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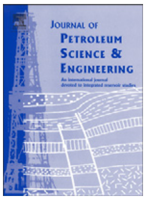

An approach for minimizing errors in computing effective porosity in reservoir of shaly nature in view of Wyllie–Raymer–Raiga relationship

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ABSTRACT

Effective porosity (ϕ_E) in clean and shaly formations is evaluated based on a newly introduced formulation that requires acoustic-radiation data and provides superior accuracy over other commonly used formulae based on various porosity tools. This equation, which is termed as a second order *acoustic porosity-V_{sh}* model, relates acoustic transit time to matrix and fluid in addition to shale parameters. In both clean and shaly formations, this relationship can be applied safely for evaluating effective porosity specifically for old wells that suffer from insufficient logging data such as density-neutron logs and in the absence of core regardless the differences observed. It can also be considered as a *quick-look* approach in interpreting the recent exploratory wells before the decision of completing the logging measurements. Successfulness of applying the proposed equation for evaluating "ϕ_E" to real field data is justified as far as reliability and range of accuracies are concerned.

1. Introduction

In quantitative well log interpretation, good porosity values are always evaluated either from core measurements or from well log analysis of porosity tools (density; ρ_b, neutron; ϕ_N, and acoustic; Δ_t). In the absence of core measurements, combination of at least two porosity tools is helpful for evaluating porosity. In old wells, lacking of these tools obstacle such determination and the use of empirical formula is highly demanding. There are several empirical formulas for computing porosity from sonic transit time has been derived by many investigators. Each within limit can give reasonable values for porosity and takes into consideration the effect of specific parameter. For example, Wyllie et al. (1956) and Raymer et al. (1980) in *clean* formations considered the effect of fluid and matrix in their equations utilizing the data derived from sonic logs. These are:

$$\phi_s = \frac{\Delta_t - \Delta_{tma}}{\Delta_{tf} - \Delta_{tma}} \tag{1}$$

(Wyllie et al., 1956).

In unconsolidated sand, Tixier et al. (1959) introduced the compaction factor into Eq. (1) in the form

$$\phi_S = \frac{\Delta_t - \Delta_{tma}}{\Delta_{tf} - \Delta_{tma}} \times \frac{1}{C_P} \tag{2}$$

where $C_P = (\Delta_{tsh} \times C)/100$ is the Compaction factor (Δ_{tsh} is the transit time in adjacent shales in µs/ft. and C is constant which is normally 1.0 (Hilchie, 1978)).

$$\Delta_t = \left[\frac{(1-\phi_S)^2}{\Delta_{tma}} + \frac{\phi_S}{\Delta_{tf}}\right]^{-1} \tag{3}$$

(Raymer et al., 1980)where ϕ_S is the sonic-derived porosity, Δ_t is the interval transit time of a compressional sound wave, Δ_{tma} is the matrix transit time, and Δ_{tf} is the fluid transit time.

Another approach taken by Raiga-Clemenceau et al. (1988) that sonic transit time (Δ_t) and porosity (ϕ_S) better than Wyllie in the form:

$$\phi_S = 1 - \left(\frac{\Delta_{tma}}{\Delta_t}\right)^{1/x} \tag{4}$$

where x is the matrix exponent related to the matrix nature. Eq. (4) considers only the effect of matrix parameter (Δ_{tma}) and exponent (x).

Table 1 summarizes the advantage and limitations of each approaches used for derived porosity from sonic transit time in clean formations (i.e., $V_{sh} < 10\%$).

Kamel et al. (2002) noticed that matrix transit time Δ_{tma} is inversely proportional to the matrix exponent x indicated in Table 1. Accordingly, he introduced an empirical equation for computing the matrix exponent (x) once the value of the matrix transit time is obtained. This equation expressed as:

$$x = 55.196\Delta_{tma}^{-0.8843} \tag{5}$$

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Table 1
Different approaches for derived porosity from sonic transit time in clean formation.

Reference	Working equation	Eq. no.	Remarks
Wyllie et al., 1956	$\phi_S = \frac{\Delta_t - \Delta_{tma}}{\Delta_{tf} - \Delta_{tma}}$	1	* Very popular * Works with consolidated sandstones and carbonates with intergranular porosity * Gives good correlations between porosity and acoustic interval transit time * It uses 55.5 µs/ft. for sand, 47.6 µs/ft. for limestone, and 43.5 µs/ft. for dolomite.
Tixier et al., 1959	$\phi_S = \frac{\Delta_t - \Delta_{tma}}{\Delta_{tf} - \Delta_{tma}} \times \frac{1}{C_P}$ $C_P = \frac{\Delta_{tsh} \times C}{100}$	2	* In unconsolidated sand * It uses 55.5 µs/ft. for sand, 47.6 µs/ft. for limestone, and 43.5 µs/ft. for dolomite
Raymer et al., 1980	$\Delta_t = \left[\frac{(1-\phi_S)^2}{\Delta_{tma}} + \frac{\phi_S}{\Delta_{tf}}\right]^{-1}$	3	* Essentially empirical * Assumes the fluid is a liquid, not a gas. * It uses 54 µs/ft. for sand, 49 µs/ft. for limestone, and 44 µs/ft. for dolomite.
Raiga-Clemenceau et. al., 1988..	$\phi_S = 1 - \left(\frac{\Delta_{tma}}{\Delta_t}\right)^{1/x}$	4	* Does not account for any effect of the pore fluids. * Uses 55.5 µs/ft. for sand, 47.5 µs/ft. for calcite, and 43.5 µs/ft. for dolomite (Schlumberger, 1972). * x is the exponent related to the matrix nature. 1.6 for sand, 1.76 for calcite and 2 for dolomite.

He was also able to classify the area under the curve into zones in which the ranges of each lithologic type (including salt, sandstone, gypsum, anhydrite, limestone, and dolomite) can be clearly identified and nearly separated on the matrix exponent scale shown in Fig. 1 and listed in Table 2.

In 1979, Dresser Atlas introduced formula for evaluating porosity in shaly formation from sonic logs takes into account the effect of matrix (Δ_{tma}), fluid (Δ_{tf}) in addition to shale volume V_{sh} ($10\% < V_{sh} < 33\%$) and shale transit time (Δ_{tsh}).

$$\phi_e = \left[\frac{\Delta_t - \Delta_{tma}}{\Delta_{tf} - \Delta_{tma}} \times \frac{1}{C_P}\right] - V_{sh}\left[\frac{\Delta_{tsh} - \Delta_{tma}}{\Delta_{tf} - \Delta_{tma}}\right] \qquad (6)$$

where $C_P = (\Delta_{tsh} \times C)/100$ is the Compaction factor (Δ_{tsh} is the transit time in adjacent shales in µs/ft. and C is constant which is normally 1.0 (Hilchie, 1978)).

The interval transit time (Δ_t) of a formation is increased due to the presence of hydrocarbons (i.e. hydrocarbon effect). If the effect of hydrocarbons is not corrected, the sonic derived porosity will be too high. Hilchie (1978) suggests the following empirical corrections for hydrocarbon effect:

$$\phi_{Sc} = \phi_{Sonic} \times 0.7(\text{gas}) \qquad (7)$$

$$\phi_{Sc} = \phi_{Sonic} \times 0.9(\text{oil}) \qquad (8)$$

Dresser atlas treats the effect of shaliness as a correction term subtracted from the clean-sand term, which indicates the practical aspect of the shale effect and its correction.

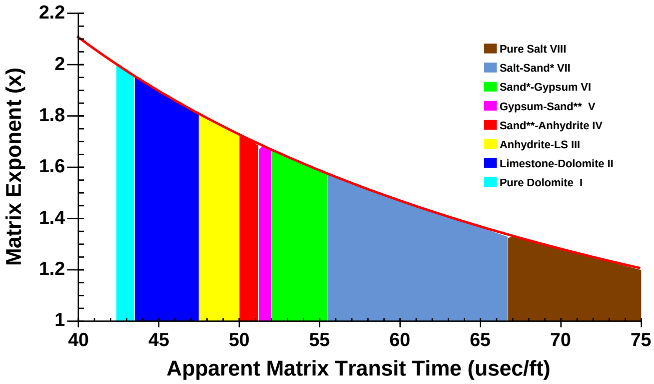

Fig. 1. Chart for determining matrix exponent (x) and identifying the type of lithology using apparent matrix transit time (after Kamel et al. 2002).

Kamel et al., 2002 introduce an equation relating acoustic travel time to the matrix and fluid times with the aim of quantifying sonic porosity is introduced

$$\phi_S = \sqrt{\frac{(\Delta_t - \Delta_{tma})\left(\Delta_t^{1/x} - \Delta_{tma}^{1/x}\right)}{\Delta_t^{1/x}\left(\Delta_{tf} - \Delta_{tma}\right)}} \qquad (9)$$

Kamel and Mohamed 2006 are to introduce an equation for estimating effective porosity in clean/shaly formations from sonic logs of the form as:

$$\phi_s = A\left[V_P - B\left(C + \frac{DV_P}{B}\right)^2\right] \qquad (10)$$

where $A = \Delta_{tf}$ (µs/ft.), $B = \frac{1}{\Delta_{tma}}$ (µs/ft.)$^{-1}$, $C = 1 - \frac{1}{\rho_{ma} - \rho_f}$ (g/cc)$^{-1}$, and $D = \frac{1}{\rho_{ma} - \rho_f}$ (g/cc)$^{-1}$, are constants to be determined according to the type of matrix and fluid. The values of these constants are listed in Table 3.

Table 2
Classification of different lithologic types illustrated in Fig. 1 (after Kamel et al. 2002).

	Name	Δ_{tma} (µs/ft.)	x
I	Pure salt	67	1.33
II	Salt–sand*	67–55.5	1.33 to 1.58
III	Sand*–gypsum	55.5–52	1.58 to 1.66
IV	Gypsum–sand**	52–51.2	1.66 to 1.68
V	Sand**–anhydrite	51.2–50	1.68 to 1.735
VI	Anhydrite–limestone	50–47.5	1.735 to 1.81
VII	Limestone–dolomite	47.5–43.5	1.81 to 1.955
VIII	Pure dolomite	43.5	1.955

Sand*: $V_{ma} = 18{,}000$, $\phi > 10\%$, Sand**: $V_{ma} = 19{,}500$, $\phi < 10\%$.

Table 3
The values of constants used in Eq. (10) for different matrices and fluids.

	Sandstone		Limestone		Dolomite	
	$\Delta_{tma} = 55.5$ µs/ft., $\rho_{ma} = 2.65$ g/cc		$\Delta_{tma} = 47.5$ µs/ft., $\rho_{ma} = 2.71$ g/cc		$\Delta_{tma} = 43.5$ µs/ft., $\rho_{ma} = 2.82$ g/cc.	
	Fresh $\rho_f = 1$ g/cc	Saline $\rho_f = 1.1$ g/cc	Fresh $\rho_f = 1$ g/cc	Saline $\rho_f = 1.1$ g/cc	Fresh $\rho_f = 1$ g/cc	Saline $\rho_f = 1.1$ g/cc
A	189	185	189	185	189	185
B	0.018018		0.021053		0.022989	
C	0.39	0.35	0.42	0.38	0.45	0.42
D	0.601	0.645	0.585	0.621	0.55	0.581

Table 4
Comparison of the observed and calculated porosities using different approaches including the proposed equation. Compiled from Tosaya, 1982; Gregory 1976; Johnston, 1978).

V_P ft./s	ϕ Core	Δ_t μs/ft.	ϕ Wyllie	Diff. %	ϕ Raymer	Diff. %	ϕ Raiga	Diff. %	ϕ Kamel et al., 2002	Diff. %	ϕ Kamel and Mohamed 2006	Diff. %	ϕ proposed	Diff. %
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
12,756.52	0.184	78.391	0.177	0.723	0.18	0.405	0.194	−1.013	0.19	−0.12	0.19	−0.70	0.18	0.86
13,586.62	0.164	73.602	0.14	2.422	0.151	1.334	0.162	0.226	0.15	1.36	0.18	−1.80	0.14	2.70
11,161.96	0.268	89.59	0.263	0.476	0.236	3.179	0.259	0.935	0.26	0.71	0.22	4.80	0.27	−0.25
11,591.77	0.268	86.268	0.238	3.041	0.221	4.696	0.241	2.706	0.24	2.87	0.25	1.80	0.24	2.66
15,525.69	0.0445	64.409	0.069	−2.43	0.082	−3.775	0.089	−4.435	0.08	−3.37	0.07	−2.05	0.07	−2.15
16,372.19	0.0445	61.079	0.043	0.142	0.052	−0.789	0.058	−1.361	0.05	−0.55	0.07	−2.75	0.04	0.35
14,246.1	0.0802	70.195	0.113	−3.327	0.127	−4.72	0.137	−5.634	0.12	−4.43	0.05	3.22	0.11	−3.01
12,497.32	0.179	80.017	0.189	−1.032	0.189	−1.009	0.204	−2.54	0.20	−1.77	0.19	−1.40	0.19	−0.97
12,671.22	0.21	78.919	0.181	2.916	0.183	2.704	0.198	1.25	0.19	2.10	0.19	1.80	0.18	3.03
12,677.78	0.16	78.878	0.181	−2.053	0.183	−2.273	0.197	−3.724	0.19	−2.87	0.19	−3.20	0.18	−1.94
12,270.94	0.17	81.493	0.201	−3.072	0.197	−2.708	0.213	−4.344	0.21	−3.70	0.19	−2.40	0.20	−3.10
16,733.1	0.066	59.762	0.033	3.309	0.04	2.634	0.045	2.081	0.04	2.74	0.07	−0.80	0.03	3.48
16,011.28	0.072	62.456	0.054	1.829	0.065	0.688	0.071	0.086	0.06	1.02	0.07	0.20	0.05	2.07
14,436.4	0.11	69.269	0.106	0.367	0.121	−1.068	0.129	−1.935	0.12	−0.73	0.16	−5.00	0.10	0.69
14,436.4	0.075	69.269	0.106	−3.133	0.121	−4.568	0.129	−5.435	0.12	−4.23	0.05	2.40	0.10	−2.81
12,959.95	0.15	77.161	0.167	−1.726	0.173	−2.277	0.186	−3.612	0.18	−2.64	0.19	−3.90	0.17	−1.54
11,811.6	0.196	84.663	0.225	−2.919	0.213	−1.729	0.232	−3.597	0.23	−3.26	0.19	0.20	0.23	−3.16
17,028.39	0.05	58.725	0.025	2.509	0.029	2.076	0.035	1.531	0.03	2.06	0.08	−2.50	0.02	2.64
17,356.49	0.05	57.615	0.016	3.367	0.018	3.234	0.023	2.689	0.02	3.06	0.08	−2.50	0.02	3.46
11,549.12	0.28	86.587	0.24	3.995	0.223	5.745	0.243	3.731	0.24	3.86	0.24	4.00	0.24	3.58
11,221.02	0.3	89.118	0.26	4.04	0.234	6.588	0.256	4.38	0.26	4.21	0.35	−5.00	0.27	3.37
11,319.45	0.28	88.344	0.254	2.638	0.231	4.935	0.252	2.787	0.25	2.71	0.33	−5.00	0.26	2.05
10,499.2	0.32	95.245	0.307	1.309	0.26	6.041	0.286	3.352	0.30	2.35	0.30	2.00	0.32	−0.20
10,925.73	0.26	91.527	0.278	−1.82	0.245	1.546	0.268	−0.85	0.27	−1.33	0.31	−5.00	0.29	−2.78

Eq. (10) can be applied in shaly formations ($10\% < V_{sh} < 33\%$) if the shale term of the form (Dresser Atlas, 1979)

$$V_{sh}\left(\frac{\Delta_{tsh}-\Delta_{tma}}{\Delta_{tf}-\Delta_{tma}}\right)$$

is subtracted from it. Then, the final form of Eq. (10) becomes

$$\phi_s = A\left[V_P - B\left(C + \frac{DV_P}{B}\right)^2\right] - V_{sh}\left(\frac{\Delta_{tsh}-\Delta_{tma}}{\Delta_{tf}-\Delta_{tma}}\right) \tag{11}$$

where Δ_{tma} is the sonic transit time of matrix (μs/ft.), Δ_{tf} is the sonic transit time of a fluid (μs/ft.); Δ_{tsh}, transit time for shale (μs/ft.), can be selected corresponding to the shale streaks (>5 ft. thick) and for the situations where many shale streaks exist within the formation, variable Δ_{tsh} must be selected based on the experience of the user. V_{sh} is the shale volume (% or fraction), V_P is the compressional velocity (ft./μs), and the coefficients A, B, C, and D were previously defined in Table 3.

The question now is what would happen if the effect of the parameters shown in the previous equations is compiled together in one equation? What would be the result if merging the three equations together; Eqs. (1), (3) and (4), and subtract the correction term of Eq. (6) from the equation derived after merging?

The second order *acoustic porosity-V_{sh}* model.

To answer the above questions, let us start with merging the three equations mathematically and see what the result is.

First, equating Eq. (1) with Eq. (4) and rearrange, we get

$$\frac{\Delta_{tma}}{\Delta_t} = \left(1 - \frac{\Delta_t - \Delta_{tma}}{\Delta_{tf} - \Delta_{tma}}\right)^x \tag{12}$$

Re-write Eq. (12) in terms of Δ_t, the following form is obtained:

$$\Delta_t = \frac{\Delta_{tma}}{\left(\frac{\Delta_{tf} - \Delta_t}{\Delta_{tf} - \Delta_{tma}}\right)^x} \tag{13}$$

Again, equating Eq. (13) with Eq. (3) of Raymer et al., (1980) and rearrange, we get:

$$\frac{(1-\phi_s)^2}{\Delta_{tma}} + \frac{\phi_s}{\Delta_{tf}} = \frac{\left(\frac{\Delta_{tf} - \Delta_t}{\Delta_{tf} - \Delta_{tma}}\right)^x}{\Delta_{tma}} \tag{14}$$

Table 5
Show r-squared between measured and calculated porosities using Eqs. (1), (3), (4), (9), (10), and (17).

	Measured & Wyllie et al., 1956	Measured & Raymer et al., 1980	Measured & Raiga-Clemenceau et. al., 1988.	Measured & Kamel et al., 2002	Measured & Kamel and Mohamed 2006	Measured & Proposed
r-squared	0.91	0.87	0.88	0.9	0.89	0.93

Table 6
Minimum, maximum, and average values of the difference between measured and calculated porosities using Eqs. (1), (3), (4), (9), (10), and (17).

	Measured & Wyllie et al., 1956	Measured & Raymer et al., 1980	Measured & Raiga-Clemenceau et. al., 1988.	Measured & Kamel et al., 2002	Measured & Kamel and Mohamed 2006	Measured & proposed
Minimum	−3.33	−4.72	−5.63	−4.43	−5	−3.58
Minimum	4.04	6.59	4.38	4.21	4.8	3.16
Average	0.48	0.87	−0.53	0.003	−0.98	−0.38

Multiplying Eq. (14) by Δ_{tma} and rearrange, we get the following second order equation:

$$\phi_s^2 + \left(\frac{\Delta_{tma}}{\Delta_{tf}} - 2\right)\phi_s + 1 - \left(\frac{\Delta_{tf} - \Delta_t}{\Delta_{tf} - \Delta_{tma}}\right)^x = 0 \tag{15}$$

which yields an expression of the type

$$Ax^2 + Bx + C = 0 \tag{16}$$

and the roots of Eq. (16) are

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \tag{17}$$

where $x = \phi_s$, $A = 1$, $B = \left(\frac{\Delta_{tma}}{\Delta_{tf}} - 2\right)$, $C = 1 - \left(\frac{\Delta_{tf} - \Delta_t}{\Delta_{tf} - \Delta_{tma}}\right)^x$

To make Eq. (17) reasonable for shaly formation, the shale term of Dresser Atlas, 1979–Eq. (6) is subtracted from Eq. (17) to end up with the form:

$$x_A = \frac{-B \pm \sqrt{B^2 - 4C}}{2} - V_{sh}\frac{\Delta_{tsh} - \Delta_{tma}}{\Delta_{tf} - \Delta_{tma}} \tag{18}$$

where $x_A = \phi_E$, $B = \left(\frac{\Delta_{tma}}{\Delta_{tf}} - 2\right)$, $C = 1 - \left(\frac{\Delta_{tf} - \Delta_t}{\Delta_{tf} - \Delta_{tma}}\right)^x$, Δ_{tsh}, transit time for shale (µs/ft.), can be selected corresponding to the shale streaks (>5 ft. thick) and for the situations where many shale streaks exist within the formation, variable Δ_{tsh} must be selected based on the experience of the user. Δ_{tma} is the sonic transit time of matrix (µs/ft.), Δ_{tf} is the sonic transit time of a fluid (µs/ft.) and V_{sh} is the shale volume (% or fraction).

Note that in very shaly sands the magnitude of V_{sh} is close to 1, and effective porosity (ϕ_E) is close to zero. From Eq. (18)it follows that in this case $\phi_{sh} = \phi_T$ (Tenchov, 1998). To that end, the remaining is to test the validity of the suggested second order equation to see how far the results are from the actual values of porosity within the same field.

Testing the second order acoustic porosity model.

In the following, a number of samples compiled from the literatures (Tosaya, 1982; Gregory 1976; Johnston, 1978) were used to test the validity of Eq. (17). These samples, listed in Table 4, included in-situ measurements of compressional wave velocity (V_P) and porosity (ϕ).

A comparison was made between the measured porosity values, listed in Table 4, and those computed using different approaches including the Wyllie et al., 1956 – Eq. (1), Raymer et al., 1980– Eq. (3), Raiga-Clemenceau et al., 1988 – Eq. (4), Kamel et al., 2002 – Eq. (9), Kamel and Mohamed, 2006 – Eq. (10), and the new derived Eq. (17) with the following parameters:

1. For all equations, the sonic transit time (Δ_t) can be obtained from the reciprocal of the available compressional wave velocity (V_P) listed in Table 4).
2. In the Time average equation of Wyllie et al. (1956), Δ_{tma} is taken equal to 55.5 µs/ft. for sandstone and $\Delta_{tf} = 185$ µs/ft. for saline media.
3. Raymer et al. (1980) equation, on the other hand, using Δ_{tma} equal to 56 µs/ft. for sandstone.
4. Raiga-Clemenceau equation, on the other hand, using Δ_{tma} equal to 55.5 µs/ft. for sandstone, the exponent x equal to 1.6; computed via Eq. (5).
5. Kamel et al., 2002 equation, on the other hand, using Δ_{tma} is taken equal to 55.5 µs/ft. for sandstone, $\Delta_{tf} = 185$ µs/ft. for saline media and the exponent x equal to 1.6; computed via Eq. (5).
6. Kamel and Mohamed 2006, on the other hand, using their constants from Table 3.
7. In the proposed Eq. (17), the matrix transit time is taken to be equal 55.5 µs/ft. for sandstone, $\Delta_{tf} = 185$ µs/ft. for saline media, and the exponent (x) is taken to be equal 1.6. The Δ_t is computed from the reciprocal of V_P.

The results are listed in Table 4 and illustrated in Tables 5 and 6 from whom one can easily notice the following remarks:

1. Table 4 (column 2 & 14) and Table 5 reflect the reliability of the proposed equation in which it gives very close and reliable values

Table 7

Represent input/output for computing porosity using different approaches.

Depth (ft.)	Δ_t (µs/ft.)	ϕ-observed	V_{sh}	ϕ-Atlas	Error %	ϕ-proposed	Error %
4000	90.97	0.20	0.31	0.19	7.11	0.20	2.31
4005	74.20	0.10	0.30	0.10	−3.15	0.10	−0.59
4010	73.22	0.10	0.30	0.10	−4.78	0.10	−2.06
4015	92.68	0.21	0.30	0.19	7.61	0.20	1.69
4020	72.36	0.09	0.24	0.10	−4.65	0.09	−1.61
4025	77.62	0.13	0.18	0.13	−2.27	0.13	−1.01
4030	79.36	0.14	0.17	0.15	−1.35	0.14	−0.74
4035	76.61	0.13	0.22	0.13	−3.36	0.13	−1.89
4040	76.61	0.12	0.15	0.13	−3.14	0.12	−1.56
4045	91.75	0.21	0.32	0.19	7.80	0.20	2.51
4050	74.62	0.12	0.15	0.13	−3.38	0.12	−1.35
4055	90.11	0.19	0.31	0.18	7.53	0.18	2.93
4060	74.52	0.12	0.14	0.12	−4.84	0.12	−2.80
4065	91.52	0.21	0.32	0.19	7.43	0.20	2.35
4070	73.32	0.11	0.13	0.12	−4.99	0.11	−2.60
4075	74.72	0.12	0.10	0.12	−3.55	0.12	−1.52
4080	76.01	0.14	0.12	0.14	−2.50	0.14	−0.97
4085	76.91	0.15	0.12	0.15	−2.23	0.15	−0.97
4090	78.54	0.15	0.22	0.15	−1.41	0.15	−0.58
4095	79.89	0.17	0.16	0.17	−1.13	0.17	−0.76
4100	81.11	0.19	0.12	0.18	1.76	0.18	1.75
4105	79.99	0.15	0.19	0.15	−2.90	0.15	−2.51
4110	90.56	0.20	0.31	0.19	6.59	0.20	2.07
4115	79.89	0.17	0.14	0.17	−1.53	0.17	−1.16
4120	78.48	0.14	0.21	0.14	−2.24	0.14	−1.33
4125	75.85	0.11	0.25	0.12	−3.59	0.11	−1.68
4130	74.89	0.11	0.27	0.11	−3.70	0.11	−1.43
4135	76.51	0.13	0.22	0.13	−4.30	0.13	−2.81
4140	83.45	0.19	0.14	0.18	3.40	0.18	2.53
4145	80.89	0.16	0.20	0.16	−0.92	0.16	−0.85
4150	81.35	0.11	0.23	0.10	2.53	0.10	2.38
4155	79.91	0.16	0.17	0.16	−0.82	0.16	−0.43
4160	79.91	0.14	0.21	0.14	−2.01	0.14	−1.58
4165	79.41	0.16	0.18	0.17	−2.37	0.16	−1.85
4170	77.51	0.12	0.26	0.12	−3.78	0.12	−2.46
4175	77.21	0.12	0.26	0.12	−2.02	0.12	−0.54
4180	81.91	0.13	0.19	0.13	0.64	0.13	0.26
4185	76.02	0.12	0.20	0.13	−3.46	0.13	−1.76
4190	77.26	0.12	0.21	0.12	−3.82	0.12	−2.38
4195	82.54	0.14	0.22	0.13	3.47	0.13	2.77
4200	76.92	0.11	0.23	0.11	−3.42	0.11	−1.78
4205	81.74	0.17	0.15	0.17	3.13	0.17	2.88
4210	78.16	0.13	0.19	0.14	−3.60	0.13	−2.57
4215	81.14	0.14	0.18	0.14	1.52	0.14	1.49
4220	79.30	0.13	0.18	0.14	−2.99	0.14	−2.33
4225	82.89	0.19	0.11	0.18	3.27	0.18	2.64
4230	79.55	0.12	0.22	0.12	−3.35	0.12	−2.71
4235	80.05	0.11	0.28	0.11	−3.30	0.11	−2.82
4240	81.89	0.14	0.17	0.14	3.08	0.14	2.72
4245	80.35	0.10	0.25	0.10	−1.81	0.10	−1.43
4250	76.92	0.10	0.26	0.11	−3.51	0.11	−1.76
4255	76.97	0.10	0.20	0.11	−3.02	0.11	−1.28
4260	80.55	0.12	0.24	0.12	−0.59	0.12	−0.36
4265	75.22	0.11	0.18	0.11	−3.00	0.11	−0.88
4270	81.95	0.19	0.11	0.18	2.95	0.18	2.65
4275	81.34	0.18	0.14	0.17	2.14	0.17	2.05
4280	77.02	0.10	0.18	0.10	−3.84	0.10	−2.08
4285	83.46	0.20	0.12	0.19	3.36	0.19	2.53
4290	80.35	0.12	0.23	0.12	−3.25	0.12	−2.92
4295	80.75	0.15	0.17	0.15	−2.84	0.15	−2.72
4300	79.85	0.12	0.22	0.13	−1.74	0.13	−1.22
4305	79.15	0.13	0.24	0.13	−3.61	0.13	−2.89
4310	80.45	0.15	0.18	0.16	−2.41	0.16	−2.19

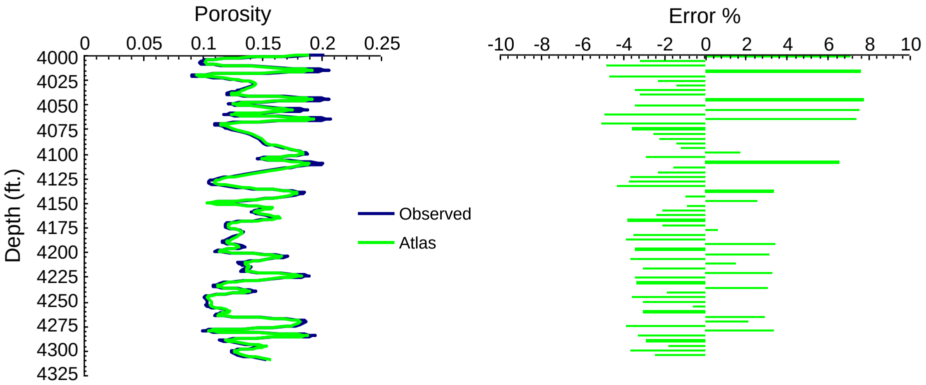

Fig. 2. Vertical variation between porosity from observed and Eq. (6) with errors, Gulf of Suez, Egypt.

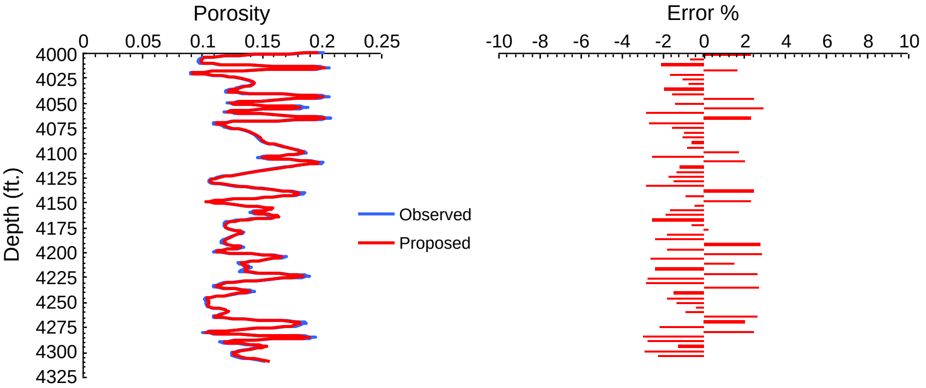

Fig. 3. Vertical variation between porosity from observed and proposed Eq. (18) with errors, Gulf of Suez, Egypt.

with those measured regardless the discrepancies observed. This is attributed to the integration of all parameters included in other approaches, separately.

2. Table 6 represents the minimum, maximum and average values of the difference between the measured and computed porosities using different approaches from which one can easily notice that the lowest value is, of course, belongs to the proposed Eq. (17). This is attributed to the fact that this equation considers several parameters not actually included in other approaches. In other words, since the proposed Eq. (17) takes into consideration the effect of matrix nature (x), matrix transit time (Δ_{tma}) and fluid transit time (Δ_{tf}), Raymer et al. (1980) equation includes only the effect of Δ_{tf} and Δ_{tma} whereas the matrix nature (x) and Δ_{tma} are included in Raiga's equation.

2. Application

The successfulness of testing Eq. (17) in clean formation ($V_{sh}<10\%$) encourage the present authors to compare the proposed Eq. (18) with Dresser Atlas Eq. (6) in shaly formation ($10\%<V_{sh}<33\%$) to one well from depth 4000 to 4310 ft. within the north Gulf of Suez Basin of Egypt. This interval consists of shaly sand with few amount of limestone with shale volume ranging from 10% to 32% and porosity from 9% to 21%. For this interval, the parameters required for Eq. (18) are $\Delta_{tma}=55.5$ μs/ft., $\Delta_{tf}=185$ μs/ft., Δ_{tsh} variable (Appendix A) and $x=1.58$. The interval transit time (Δ_t) is measured using long spacing sonic logs after calibration and the shale volume is computed from Gamma ray readings using the equation adopted by Schlumberger (1975). The porosity computed using Dresser Atlas and proposed equation is compared with the computed Neutron–Density combination (ϕ_{ND}) within this interval. The results of this comparison are shown in Table 7.

Figs. 2 and 3 represent the vertical variation of different approaches for computing ϕ_{E} with depth and errors. Table 8 shows the minimum, maximum and average errors between different approaches for computing ϕ_{E} that reflect that our approaches gives the lowest errors not exceeded ±3% in comparison with Dresser Atlas owing that our approach integrate parameters that not included in Dresser Atlas. Table 9 shows lowest standard deviation ±1.9% (st. Dev.) and root mean square (RMS) of errors of our approach in comparison with Dresser Atlas.

The successfulness of testing Eq. (18) encourage the present authors to make comparison between performance of Eq. (11) (Kamel and Mohamed 2006) and the proposed equation to one well from depth 4800 to 5010 ft. within the central Gulf of Suez Basin of Egypt.

Table 8
Shows the minimum, maximum and average errors between porosity from observed and other approaches, Gulf of Suez, Egypt.

	Observed & Atlas	Measured & Proposed
Minimum	−4.99	−2.92
Maximum	7.80	2.93
Average	−0.88	0.56

Table 9
Show standard deviation of errors and root mean square of errors between porosity from observed and other approaches, Gulf of Suez, Egypt.

	Observed &Atlas	Measured & Proposed
Standard deviation	3.59	1.93
Root mean square	0.0052	0.0027

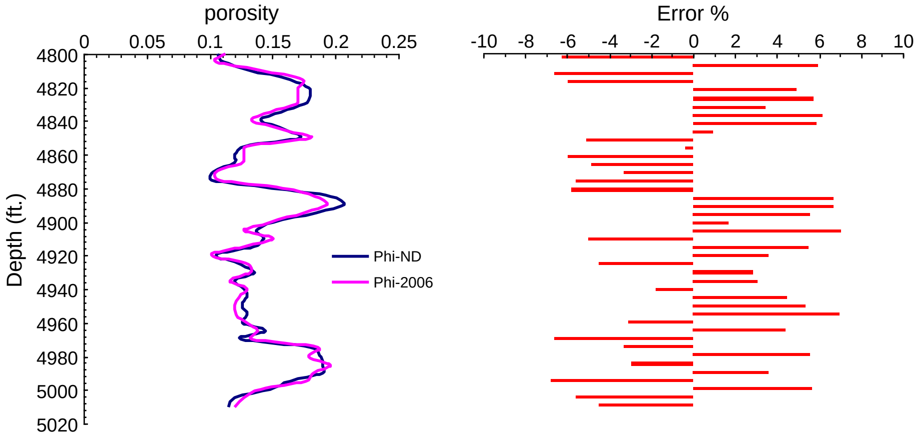

Fig. 4. Vertical variation between porosity computed from Neutron–Density combination and Eq. (11) with errors, Gulf of Suez, Egypt.

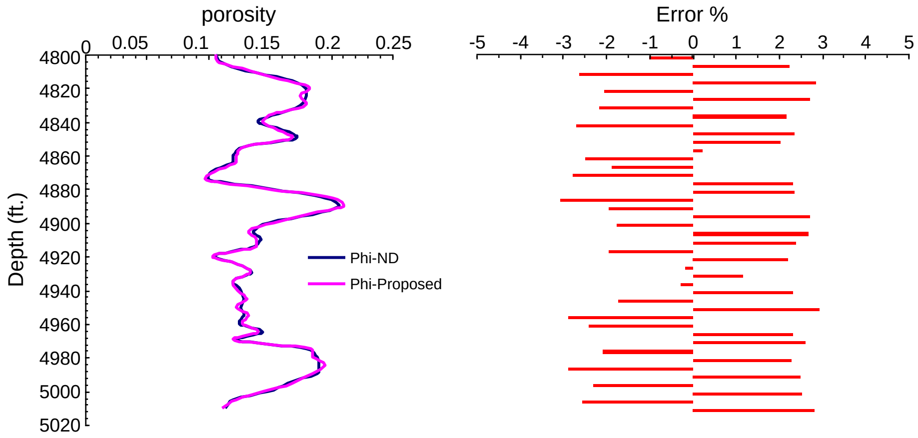

Fig. 5. Vertical variation between porosity computed from Neutron–Density combination and proposed Eq. (18) with errors, Gulf of Suez, Egypt.

This interval consists of shaly sand with few amount of limestone and devoid from anhydrite with shale volume ranging from 10% to 42% and porosity from 10% to 21%. For this interval, the parameters required for both Eq. (11) is taken from Table 3 and Eq. (18) are $\Delta_{tma} = 55.5$ μs/ft., $\Delta_{tf} = 185$ μs/ft., Δ_{tsh} variable from 75 to 110 μs/ft. (Appendix A) and $x = 1.6$. The interval transit time (Δ_t) is measured using long spacing sonic logs after calibration and the shale volume is computed from Gamma ray readings using the equation adopted by Schlumberger (1975). The porosity computed using both Eqs. (11) and (18) are compared with the computed Neutron–Density combination (ϕ_{ND}) within this interval. The vertical variation with depth of different values of ϕ computed using different approaches (Figs. 4 and 5) showing good agreement with an acceptable range of error not exceeding ±3% for Eq. (18) than Eq. (11). Table 10 shows the minimum, maximum, and average errors between the porosity computed Neutron–Density combination (ϕ_{ND}) and those computed using different approaches (Eqs. (11) and (18)). Table 11 demonstrates the r-squared existing between ϕ_{N-D}, Eqs. (11), and (18).

Table 10
Shows the minimum, maximum and average errors between porosity computed from Neutron–Density combination and different approaches (Eqs. (11) and (18)), Gulf of Suez, Egypt.

	Phi-ND & Eq. (11)	Phi-ND & Eq. (18)
Minimum	−6.73	−3.13
Maximum	7.03	3.06
Average	0.42	0.14

3. Conclusion

To this end, in this paper that resulted from merging different known equations (i.e. Wyllie et al. (1956), Dresser Atlas (1979), Raymer et al. (1980) and Raiga-Clemenceau et. al. (1988)), that the proposed second order-V_{sh} model capable of evaluation porosity from sonic and gamma ray logs and treats the effect of shaliness as a correction term subtracted from the clean-sand term.

From a comparison with different shaly sand samples the standard deviation of errors and root mean square errors give the lowest values that indication that our approach integrated parameters that not included in Dresser Atlas (matrix exponent).

Nevertheless, a proposed model can still be applied with high accuracy for evaluating effective porosity in old wells that suffer from insufficient logging data regardless the differences observed. Also, it can be considered as a quick-look approach in interpreting the recent

Table 11
Show r-squared between porosity computed from Neutron–Density combination and different approaches (Eqs. (11) and (18)), Gulf of Suez, Egypt.

	Phi-ND & Eq. (11)	Phi-ND & Eq. (18)
r-squared	0.93	0.98

exploratory wells before the decision of completing the logging measurements.

Appendix A

Adjusting D_{tsh} to be variable

1. For clean sand layer ($V_{sh} < 10\%$) substitute Δ_{tsh} with Δ_t sand of this layer.
2. For shaly sand, read and use Δ_{tsh} shale corresponding to shale streaks (>5 ft. thick).
3. For situations where many shale streaks exist within the formation, calculate the gradient between Δ_{tsh} shale at the top and the bottom of the formation (using streaks >5 ft. thick) and calculate Δ_{tsh} shale gradient then calculate Δ_t for thin shale streaks.

The following mathematics explains the third situation:

- Select Δ_{t1} at depth Z_1 (near the top)
- Select Δ_{t2} at depth Z_2 (near the bottom)
- Select Δ_{tc} at a control depth c in between Z_1 and Z_2
- Calculate Δ_t gradient $G\Delta_t$ as follows: $G\Delta_t = \Delta_{t2} - \Delta_{t1} / (Z_2 - Z_1)$
- Calculate Δ_{tc} theoretically as follows: $\Delta_{t1} + (Z_c - Z_1) * G\Delta_t$
- Compare the calculated Δ_{tct} and Δ_{tc} if error acceptable (within 3–5%_ applies the gradient to calculate Δ_{tsh} at any desired depth and use the calculated in the estimated equation for porosity.
- If not acceptable calculate two gradient using same concepts $G\Delta_{t1}$ between Δ_{t1} and Δ_{tc}, and $G\Delta_{t2}$ between Δ_{tc} and Δ_{t2} using respective depths.

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