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Viscous Quark-Gluon Plasma in the Early Universe

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Abstract

We consider the evolution of a flat, isotropic and homogeneous Friedmann-Robertson-Walker Universe, filled with a causal bulk viscous cosmological fluid, that can be characterized by an ultra-relativistic equation of state and bulk viscosity coefficient obtained from recent lattice QCD calculations. The basic equation for the Hubble parameter is derived under the assumption that the total energy in the Universe is conserved. By assuming a power law dependence of bulk viscosity coefficient, temperature and relaxation time on energy density, an approximate solution of the field equations has been obtained, in which we utilized equations of state from recent lattice QCD simulations QCD and heavy-ion collisions to derive an evolution equation. In this treatment for the viscous cosmology, we found no evidence for singularity. For example, both Hubble parameter and scale factor are finite at $t = 0$, t is the comoving time. Furthermore, their time evolution essentially differs from the one associated with non-viscous and ideal gas. Also thermodynamic quantities, like temperature, energy density and bulk pressure remain finite as well. In order to prove that the free parameter in our model does influence the final results, qualitatively, we checked out other particular solutions.

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I. INTRODUCTION

The dissipative effects, including both bulk and shear viscosity, are supposed to play a very important role in the early evolution of the Universe. The first attempts at creating a theory of relativistic fluids were those of Eckart [1] and Landau and Lifshitz [2]. These theories are now known to be pathological in several respects. Regardless of the choice of equation of state, all equilibrium states in these theories are unstable and in addition signals may be propagated through the fluid at velocities exceeding the speed of light. These problems arise due to the first order nature of the theory, that is, it considers only first-order deviations from the equilibrium leading to parabolic differential equations, hence to infinite speeds of propagation for heat flow and viscosity, in contradiction with the principle of causality. Conventional theory is thus applicable only to phenomena which are quasi-stationary, i.e. slowly varying on space and time scales characterized by mean free path and mean collision time.

A relativistic second-order theory was found by Israel [3] and developed by Israel and Stewart [4], Hiscock and Lindblom [5] and Hiscock and Salmonson [6] into what is called "transient" or "extended" irreversible thermodynamics. In this model deviations from equilibrium (bulk stress, heat flow and shear stress) are treated as independent dynamical variables, leading to a total of 14 dynamical fluid variables to be determined. For general reviews on causal thermodynamics and its role in relativity see [7].

Causal bulk viscous thermodynamics has been extensively used for describing the dynamics and evolution of the early Universe or in an astrophysical context. But due to the complicated character of the evolution equations, very few exact cosmological solutions of the gravitational field equations are known in the framework of the full causal theory. For a homogeneous Universe filled with a full causal viscous fluid source obeying the relation $\xi \sim \rho^{1/2}$, with ρ the energy density of the cosmological fluid, exact general solutions of the field equations have been obtained in [8–12]. It has also been proposed that causal bulk viscous thermodynamics can model on a phenomenological level matter creation in the early Universe [8]. Exact causal viscous cosmologies with $\xi \sim \rho^{s}$, $s \neq 1/2$ have been considered in Ref. [9].

Because of technical reasons, most investigations of dissipative causal cosmologies have assumed Friedmann-Robertson-Walker (FRW) symmetry (i.e. homogeneity and isotropy) or small perturbations around it [12]. The Einstein field equations for homogeneous models with dissipative fluids can be decoupled and therefore are reduced to an autonomous system of first order ordinary differential equations, which can be analyzed qualitatively [13].

The role of a transient bulk viscosity in a FRW space-time with decaying vacuum has been discussed in [14]. Models with causal bulk viscous cosmological fluid have been considered recently [15]. They obtained both power-law and inflationary solutions, with the gravitational constant an increasing function of time. The dynamics of a viscous cosmological fluids in the generalized Randall-Sundrum model for an isotropic brane were considered in [16]. The renormalization group method was applied to the study of homogeneous and flat FRW Universes, filled with a causal bulk viscous cosmological fluid, in [17]. A generalization of the Chaplygin gas model, by assuming the presence of a bulk viscous type dissipative term in the effective thermodynamic pressure of the gas, was investigated recently in [18].

Recent RHIC results give a strong indication that in the heavy-ion collisions experiments, a hot dense matter can be formed [19]. Such an experimental evidence might agree with the "new state of matter" as predicted in the Lattice QCD simulations [20]. However, the experimentally observed elliptic flow in peripheral heavy-ion collisions seems to indicate that a thermalized collective QCD matter has been produced. In a addition to that, the success of ideal fluid dynamics in explaining several experimental data e.g. transverse momentum spectra of identified particles, elliptic flow [21], together with the string theory motivated that the shear viscosity η to the entropy s would have the lower limit $\approx 1/4\pi$ [22] leading to a paradigm that in heavy- ion collisions, that a *nearly* perfect fluid likely be created and the quarks and gluons likely go through relatively rapid equilibrium characterized with a thermalization time less than 1 fm/c [23].

According to recent lattice QCD simulations [24], the bulk viscosity ξ is not negligible near the QCD critical temperature T_c. It has been shown that the bulk and shear viscosity at high temperature T and weak coupling α_s, $\xi \sim \alpha_s^2 T^3/\ln\alpha_s^{-1}$ and $\eta \sim T^3/(\alpha_s^2\ln\alpha_s^{-1})$ [26]. Such a behavior obviously reflects the fact that near T_c QCD is far from being conformal. But at high T, QCD approaches conformal invariance, which can be indicated by low trace anomaly $(\epsilon - 3p)/T^4$ [27], where ϵ and p are energy and pressure density, respectively. In the quenched lattice QCD, the ratio ζ/s seems to diverge near T_c [28].

To avoid the mathematical difficulties accompanied with the Abel second type non-homogeneous and non-linear differential equations [29], one used to model the cosmological fluid as an ideal (non-viscous) fluid. No doubt that the viscous treatment of the cosmological background should have many essential consequences [30]. The thermodynamical ones, for instance, can profoundly modify the dynamics and configurations of the whole cosmological background [31]. The reason is obvious. The bulk viscosity is to be expressed as a function of the Universe energy density ρ [32]. Much progress has been achieved in relativistic thermodynamics of dissipative fluids. The pioneering theories of Eckart [1] and Landau and Lifshitz [2] suffer from lake of causality constrains. The currently used theory is the Israel and Stewart theory [3, 4], in which the causality is conserved and theory itself seems to be stable [5, 7].

In this article, we aim to investigate the effects that bulk viscosity has on the Early Universe. We consider a background corresponding to a FRW model filled with ultra-relativistic viscous matter, whose bulk viscosity and equation of state have been deduced from recent heavy-ion collisions experiments and lattice QCD simulations.

The present paper is organized as follows. The basic equations of the model are written down in Section II. In Section III we present an approximate solution of the evolution equation. Section IV is devoted to one particular solution, in which we assume that $H = const$. The results and conclusions are given in Sections VI and VII, respectively.

II. EVOLUTION EQUATIONS

We assume that geometry of the early Universe is filled with a bulk viscous cosmological fluid, which can be described by a spatially flat FRW type metric given by

$$ds^2 = dt^2 - a^2\left(t\right)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]. \tag{1}$$

The Einstein gravitational field equations are:

$$R_{ik} - \frac{1}{2}g_{ik}R = 8\pi G T_{ik}. \tag{2}$$

In rest of this article, we take into consideration natural units, i.e., $c = 1$, for instance.

The energy-momentum tensor of the bulk viscous cosmological fluid filling the very early Universe is given by

$$T_i^k = \left(\rho + p + \Pi\right)u_i u^k - \left(p + \Pi\right)\delta_i^k, \tag{3}$$

where i, k takes $0, 1, 2, 3$, ρ is the mass density, p the thermodynamic pressure, Π the bulk viscous pressure and u_i the four velocity satisfying the condition $u_i u^i = 1$. The particle and entropy fluxes are defined according to $N^i = nu^i$ and $S^i = sN^i - \left(\tau\Pi^2/2\xi T\right)u^i$, where n is the number density, s the specific entropy, $T \geq 0$ the temperature, ξ the bulk viscosity coefficient, and $\tau \geq 0$ the relaxation coefficient for transient bulk viscous effect (i.e. the relaxation time), respectively.

The evolution of the cosmological fluid is subject to the dynamical laws of particle number conservation $N^i_{;i} = 0$ and Gibbs' equation $Td\rho = d\left(\rho/n\right) + pd\left(1/n\right)$. In the following we shall also suppose that the energy-momentum tensor of the cosmological fluid is conserved, that is $T^k_{i;k} = 0$.

The bulk viscous effects can be generally described by means of an effective pressure Π, formally included in the effective thermodynamic pressure $p_{eff} = p + \Pi$ [7]. Then in the comoving frame the energy momentum tensor has the components $T^0_0 = \rho, T^1_1 = T^2_2 = T^3_3 = -p_{eff}$. For the line element given by Eq. (1), the Einstein field equations read

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}G\,\rho, \tag{4}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3}G\,\left(3p_{eff} + \rho\right), \tag{5}$$

where one dot denotes derivative with respect to the time t, G is the gravitational constant and a is the scale factor.

Assuming that the total matter content of the Universe is conserved, $T^j_{i;j} = 0$, the energy density of the cosmic matter fulfills the conservation law:

$$\dot{\rho} + 3H\left(p_{eff} + \rho\right) = 0, \tag{6}$$

where we introduced the Hubble parameter $H = \dot{a}/a$. In presence of bulk viscous stress Π, the effective thermodynamic pressure term becomes $p_{eff} = p + \Pi$. Then Eq. (6) can be written as

$$\dot{\rho} + 3H\left(p + \rho\right) = -3\Pi H. \tag{7}$$

For the evolution of the bulk viscous pressure we adopt the causal evolution equation [7], obtained in the simplest way (linear in Π) to satisfy the H-theorem (i.e., for the entropy production to be non-negative, $S^i_{;i} = \Pi^2/\xi T \geq 0$ [3, 4]). According to the causal relativistic Israel-Stewart theory, the evolution equation of the bulk viscous pressure reads [7]

$$\tau\dot{\Pi} + \Pi = -3\xi H - \frac{1}{2}\tau\Pi\left(3H + \frac{\dot{\tau}}{\tau} - \frac{\dot{\xi}}{\xi} - \frac{\dot{T}}{T}\right). \tag{8}$$

In order to have a closed system from equations (4), (7) and (8) we have to add the equations of state for p and T.

As shown in Appendix A, the equation of state, the temperature and the bulk viscosity of the quark-gluon plasma (QGP), can be determined approximately at high temperatures [25] from recent lattice QCD calculations [34], as

$$P = \omega\rho, \qquad T = \beta\rho^{r}, \qquad \xi = \alpha\rho + \frac{9}{\omega_0}T_c^4, \tag{9}$$

with $\omega = (\gamma - 1)$, $\gamma \simeq 1.183$, $r \simeq 0.213$, $\beta \simeq 0.718$,

$$\alpha = \frac{1}{9\omega_0}\frac{9\gamma^2 - 24\gamma + 16}{\gamma - 1}, \tag{10}$$

and $\omega_0 \simeq 0.5 - 1.5$ GeV. In the following we assume that $\alpha\rho >> 9/\omega_0 T_c^4$, and therefore we take $\xi \simeq \alpha\rho$. In order to close the system of the cosmological equations, we have also to give the expression of the relaxation time τ, for which we adopt the expression [7],

$$\tau = \xi\rho^{-1} \simeq \alpha. \tag{11}$$

Eqs. (9) are standard in the study of the viscous cosmological models, whereas the equation for τ is a simple procedure to ensure that the speed of viscous pulses does not exceed the speed of light. Eq. (11) implies that the relaxation time in our treatment is constant but strongly depends on EoS. These equations are without sufficient thermodynamical motivation, but in the absence of better alternatives, we shall follow the practice of adopting them in the hope that they will at least provide some indication of the range of bulk viscous effects. The temperature law is the simplest law guaranteeing positive heat capacity.

With the use of Eqs. (8), (9) and (11), respectively, we obtain the following equation describing the cosmological evolution of the Hubble function H

$$\ddot{H} + \frac{3}{2}[1 + (1 - r)\gamma]H\dot{H} + \frac{1}{\alpha}\dot{H} - (1 + r)H^{-1}\dot{H}^2 + \frac{9}{4}(\gamma - 2)H^3 + \frac{3}{2}\frac{\gamma}{\alpha}H^2 \ = \ 0. \tag{12}$$

III. AN APPROXIMATE SOLUTION

We introduce the transformation $u = \dot{H}$, so that Eq. (12) is transformed into a first order ordinary differential equation,

$$u\frac{du}{dH} - (1 + r)H^{-1}u^2 + \left(\frac{3}{2}[1 + (1 - r)\gamma]H + \alpha^{-1}\right)u + \frac{9}{4}\frac{1}{(\gamma)}H^3 + \frac{3}{2}\frac{\gamma}{\alpha}H^2 = 0. \tag{13}$$

We can rewrite Eq. (13) in the form

$$\Omega \frac{d\Omega}{dH} = F_1(H)\Omega + F_0(H), \tag{14}$$

where

$$\begin{aligned}
\Omega &= u\,E \;\; = u\,\exp\left(-\int \frac{1+r}{H} dH\right), \\
F_1(H) &= -\left(\frac{3}{2}[1+(1-r)\gamma]H + \frac{1}{\alpha}\right) E, \\
F_0(H) &= -\left(\frac{9}{4}(\gamma-2)H^3 + \frac{3}{2}\frac{\gamma}{\alpha}H^2\right) E^2.
\end{aligned}$$

By introducing a new independent variable $z = \int F_1(H)\, dH$, we obtain

$$\Omega \frac{d\Omega}{dz} - \Omega = g(z), \tag{15}$$

with $g(z)$ is defined parametrically as,

$$g(z) = \frac{F_0}{F_1}. \tag{16}$$

As shown in Appendix B, $g(z)$ can be approximated as a simple function of z

$$g(z) \approx \mathcal{C}\, z, \tag{17}$$

where $\mathcal{C}$ is a constant. We proceed with this approximation to get solvable differential equations. Keeping the parametric solution of $g(z)$, Eq. (50), results in much more complicated differential equations. This would be the subject of a future work.

From the definitions of Ω and z we have

$$\Omega = H^{1+r}\dot{H}, \tag{18}$$

$$z = H^{2+r}\left(\frac{-3[1+(1-r)\gamma]H}{2(1-r)} + \frac{1}{\alpha r}\right), \tag{19}$$

Analogous to the solution of reduced Abel type canonical equation,

$$y\frac{dy}{dx} - y = ax \tag{20}$$

(see Appendix C) we obtain the relation $\Omega = z/\mathcal{P}$. Therefore, from Eqs. (18) and (19) we obtain the following first order differential equation for Hubble parameter H,

$$\mathcal{P}\dot{H} = \frac{-3[1+(1-r)\gamma]}{2(1-r)}H^2 + \frac{1}{\alpha r}H \tag{21}$$

with the solution

$$H(t) = \frac{B}{\exp(-Bt/\mathcal{P}) - A} \tag{22}$$

where

$$A = \frac{-3[1+(1-r)\gamma]}{2(1-r)}, \qquad B = \frac{1}{\alpha r}, \tag{23}$$

and $\mathcal{P}$ is taken as a free parameter. We can assign any real value to $\mathcal{P}$. For the results presented in this work, we used a negative value. This negative sign is necessarily to overcome the sign from the integral limits. The geometric and thermodynamic quantities of the Universe read

$$a(t) = a_0 \left(\frac{\exp(-Bt/\mathcal{P})}{\exp(-Bt/\mathcal{P}) - A}\right)^{\mathcal{P}/A}, \tag{24}$$

$$\rho(t) = 3\,H^2 = 3\left(\frac{B}{\exp(-Bt/\mathcal{P}) - A}\right)^2, \tag{25}$$

$$T(t) = \beta\rho^r = \beta\left(3\frac{B^2}{[\exp(-Bt/\mathcal{P}) - A]^2}\right)^r, \tag{26}$$

$$\Pi(t) = -2\dot{H} - 3\gamma H^2 = -\frac{B^2}{\mathcal{P}}\left(\frac{2\exp(-Bt/\mathcal{P}) + 3\gamma\mathcal{P}}{[\exp(-Bt/\mathcal{P}) - A]^2}\right), \tag{27}$$

$$q(t) = \frac{d}{dt}H^{-1} - 1 = -\frac{1}{\mathcal{P}}\exp(-Bt/\mathcal{P}) - 1. \tag{28}$$

a_0 is an arbitrary constant of the integration. The sign of q indicates whether the Universe decelerates (positive) or accelerates (negative). q can also be given as a function of the thermodynamic, gravitational and cosmological quantities $q(t) = [\rho(t)+3p(t)+3\Pi(t)]/2\rho(t)$ [33].

IV. DE SITTER UNIVERSE

Besides the approximation in $g(z)$, previous solution apparently depends on the free parameter $\mathcal{P}$. In this section, we suggest a particular solution to overcome $\mathcal{P}$. Eq. (12) can easily be obtained by assuming that H doesn't depend one t, i.e, de Sitter Universe. With a simple calculation, we get an estimation for H

$$H = \frac{4}{9}\frac{\alpha^{-1}\gamma}{2-\gamma}. \tag{29}$$

The geometric and thermodynamic parameters of the Universe are given by

$$a(t) = a_0 \exp\left[\frac{4\alpha^{-1}\gamma}{9(2-\gamma)} t\right], \tag{30}$$

$$\rho(t) = 3\left[\frac{4\alpha^{-1}\gamma}{9(2-\gamma)}\right]^2, \tag{31}$$

$$T(t) = 3^r \beta \left[\frac{4\alpha^{-1}\gamma}{9(2-\gamma)}\right]^{2r}, \tag{32}$$

$$\Pi(t) = -3\gamma \left[\frac{4\alpha^{-1}\gamma}{9(2-\gamma)}\right]^2, \tag{33}$$

$$q(t) = -1. \tag{34}$$

Although we have assumed here that the cosmic background is filled with viscous matter, the assumption that $H = const$ results in an exponential scale parameter, Eq. (30). This behavior characterizes the de Sitter space, when $\Lambda = k = 0$. ρ and T are finite at small t as given in Fig. 2.

V. PARTICULAR SOLUTION

Another particular solution for Eq. (12) can be obtained, when assuming that the dependence of u on H can be given by the polynomial in Eq. (21)

$$u = b_1 H^2 + b_2 H, \tag{35}$$

where b_1 and b_2 are constants. Some simple calculations show that this form is a solution of the initial equation, Eq. (13), if

$$b_1 = -\frac{3}{2}\frac{1+\gamma}{1-r}, \tag{36}$$

$$b_2 = \frac{1}{r\alpha}. \tag{37}$$

b_2 is identical to B in Eq. (23). r and γ have to satisfy the compatibility relation

$$r = \frac{2-\gamma}{2+\gamma^2}. \tag{38}$$

Integrating Eq. (35) results in

$$H(t) = \frac{b_2 \exp(-b_2 t)}{1 - b_1 \exp(-b_2 t)}, \tag{39}$$

where minus sign in the exponential function refers to flipping the integral limits. This was not necessary while deriving the expressions given in Section III. The free parameter P compensates it. The geometric and thermodynamic quantities of the Universe read

$$a(t) = a_0 \left(\frac{\exp(b_2 t) - b_1}{\exp(b_2 t)}\right)^{1/b_1}, \tag{40}$$

$$\rho(t) = 3 \left(\frac{b_2 \exp(-b_2 t)}{1 - b_1 \exp(-b_2 t)}\right)^2, \tag{41}$$

$$T(t) = 3^r \, \beta \left(\frac{b_2 \exp(-b_2 t)}{1 - b_1 \exp(-b_2 t)}\right)^{2r}, \tag{42}$$

$$\Pi(t) = \frac{b_2^2 \left[2 \exp(b_2 t) - 3\gamma\right]}{\left[\exp(b_2 t) - b_1\right]^2}, \tag{43}$$

$$q(t) = \exp(b_2 t) - 1. \tag{44}$$

Obviously , we notice that the scale parameter in Eq. (40) looks like Eq. (24), which strongly depends on the free parameter $\mathcal{P}$. The other geometric and thermodynamic quantities find similarities in Eq. (25) - (28), respectively. Deceleration parameter q seems to be positive everywhere.

VI. RESULTS

In present work, we have considered the evolution of a full causal bulk viscous flat, isotropic and homogeneous Universe with bulk viscosity parameters and equation of state taken from recent lattice QCD data and heavy-ion collisions. Three classes of solutions of the evolution equation have been obtained.

In Fig. 1, $H(t)$ and $a(t)$ are depicted in dependence on the comoving time t. We compare $H(t)$, given by Eq. (22), and $a(t)$, given by Eq. (24), with the counterpart parameters obtained in the case when the background matter is assumed to be an ideal and non-viscous fluid, described by the equations of state of the non-interacting ideal gas,

$$H(t) = \frac{1}{2t}, \tag{45}$$

$$a(t) = \sqrt{t}. \tag{46}$$

In the left panel of Fig. 1, $H(t) = \dot{a}/a$ has an exponential decay, whereas in the non-viscous case, $H(t)$ is decreasing according to Eq. (45). The latter is much slower than the former, reflecting the nature of the exponential and linear dependencies. The other

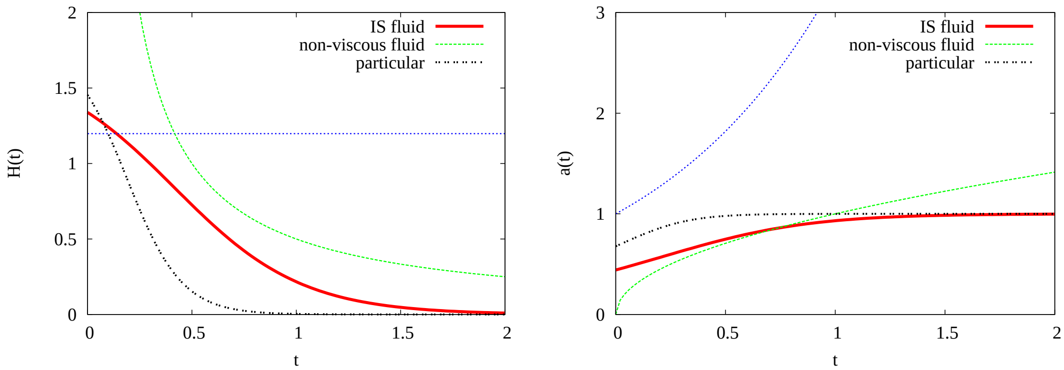

Fig. 1: Left panel: $H(t)$ vs. t for viscous (solid line) and non-viscous fluid (dashed line). Contrary to non-viscous fluid, $H(t)$ is our solution shows no singularity. The straight line gives the results from the particular solution, Eq. (29). Double dotted curve represent the particular solution, Eq. (39). Right panel: $a(t)$ vs. t. The approximate solution gives finite $a(t)$ at $t = 0$. (solid line). Dotted straight line represents the results from Eq. (30). The particular solution is given by the double dotted line, Eq. (40).

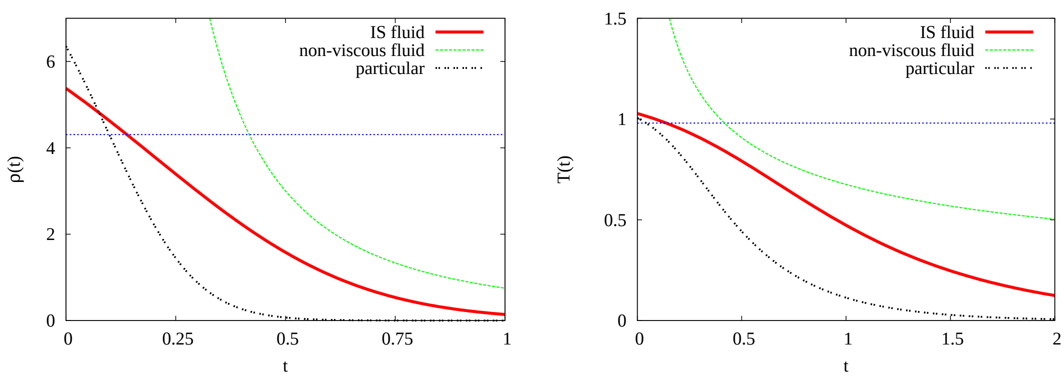

Fig. 2: Energy density $\rho(t)$ as a function of t (left panel). The dependence of T on t is given in the right panel. The curves are as in Fig. 1. In both case, viscous fluid gives no singularity at vanishing t. Straight curve are from Eq. (31) and (32), respectively.

difference between the two cases is obvious at small t. We notice a divergence, or singularity, associated with the ideal non-viscous fluid, Eq. (45). The viscous fluid results in finite H even at vanishing t, as can be seen from Eq. (22).

The scale factor $a(t)$ also shows differences in both cases. $a(t)$ in a Universe with an ideal and non-viscous background matter depends on t according to Eq. (46), which simply implies that $a(t)$ is directly proportional to t, and $a(t)$ vanishes at $t = 0$, which shows the existence of a singularity of H. Assuming that the background matter is described by a

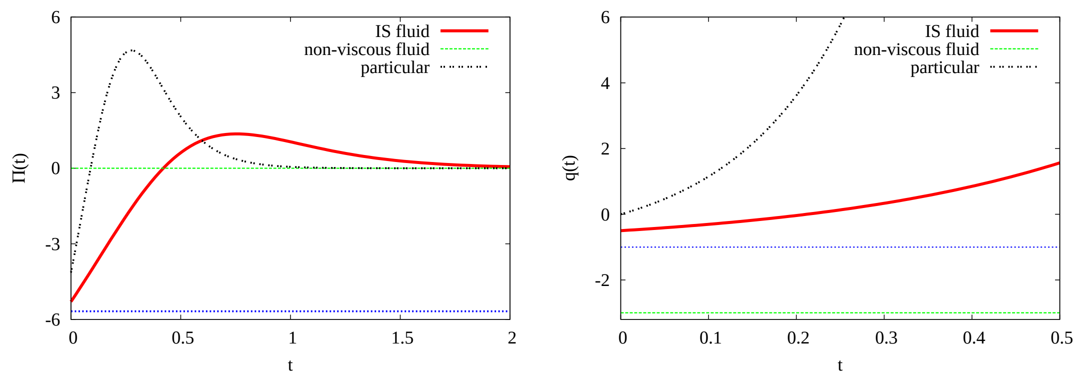

Fig. 3: Left panel: bulk pressure Π depicted in dependence on t. At small t, Π jumps from negative to positive values. At larger t, Π vanishes as the case in the non-viscous fluid (dashed line). The bottom line gives Π according to first particular solution, Eq. (33). Right panel: deceleration parameter q is depicted with t. Straight and dotted lines represent non-viscous fluid and first particular solution, Eq. (34), respectively. Top curve represents second particular solution, Eq. (44).

viscous fluid results in different $a(t)$-behaviors with increasing t. At $t = 0$, $a(t)$ remains finite. Correspondingly, $H(t)$ remains also finite. In general, the dependence on t is much more complicated than in Eq. (46). Here we have an $A/\mathcal{P}$ root of an exponential function. If $\exp(-Bt/\mathcal{P}) >> A$, a remains constant.

Fig. 2 illustrates the dependence of the two thermodynamical quantities, ρ and T, on the comoving time. The non-viscous Universe shows a singular behavior in ρ at vanishing t, as shown in the left panel of Fig. 2. This is not obvious in the case where we have taken into consideration a finite viscosity coefficient, i.e., ρ is finite at $t = 0$. In both cases, ρ is decreasing with increasing t, reflecting that the Early Universe was likely expanding. Also the life time of the thermal viscous Universe seems to be shorter than for the non-viscous Universe. Almost the same behavior is observed in the right panel of Fig. 2. The temperature T seems to be finite at vanishing t in the viscous Universe. The T-singularity is only present, if we assume that the background matter is non-viscous ideal gas.

In left panel of Fig. 3, we show the dependence of the bulk viscous pressure Π on t. Π takes negative values at very small t. Then it switches to positive values at some values of t. After reaching the maximum value, Π decays exponentially with increasing t. At larger t, Π entirely vanishes. The deceleration parameter q, given by Eq. (28), is depicted in the right panel of Fig. 3, and it is compared with q for a non-viscous fluid, $q = -3$.

The approximate solution, given by Eq. (28), results in negative q at small t, referring to expansion era. q from the particular solution, Eq. (34) is negative everywhere.

For the particular solution, only the scale factor depends on t, Eq. (30). The results are given in the right panel of Fig. 1. All cosmological and thermodynamical quantities given by Eq. (29) and Eqs. (31)-(34) are constant in time.

VII. CONCLUSIONS

It is obvious that the bulk viscosity plays an important role in the evolution of the Early Universe. Despite of the simplicity of our model, it shows that a better understanding of the dynamics of our Universe is only accessible, if we use reliable equation of state to characterize the matter filling out the cosmic background.

We conclude that the causal bulk viscous Universe described by the approximate solution starts its evolution from an initial non-singular state with a non-zero initial value of Hubble parameter $H(t)$ and scale factor $a(t)$, where t is the comoving time. In this treatment, t is given in GeV^{-1}. Also the thermodynamical quantities, energy density ρ for instance, are finite at vanishing t. Even the temperature T itself shows no singularity at $t = 0$. The Hubble parameter H decreases monotonically with T similar to ρ. The bulk viscous pressure Π likely satisfies the condition that $\Pi < 0$ at very small t indicating to inflationary era. Then Π switches to positive value. It reaches a maximum value and then decays and vanishes, exponentially, at large t. The deceleration parameter q shows an expanding behavior in the case of non-viscous ideal gas and first particular solution. For second particular solution, q starts from zero and increases, exponentially. According to this solution, the Universe was decelerating. The approximate solution shows an interesting behavior in $q(t)$, Eq. (28). At small t, the values of q are negative, i.e. the Universe was accelerating (expansion). At larger t, a non-inflationary behavior sets on, $q > 0$, i.e., the Universe switched to a decelerating evolution.

In this treatment, we assumed that the Universe is flat, $k = 0$, and the background geometry is filled out with QCD matter (QGP) with a finite viscosity coefficient. The resulting Universe is obviously characterized by a shortly increasing and afterward constant scale factor and a fast vanishing Hubble parameter. At $t = 0$, both $a(t)$ and $H(t)$ remain finite, i.e., there is no singularity. The validity of our treatment depends on the validity of the equations of states, Eq. 9, which we have deduced from the lattice QCD simulations at temperatures larger than $T_c \approx 0.19$ GeV. Below T_c, as the Universe cooled down, not only the degrees of freedom suddenly increase [36] but also the equations of state turn to be the ones characterizing the hadronic matter. Such a phase transition - from QGP to hadronic matter - would characterize one end of the validity of our treatment. The other limitation is the very high temperatures (energies), at which the strong coupling α_s entirely vanishes.

[1] C. Eckart, Phys. Rev. **58**, 919 (1940).

[2] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics*, Butterworth Heinemann (1987).

[3] W. Israel, Ann. Phys. **100**, 310 (1976).

[4] W. Israel and J. M. Stewart, Phys. Lett. **A58**, 213 (1976).

[5] W. A. Hiscock and L. Lindblom, Ann. Phys. **151**, 466 (1989).

[6] W. A. Hiscock and J. Salmonson, Phys. Rev. **D43**, 3249 (1991).

[7] R. Maartens, Class. Quantum Grav. **12**, 1455 (1995); R. Maartens, *Causal thermodynamics in relativity*, `astro-ph/9609119` (1996).

[8] L. P. Chimento and A. S. Jakubi, Class. Quantum Grav. **14**, 1811 (1997) ; L. P. Chimento and A. S. Jakubi, Int. J. Mod. Phys. **D7**, 177 (1998); M. K. Mak and T. Harko, Gen. Rel. Grav. **30**, 1171 (1998); Gen. Rel. Grav. **31**, 273 (1999); J. Math. Phys. **39**, 5458 (1998).

[9] T. Harko and M. K. Mak, Int. J. Theor. Phys. **38**, 1561 (1999).

[10] M. K. Mak and T. Harko, Aust. J. Phys. **52**, 659 (1999).

[11] M. K. Mak and T. Harko, Int. J. Mod. Phys. **D9**, 97 (2000); Aust. J. Phys. **53**, 241 (2000); Int. J. Mod. Phys. **D9**, 475 (2000).

[12] R. Maartens and J. Triginer, Phys. Rev. **D56**, 4640 (1997).

[13] A. A. Coley and R. J. van den Hoogen, Class. Quantum Grav. **12**, 1977 (1995) ; A. A. Coley and R. J. van den Hoogen, Phys. Rev. **D54**, 1393 (1996) ; A. Di Prisco, L. Herrera and J. Ibanez, Phys. Rev. **D63**, 023501 (2001).

[14] Abdussatar and R. G. Vishwakarma, Class. Quantum Grav. **14**, 945 (1997).

[15] A. I. Arbab and A. Beesham, Gen. Rel. Grav. **32**, 615 (2000).

[16] C. M. Chen, T. Harko, and M. K. Mak, Phys. Rev. **D64**, 124017 (2001); T. Harko and M. K. Mak, Class. Quant. Grav. **20**, 407 (2003).

[17] J. A. Belinchon, T. Harko, and M. K. Mak, Class. Quant. Grav. **19**, 3003 (2002).

[18] C. S. J. Pun, L. A. Gergely, M. K. Mak, Z. Kovacs, G. M. Szabo, and T. Harko, Phys.Rev. **D77**, 063528 (2008).

[19] BRAHMS Collaboration, I. Arsene *et al.*, Nucl. Phys. A **757**, 1 (2005).
PHOBOS Collaboration, B. B. Back *et al.*, Nucl. Phys. A **757**, 28 (2005).
PHENIX Collaboration, K. Adcox *et al.*, Nucl. Phys. A **757** (2005)

STAR Collaboration, J. Adams *et al.*, Nucl. Phys. A **757** (2005)

[20] F. Karsch, E. Laermann, P. Petreczky, S. Stickan and I. Wetzorke, 2001 *Proccedings of NIC Symposium*, Ed. H. Rollnik and D. Wolf, John von Neumann Institute for Computing, Jülich, NIC Series, **9**, (2002).

[21] P. F. Kolb and U. Heinz, in *Quark-Gluon Plasma 3*, edited by R. C. Hwa and X.-N. Wang, World Scientific, Singapore, (2004).

[22] G. Policastro, D. T. Son and A. O. Starinets, Phys. Rev. Lett. **87**, 081601 (2001); JHEP **0209**, 043 (2002)

[23] R. J. Fries, J. Phys. G **34**, S851 (2007).

[24] D. Kharzeev and K. Tuchin, JHEP **0809**, 093 (2008).

[25] F. Karsch, D. Kharzeev, and K. Tuchin, Phys. Lett. B **663**, 217 (2008).

[26] P. Arnold, C. Dogan, and G. D. Moore, Phys. Rev. D **74**, 085021 (2006).

[27] A. Bazavov, *et al.*, Phys. Rev. **D80**, 014504 (2009).

[28] H. Meyer, Phys. Rev. Lett. **100**, 162001 (2008).

[29] A. Tawfik, H. Mansour and M. Wahba, Invited talk given at the 7th international conference on "Modern Problems of Nuclear Physics", 22-25 September 2009, Tashkent-Uzbekistan, arXiv:0911.4105 [gr-qc];
Talk given at 12th Marcel Grossmann Meeting on "General Relativity", Paris-France, 12-18 July 2009, arXiv:0912.0115 [gr-qc].

[30] A. Tawfik, AIP Conf. Proc. **1115**, 239 (2009), arXiv:0809.3825 [hep-ph].

[31] A. Di Prisco, L. Herrera, J. Ibanez, Phys. Rev. D **63**, 023501 (2001).

[32] V. Belinskii, E. Nikomarov, I. Khalantikov, Sov. Phys. JETP, **50**, 213 (1979).

[33] E. W. Kolb and M. S. Turner, *The Early Universe*, Addison-Wesley Publ. Co. (1990).

[34] M. Cheng *et al.*, arXiv:0710.0354 [hep-lat].

[35] A. Tawfik and D. Toulban, Phys. Lett. B **623**, 48 (2005).

[36] F. Karsch, K. Redlich and A. Tawfik, Eur. Phys. J. **C29** 549 (2003), e-Print: hep-ph/0303108; Phys. Lett. **B571** 67 (2003), e-Print: hep-ph/0306208;
K. Redlich, F. Karsch and A. Tawfik, J. Phys. **G30** S1271 (2004), e-Print: nucl-th/0404009;
A. Tawfik, Phys. Rev. **D71** 054502, (2005), e-Print: hep-ph/0412336.

Appendix A: Viscosity coefficient $\xi(T)$ from LQCD

Following the discussion presented in [34], the bulk viscosity of QGP can be calculated from the lattice QCD by Eq. (13) in that paper. We assume that the decay factors for pions and kaons are vanishing above the critical temperature of the phase transition QGP-hadrons. The quark-antiquark condensates can be neglected at temperatures higher than the critical one [35]. Therefore, Eq. (22) of Ref. [34] would be reduced to

$$9\,\omega_0\,\xi = T\,s\,\left(\frac{1}{c_s^2}-3\right) - 4(\rho-3p) + 16|\epsilon_v| \tag{47}$$

where ρ is the energy density and $c_s^2 = dp/d\rho$ is the square of the speed of sound. The parameter ω_o is a scale depending on the temperature T, and defines the validity of the underlying perturbation theory. In this relation, the viscosity is assumed to have a thermal part which can be determined through lattice calculations, and a vacuum contributing part, which can be fixed using quark and gluon condensates. The vacuum part would take the value

$$16|\epsilon_v|(1+\frac{3}{8}\cdot 1.6) \simeq (560\text{ MeV})^4 \simeq (3\,T_c)^4 \tag{48}$$

Our algorithm is the following. Using lattice QCD results on trace anomaly, $(\epsilon-3p)/T^4$, and other thermodynamical quantities, we can determine the bulk viscosity. To make use of the lattice QCD results, it is useful to make a suitable fit to the data at high temperatures. Then we obtain the following equations of state

$$p \,=\, \omega\rho, \qquad\qquad T = \beta\rho^r, \qquad\qquad c_s^2 = \omega$$

where $\omega = 0.319$, $\beta = 0.718 \pm 0.054$ and $r = 0.23 \pm 0.196$. Using the equations of state, Eq. (49) in Eq. (47), we obtain

$$\xi(\epsilon) = \frac{1}{9\omega_o}\frac{9\gamma^2-24\gamma+16}{\gamma-1}\rho + \frac{9}{\omega_o}T_c^4. \tag{49}$$

Appendix B: Estimations of $g(z)$

For analytical purposes, the function $g(z)$, which is defined in z parameter as $g(z) = F_0/F_1$ in Eq. (16), can be numerically estimated depending on the parameter z by using the following procedure. First, we plot it parametrically depending on the parameter H, Fig.

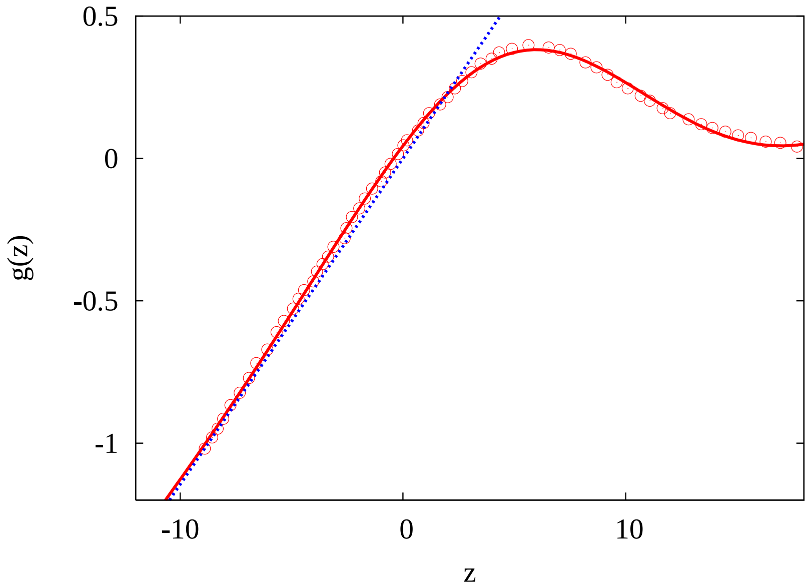

Fig. 4: The parametric dependence of $g(z)$ on z is given by open symbols. Eq. (50) is depicted as solid line. The dotted line represents the linear fit, Eq. (51).

(4). Then we fit the resulting curve to various functions. Based on least-square fit, best choice would be a mixture of polynomial and exponential functions,

$$g(z) = a + b\,z + c\frac{\exp(d\,z) + e}{[\exp(d\,z) + f]^2}, \tag{50}$$

where the coefficients read $a = -2.078 \pm 0.117$, $b = 0.091 \pm 0.007$ and $c = 17.332 \pm 1.553$, $d = 0.189 \pm 0.003$, $e = -0.814 \pm 0.162$ and $f = 2.849 \pm 0.02$. At small values of z, it is clear that the dependence is linear,

$$g(z) = c + \mathcal{C} z. \tag{51}$$

Obviously, the intersect c is much smaller than the slope $\mathcal{C}$. The sign of $g(z)$ can be flipped regarding to the sign of its independent variable z. Accordingly, we get

$$g(z) \approx \mathcal{C} z. \tag{52}$$

To prove this dependence, algebraically, we try to estimate $g(z)$ directly from the division of F_0 by F_1, which can be approximated by including their first terms only, i.e.

$$g(H) \approx \frac{3(\gamma - 2)}{2[1 + (1-r)\gamma]}\, H^{1-r}, \tag{53}$$

Then, we approximate $z(H)$ to the form,

$$z(H) \approx -\frac{3[1 + (1-r)\gamma]}{2(1-r)}\, H^{1-r}. \tag{54}$$

Finally, we now able to derive an approximate estimation for $g(z)$. According to Eq. (53) and (54), we get

$$g(z) \approx \frac{(1-r)(\gamma-2)}{[1+(1-r)\gamma]^2}\, z \tag{55}$$

Amazingly, this expression looks the same as the one we obtained from the numerical approximation with

$$\mathcal{C} = \frac{(1-r)(\gamma-2)}{[1+(1-r)\gamma]^2}. \tag{56}$$

Appendix C: Solution of Abel equation $y\dot{y} - y = ax$

To solve Eq. (20) we divide the whole equation by y^3 and introduce a new variable $v = 1/y$. Then Eq. (20) reads

$$\frac{dv}{dx} + v^2 + axv^3 = 0. \tag{57}$$

We then introduce the function $v = w/x$.

$$x\frac{dw}{dx} = w - w^2 - aw^3, \tag{58}$$

Previous differential equation can be solved by separation of variables

$$\int \frac{dw}{w - w^2 - aw^3} = \ln C^{-1}x, \tag{59}$$

where C is an arbitrary constant of integration. To calculate the integral, we write the function to be integrated as

$$\frac{1}{w - w^2 - aw^3} = \frac{1}{w} - \frac{aw}{aw^2 + w - 1} - \frac{1}{aw^2 + w - 1}. \tag{60}$$

Let us assume that $\Delta = 1 + 4a > 0$ (this implies that $a > 0$).

$$\int \frac{dw}{w - w^2 - aw^3} = -\frac{1}{2\sqrt{\Delta}} \ln \frac{2aw - \sqrt{\Delta} + 1}{2aw + \sqrt{\Delta} + 1} - \frac{1}{2} \ln\left(aw^2 + w - 1\right) + \ln w. \tag{61}$$

Therefore the general solution of Eq. (58) can be written as

$$x = C\frac{w}{\sqrt{aw^2 + w - 1}} \left(\frac{2aw + \sqrt{\Delta} + 1}{2aw - \sqrt{\Delta} + 1}\right)^{1/2\sqrt{\Delta}}, \tag{62}$$

leading to

$$y = \frac{1}{v} = \frac{x}{w} = C\frac{1}{\sqrt{aw^2 + w - 1}} \left(\frac{2aw + \sqrt{\Delta} + 1}{2aw - \sqrt{\Delta} + 1}\right)^{1/2\sqrt{\Delta}}. \tag{63}$$