	-0.01	0.00	-0.01	0.00	-0.01	0.00

**Table(2c):** Is the same as Table (2a) but using density dependent ( BDM3Y3) force.

R	HO	Approximation1	Error%	Approximation2	Error%	Approximation3	Error%
	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>
0	133.33	73.27	45.05	69.54	47.84	68.1	48.92
1	7.7	-16.39	312.86	-18.31	337.79	-19.02	347.01
2	-53.25	-56.33	-5.78	-56.67	-6.42	-56.77	-6.61
3	-19.94	-20.14	-1.00	-20.16	-1.10	-20.17	-1.15
4	-3.09	-3.1	-0.32	-3.1	-0.32	-3.1	-0.32
5	-0.25	-0.25	0.00	-0.25	0.00	-0.25	0.00
6	-0.01	-0.01	0.00	-0.01	0.00	-0.01	0.00

**Table(3a):** Is the same as Table (1c) but for using b=1.5 fm.

R	HO	Approximation1	Error%	Approximation2	Error%	Approximation3	Error%
	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>
0	-53.17	-59.2	-11.34	-59.68	-12.24	-59.88	-12.62
1	-51.27	-55.07	-7.41	-55.4	-8.06	-55.53	-8.31
2	-36.09	-37.09	-2.77	-37.19	-3.05	-37.23	-3.16
3	-15.05	-15.17	-0.80	-15.19	-0.93	-15.2	-1.00
4	-3.6	-3.61	-0.28	-3.61	-0.28	-3.61	-0.28
5	-0.51	-0.51	0.00	-0.51	0.00	-0.51	0.00
6	-0.04	-0.04	0.00	-0.04	0.00	-0.04	0.00

**Table(3b):** Is the same as Table (2c) but for using b=1.5 fm.

R	HO	Approximation1	Error%	Approximation2	Error%	Approximation3	Error%
	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>	U <sub>ex</sub>
0	-93.62	-100.31	-7.15	-100.71	-7.57	-100.89	-7.77
1	-92.71	-96.8	-4.41	-97.06	-4.69	-97.18	-4.82
2	-65.12	-66.12	-1.54	-66.2	-1.66	-66.23	-1.70
3	-25.81	-25.92	-0.43	-25.93	-0.46	-25.94	-0.50
4	-5.91	-5.91	0.00	-5.92	-0.17	-5.92	-0.17
5	-0.82	-0.82	0.00	-0.82	0.00	-0.82	0.00
6	-0.07	-0.07	0.00	-0.07	0.00	-0.07	0.00

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