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A Comparative Study of Experimental Design Techniques in Assisted History Matching

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Abstract

Quantifying the uncertainty of reservoir models has always been considered as a major concern. Uncertainty and sensitivity analyses provide us with information about how “incorrect” a proposed prediction scenario is. One effective approach to quantify reservoir uncertainties is to apply the concept of Experimental Design. As the name indicates, experimental design is the technique used to guide the choice of the experiments to be conducted in an efficient way. Samples are selected in the design space of the uncertain parameters in order to obtain the maximum amount of information using low number of experiments. Geoscientists and engineers are building larger models to address reservoir heterogeneity and hence reservoir simulation is usually expensive (takes significant time to run). Experimental design techniques became very useful in assisted history matching. Several experimental design techniques are introduced in literature, some must be more effective than others in typical reservoir simulation problems.

The objective of this paper is to compare different experimental design techniques and to introduce two efficient ones (Halton and Sobol sequences) to reservoir engineering problems. To show the potentiality and efficiency of the Halton and Sobol sequences design techniques, a comparative study is conducted to compare between their performance in solving assisted history matching problems and the performance of the most widely used experimental design technique, Latin hypercube. Other conventional techniques were compared too, but found to be less efficient than Latin hypercube. Two different scale reservoir models are used as test problems. A performance indicator is developed to compare between the three studied techniques in terms of the relative error between the estimated values of history matching parameters calculated using the proposed assisted history matching procedure and their exact solutions.

The results of this work indicate that the Sobol and Halton sequences experimental design techniques are superior to Latin hypercube method.

Introduction

In assisted history matching problems, reservoir uncertain parameters search space is explored using an experimental design technique. By applying the experimental design technique, samples are chosen within the assigned uncertainty ranges. The selected samples should provide the designer with the maximum amount of information according to a predefined rule. Scoping runs are built with the chosen samples and numerical or analytical simulations are conducted. Scoping runs are evaluated in terms of objective function which represents the misfit between actual and simulated data. After that, a proxy model or response surface model is built. The proxy model can be defined as a relation between the simulator input and output and it is used as the function to be optimized (usually minimized) using a proper optimization algorithm. The results of optimizing the proxy model will be the set of input parameters yielding a history matched reservoir model. This paper focuses on the experimental design aspect and introduces two very powerful experimental design techniques that aim to enhance the process of reservoir assisted history matching. Although these techniques have been used in other engineering problems, the authors are not aware of any use for these techniques in the petroleum industry. After highlighting the superiority of these techniques in this paper, the authors hope that these techniques found their well-deserved place in the commercial assisted history matching programs.

Experimental design was first developed by mathematician Ronald Fisher (1925) for agricultural applications. Experimental design is the approach used to sample a search space or domain in an efficient way. Samples are selected in a manner that assures obtaining the maximum amount of information using the lowest number of experiments. In reservoir engineering problems, an experiment could be a simulation run, material balance calculation, or simply a pressure or production rate calculation using a model. Several experimental design techniques are presented in the literature. Plackett and Burman (1946) developed the most economical two levels experimental design. Plackett-Burman design gives the lowest number of samples and hence it is very useful when the designer is interested only in the main effects. Box and Behnken (1960) introduced an incomplete three levels fractional factorial design approach. Box-Behnken design was introduced to limit the sample size as the number of parameters grows. Box-Behnken design requires at least three factors and it is suitable for construction of quadratic proxy models. The sample consists of points in the middle of the edges, mean levels, of the sample and its center.

One of the most powerful concepts of experimental design approach is the space filling. Space filling design approach is based on spreading the experiments around the search space and do not follow a specific model form. The algorithm divides the probability distribution of an uncertain variable into areas of equal probabilities. It assures that the sample values for each parameter are distributed over the entire range of that parameter. Space filling designs are not based on the concept of factor levels; the number of generated sample is predefined by the designer and doesn't depend on the number of the problem factors. The basic concept is based on the Van Der Corput (1935) sequence which subdivides the design space into sub-volumes and put an experiment in each of them. Halton sequence (1960) uses base-two Van Der Corput sequence for the first dimension, base-three sequence in the second dimension, base-five in the third dimension, and so on, using the prime numbers for base. Sobol sequence (1967) uses only one base for all dimensions and a different permutation of the vector elements for each dimension. Sobol sequence technique is more resistant to the high-dimensional degradation.

Damsleth et al. (1992) applied the experimental design and response surface methods for the first time in reservoir engineering for a field study in the North Sea. They used experimental design technique to perform a sensitivity study with minimum number of simulations. According to their work, experimental design approach helped to reduce the number of required simulations by 30-40% in comparison with the procedure that varies one parameter at a time. Watkins and Parish (1992a), Watkins and Parish (1992b), Parish et al. (1993), Parish and Little (1994), Eide et al. (1994), Parish and Little (1997), and Craig et al. (1997) all introduced different experimental design concepts and techniques in reservoir engineering and specially in assisted history matching. Our research focuses on introducing both Halton and Sobol sequences techniques into the area of assisted history matching of reservoir engineering problems as they have not been introduced before. **Fig. 1** shows a comparison between Latin hypercube, Halton sequence, and Sobol sequence space filling experimental design techniques on a case with two factors and a thousand samples. It is obvious that the Sobol sequence gives the most uniformly distributed samples.

ʘ Latin hypercube design ʘ Halton sequence design ʘ Sobol sequence design

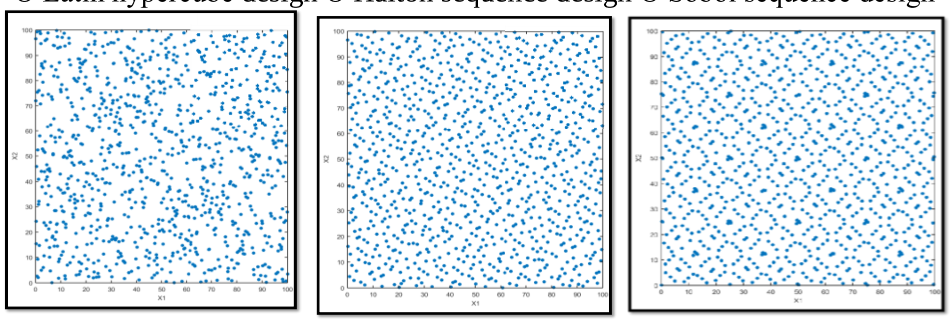

Fig. 1: A comparison between Latin hypercube, Halton, and Sobol sequences experimental design techniques for 1,000 samples in two dimensional search spaces

Methodology

MATLAB® open source codes of the three studied experimental design techniques were used in our work. Known-solution history matching problems are used to compare between the performances of the studied techniques. To fair in comparing the different algorithms, the number of selected samples and the search space are kept the same. Two different scale material balance models are used as assisted history matching test problems. For both test problems, the material balance model is constructed according to the following strategy:

1. Use MBAL® software to predict pressure and production performance using known reservoir parameters for 10 years.
2. Use the predicted pressure and production data as a historical data in the test problems.
3. Alter the original reservoir parameter to obtain unmatched pressure performance.
4. Use Latin hypercube, Halton sequence, and Sobol sequence experimental design techniques to select the samples of the history matching parameters. These selected samples are used to build the scoping runs.
5. Run the scoping runs and calculate the pressure objective function using the following formula;

$$f(x) = \frac{1}{n}\sum_i w_i \frac{(y*_i - y_i)^2}{\sigma_i^2},$$

Where n is the number of the observed data points, y_i* represents the simulated pressure data obtained using the set of parameters x and y_i are the observed pressure data; σ_i^2 is the variance of the observed values and w_i is the weight assigned to each set of data in the objective function.

6. Build proxy model that interpolates the objective function with the experimentally designed reservoir parameter samples. For fair comparison among the experimental design techniques, we built all proxy models using Artificial Neural Network technique.
7. Minimize the created proxy model using genetic optimization algorithm. This minimization process is run five times to take a more representative solution. Consistency was maintained in running the optimization algorithm for all three tested experimental design techniques.

8. Use the following performance indicator to quantify the error between the estimated and exact solutions of the reservoir uncertain parameters.

$$Performance\ Indicator = Avg.\left[Abs.\left(\frac{Estimated\ solution - Exact\ solution}{Exact\ solution}\ x\ 100\right)\right]$$

Test Problems

Two different scale material balance models are used to test the potential of Halton and Sobol sequence techniques and compare their performances with Latin hypercube method. The two test problems describe two different reservoir models; the first one is a single tank and the second one is a multiple tank material balance problem. The objective of using different scale test problems is to validate the results and check if the conclusions can be generalized. The two test problems and the comparison results are presented in the following sections.

Single Tank Material Balance Test Problem

Fig.2 shows a schematic of the single tank material balance problem with a Hurst-van Everdingen-modified aquifer model.

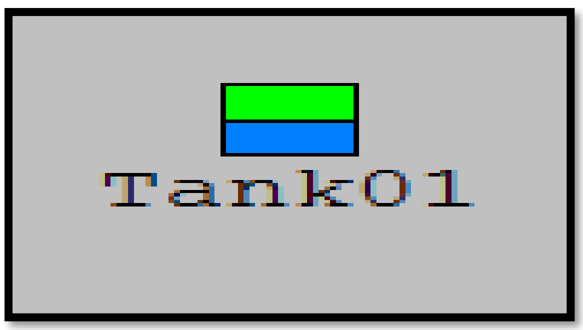

Fig.2: Single Tank Material Balance Test Problem

The values of the reservoir tank and aquifer parameters are shown in **Fig.3**. This problem configuration gives seven uncertain parameters; all are presented in **Table 1.** Three uncertain parameters (highlighted ones in Table 1) are selected as history matching parameters to test the studied experimental design techniques.

Tank Parameters (Exact Solution)

Parameter	OOIP, MMSTB	Initial pressure, psig	Temp., deg F	Porosity, fr	Swc, fr	Initial gas cap
Value	210.867	4000	250	0.23	0.15	0

Aquifer Parameters (Exact Solution)

Parameter	Aquifer Model	Res. Thick., ft	Res. Radius, ft	Outer/inner radius	Encroachment angle, deg	Aq. Perm, md
Value	Hurst-van Everdingen-Modified	250	2500	4.82266	156.589	9.36911

Fig.3: Reservoir tank and aquifer parameters values of the single MBAL problem

Table 1: Single tank MBAL problem uncertain parameters

1. Oil in Place
2. Outer/Inner Radius
3. Reservoir Radius
4. Encroachment Angle
5. Reservoir Thickness
6. Porosity
7. Aquifer Permeability

For each history matching parameter, an uncertainty range is assigned as shown in **Table 2.**

Table 2: Selected history matching parameters uncertainty range

HM Parameter	Number	Exact Solution	ML	Uncertainity Range	
				Min.	Max.
Oil in Place, MMSTB	X1	210.867	225	200	250
Encroachment Angle	X2	156.589	140	100	180
Aquifer Permeability	X3	9.36911	25.5	1	50

Tables 3, 4 and 5 show the seven samples selected by the three studied experimental design techniques, Latin hypercube, Sobol sequence, and Halton sequence respectively (sample number 7 represents the most likely values of the history matching parameters, ML). Each experiment represents the input data for a scoping run. For each scoping run, the tank pressure objective function is calculated.

Table 3: Latin hypercube samples and corresponding calculated objective function

Experiment	1. Oil in Place, MMSTB	2. Encroachment Angle	3. Aquifer Permeability	Objective Function
	X1	X2	X3	
1	200	180	1	2498
2	210	164	40	313
3	230	132	30	56
4	220	148	21	79
5	240	116	11	113
6	250	100	50	71
7	225	140	26	73

Table 4: Sobol Sequence samples and corresponding calculated objective function

Experiment	1. Oil in Place, MMSTB	2. Encroachment Angle	3. Aquifer Permeability	Objective Function
	X1	X2	X3	
1	200	100	1	3408
2	225	140	26	73
3	213	160	13	46
4	238	120	38	42
5	206	150	44	182
6	231	110	19	81
7	225	140	26	73

Table 5: Halton Sequence samples and corresponding calculated objective function

Experiment	1. Oil in Place, MMSTB	2. Encroachment Angle	3. Aquifer Permeability	Objective Function
	X1	X2	X3	
1	225	127	11	59
2	213	153	21	99
3	238	109	30	48
4	206	136	40	75
5	231	162	3	621
6	219	118	13	108
7	225	140	26	73

Multiple Tank Material Balance Test Problem

The problem configuration is shown in **Fig. 4** and can be described as follows;

- Three fault blocks; FB_A, FB_B, and FB_C.
- Each fault block has two layers; FB_A_Lay1, FB_A_Lay2, FB_B_Lay1, FB_B_Lay2, FB_C_Lay1, and FB_C_Lay2.
- Each fault block and layer is modeled with its own material balance model, which gives a total of six compartments.
- The layers are commingled (only communicating through the well bore).
- The fault blocks are communicating with some transmissibility across the boundaries of the faults.
- Two fault blocks include three wells (two producers and one water/gas injector). The third fault block includes two producers. Total number of wells is eight.
- Each well producer/injector is controlled by a productivity index value and flowing bottom hole pressure.

The values of the reservoir tank and aquifer parameters are shown in **Tables 6 and 7** respectively. This problem configuration gives thirty eight uncertain parameters; all are presented in **Table 8.** Twenty uncertain parameters are selected as history matching parameters (these are the highlighted ones in Table 8) to test studied experimental design techniques.

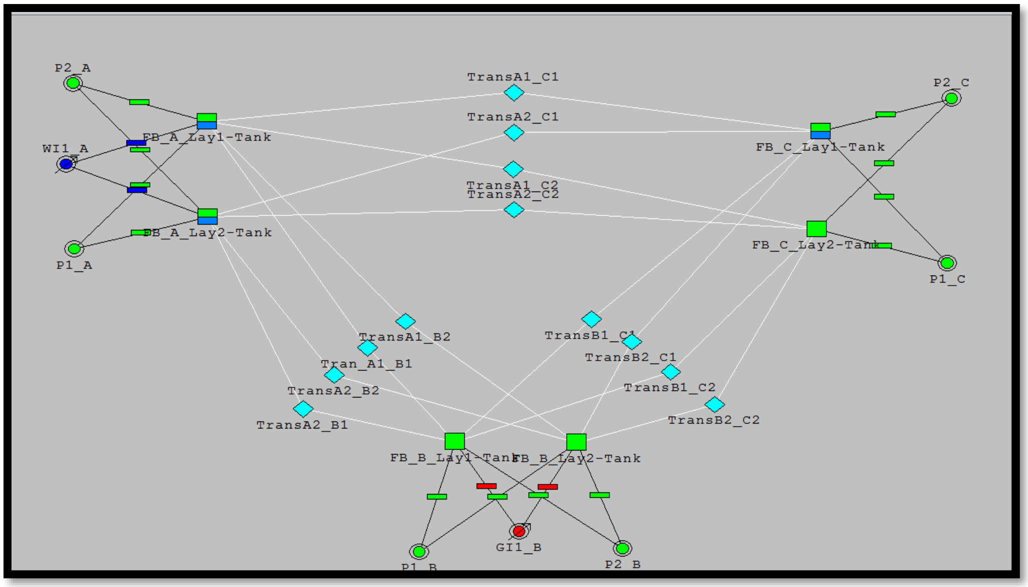

Fig.4: Multiple Material Balance Test Problem

Table 6: Reservoir tank parameters values of the multiple tank MBAL problem

Parameter	FB_A_Lay1	FB_A_Lay2	FB_B_Lay1	FB_B_Lay2	FB_C_Lay1	FB_C_Lay2
OOIP, MMSTB	200	300	250	180	170	220
Initial pressure, psig	2620	2660	2578.41	2606.19	2590	2610
Temp., deg F	240.8	250	240	250	200	220
Porosity, fr	0.1	0.12	0.2	0.15	0.18	0.18
Swc, fr	0.25	0.2	0.25	0.2	0.2	0.24
Initial gas cap	0	0	0.3	0.35	0	0

Table 7: Aquifer parameters values of the multiple tank MBAL problem

Parameter	FB_A_Lay1	FB_A_Lay2	FB_B_Lay1	FB_B_Lay2	FB_C_Lay1	FB_C_Lay2
Aquifer Model	Fetkovich Steady State	Hurst-van Everdingen-Modified	-	-	Carter-Tracy	-
Res. Thick., ft	120	100	-	-	100	-
Res. Radius, ft	14,000	1,000	-	-	12,000	-
Outer/liner radius	5	5	-	-	5	-
Encroachment angle, deg	180	200	-	-	180	-
Aq. Perm, md	300	10	-	-	200	-

Table 8: Multiple tank MBAL problem uncertain parameters

1.	FB_A_Lay2 Oil in Place	14.	FB_C_Lay1 Aquifer Permeability	27.	Trans. A1_B1
2.	FB_A_Lay2 Outer/Inner Radius	15.	FB_A_Lay1 Oil in Place	28.	Trans. A1_B2
3.	FB_A_Lay2 Reservoir Radius	16.	FB_A_Lay1 Outer/Inner Radius	29.	Trans. A1_C1
4.	FB_A_Lay2 Encroachment Angle	17.	FB_A_Lay1 Reservoir Radius	30.	Trans. A1_C2
5.	FB_A_Lay2 Reservoir Thickness	18.	FB_A_Lay1 Encroachment Angle	31.	Trans. A2_B1
6.	FB_A_Lay2 Porosity	19.	FB_A_Lay1 Reservoir Thickness	32.	Trans. A2_B2
7.	FB_A_Lay2 Aquifer Permeability	20.	FB_A_Lay1 Porosity	33.	Trans. A2_C1
8.	FB_C_Lay1 Oil in Place	21.	FB_A_Lay1 Aquifer Permeability	34.	Trans. A2_C2
9.	FB_C_Lay1 Outer/Inner Radius	22.	FB_B_Lay1 Oil in Place	35.	Trans. B1_C1
10.	FB_C_Lay1 Reservoir Radius	23.	FB_B_Lay1 Initial Gas Cap	36.	Trans. B1_C2
11.	FB_C_Lay1 Encroachment Angle	24.	FB_B_Lay2 Oil in Place	37.	Trans. B2_C1
12.	FB_C_Lay1 Reservoir Thickness	25.	FB_B_Lay2 Initial Gas Cap	38.	Trans. B2_C2
13.	FB_C_Lay1 Porosity	26.	FB_C_Lay2 Oil in Place		

For each history matching parameter, an uncertainty range is assigned as shown in **Table 9.**

Table 9: Multiple tank problem selected history matching parameters uncertainty range

HM Parameter	Number	Exact Solution	ML	Uncertainity Range	
				Min.	Max.
FB_A_Lay2 Oil in Place	X1	300	250	100	400
FB_A_Lay2 Reservoir Thickness	X2	100	95	90	110
FB_A_Lay2 Aquifer Permeability	X3	10	15	2	20
FB_C_Lay1 Oil in Place	X4	170	150	140	200
FB_C_Lay1 Encroachment Angle	X5	180	160	150	200
FB_A_Lay1 Oil in Place	X6	200	220	150	250
FB_A_Lay1 Outer/Inner Radius	X7	5	3	1	7
FB_B_Lay1 Oil in Place	X8	250	280	200	300
FB_B_Lay1 Initial Gas Cap	X9	0.3	0.5	0.2	0.6
FB_B_Lay2 Oil in Place	X10	180	150	100	200
FB_B_Lay2 Initial Gas Cap	X11	0.35	0.25	0.1	0.5
FB_C_Lay2 Oil in Place	X12	220	240	200	250
Trans. A1_B1	X13	8	10	5	15
Trans. A1_C1	X14	10	7	2	12
Trans. A1_C2	X15	5	3	0.01	6
Trans. A2_B2	X16	3	6.5	1	10
Trans. A2_C2	X17	9	10	5	15
Trans. B1_C2	X18	7	4	0.1	9
Trans. B2_C1	X19	6	9	5	12
Trans. B2_C2	X20	9	7	5	12

Tables 10, 11 and 12 show the fifty-one samples selected by the three studied experimental design techniques, Latin hypercube, Sobol sequence, and Halton sequence respectively (the sample number 51 represents the most likely values of the history matching parameters, ML). Each experiment represents the input data for a scoping run. For each scoping run the tanks pressure objective function is calculated.

Table 10: Latin hypercube samples and corresponding calculated objective function

Experiment Number	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	Objective Fun
1	112.24	105.51	11.92	198.78	161.22	203.06	1.73	255.10	0.41	161.22	0.40	246.94	13.98	4.65	0.87	5.22	7.04	6.82	11.00	5.29	**72.06**
2	179.59	103.06	15.59	157.14	162.24	223.47	3.45	251.02	0.39	177.55	0.17	236.73	5.20	11.18	3.80	2.65	10.10	6.64	9.86	9.43	**25.21**
3	167.35	97.76	2.37	147.35	167.35	205.10	5.90	295.92	0.43	138.78	0.15	230.61	10.10	7.31	0.50	2.84	7.45	7.91	11.14	10.14	**61.77**
4	155.10	109.18	7.14	180.41	154.08	239.80	2.22	240.82	0.53	155.10	0.17	221.43	11.53	4.86	0.38	5.78	11.12	6.09	6.86	10.86	**87.22**
5	210.20	104.29	18.53	196.33	180.61	184.69	6.76	253.06	0.33	181.63	0.13	238.78	11.73	4.24	5.27	8.35	10.51	1.92	7.57	6.71	**20.40**
6	302.04	91.63	13.02	175.51	175.51	215.31	4.67	269.39	0.24	191.84	0.25	228.57	7.86	8.73	2.09	4.31	12.55	7.18	8.00	10.29	**2.17**
7	240.82	106.73	3.84	144.90	153.06	219.39	6.27	285.71	0.22	159.18	0.26	208.16	6.22	2.41	1.35	4.86	10.31	4.64	9.71	8.57	**44.70**
8	320.41	103.47	18.90	141.22	151.02	229.59	4.43	263.27	0.44	148.98	0.20	219.39	8.67	5.06	0.01	6.51	12.76	1.73	8.57	5.14	**32.14**
9	204.08	105.92	7.88	170.61	190.82	152.04	2.84	242.86	0.58	187.76	0.45	229.59	12.55	2.00	3.07	6.69	9.69	4.28	6.14	5.00	**36.98**
10	338.78	90.82	12.65	164.49	158.16	198.98	2.35	267.35	0.20	114.29	0.30	214.29	10.92	6.90	4.90	7.80	6.22	4.46	10.71	9.00	**102.68**
11	222.45	101.43	16.69	151.02	166.33	188.78	4.31	238.78	0.28	142.86	0.18	233.67	14.39	10.37	4.41	8.53	9.90	5.55	9.29	11.86	**55.28**
12	100.00	105.10	19.27	163.27	188.78	211.22	1.86	259.18	0.58	112.24	0.34	217.35	6.63	10.16	4.53	1.73	8.06	8.64	6.00	11.71	**110.45**
13	369.39	95.71	7.51	190.20	179.59	170.41	4.18	244.90	0.55	130.61	0.21	210.20	12.76	7.92	0.13	3.20	5.82	3.01	11.71	7.29	**45.04**
14	283.67	110.00	16.33	176.73	192.86	213.27	6.51	210.20	0.30	120.41	0.50	226.53	10.71	5.67	5.88	8.16	7.24	4.82	7.00	11.29	**11.04**
15	308.16	104.69	6.04	187.76	194.90	186.73	2.10	248.98	0.47	175.51	0.35	213.27	9.90	3.02	1.48	5.96	14.18	8.09	7.14	9.57	**26.56**
16	381.63	100.20	4.94	184.08	156.12	225.51	2.96	279.59	0.32	151.02	0.44	212.24	6.43	8.12	3.43	7.24	6.84	0.64	9.57	5.86	**11.32**
17	191.84	96.94	20.00	160.82	189.80	158.16	3.08	291.84	0.60	163.27	0.29	244.90	5.82	11.39	2.33	5.04	7.86	0.83	10.57	7.57	**35.90**
18	314.29	108.78	8.24	173.06	163.27	166.33	3.20	208.16	0.29	165.31	0.48	240.82	14.80	2.82	3.92	7.06	10.92	2.82	11.57	6.86	**51.58**
19	234.69	97.35	11.18	140.00	164.29	243.88	3.94	230.61	0.27	193.88	0.26	209.18	5.00	6.49	4.29	3.57	11.33	7.00	8.29	5.57	**27.28**
20	259.18	98.57	9.71	193.88	159.18	176.53	5.29	224.49	0.53	102.04	0.27	231.63	13.37	3.84	4.78	6.14	10.71	4.10	8.71	11.00	**54.01**
21	344.90	108.37	17.06	181.63	150.00	237.76	6.14	212.24	0.36	110.20	0.12	204.08	5.61	9.96	1.23	9.27	11.73	0.10	5.29	6.43	**109.37**
22	136.73	107.14	17.80	148.57	184.69	196.94	1.00	228.57	0.49	179.59	0.31	211.22	9.29	6.08	2.21	1.00	9.29	2.10	5.57	8.14	**76.51**
23	289.80	92.45	2.00	197.55	187.76	217.35	1.37	283.67	0.21	153.06	0.49	242.86	9.49	8.94	1.60	3.39	7.65	2.64	6.43	9.29	**41.98**
24	326.53	94.49	6.41	159.59	186.73	247.96	5.16	236.73	0.25	100.00	0.43	216.33	12.96	6.69	5.39	5.41	8.27	7.55	9.14	5.71	**31.49**
25	357.14	93.67	5.67	171.84	178.57	207.14	6.63	271.43	0.48	183.67	0.19	207.14	10.51	9.14	1.97	7.61	15.00	3.91	7.29	7.86	**32.60**
26	351.02	109.59	10.08	152.24	185.71	190.82	3.69	287.76	0.36	124.49	0.41	203.06	15.00	3.63	5.63	10.00	12.96	1.55	7.71	5.43	**28.90**
27	363.27	107.96	8.98	146.12	193.88	178.57	1.49	204.08	0.57	106.12	0.24	218.37	10.31	9.76	1.11	4.12	11.94	0.28	5.86	8.71	**139.86**
28	130.61	94.90	3.10	185.31	183.67	160.20	4.06	222.45	0.40	167.35	0.37	241.84	13.16	11.80	0.25	4.67	14.59	3.37	6.57	10.00	**25.49**
29	118.37	102.24	15.22	149.80	198.98	250.00	1.61	257.14	0.54	144.90	0.39	250.00	12.14	7.71	1.84	7.43	14.80	1.01	6.29	6.29	**58.83**
30	265.31	95.31	19.63	154.69	196.94	235.71	5.78	214.29	0.59	134.69	0.28	205.10	7.45	10.57	4.04	7.98	8.47	5.00	9.00	11.57	**16.82**
31	400.00	100.61	10.45	142.45	182.65	209.18	1.24	206.12	0.27	132.65	0.16	232.65	12.35	5.88	1.72	9.08	5.00	5.19	5.00	9.71	**177.81**
32	246.94	96.12	9.35	165.71	160.20	180.61	6.88	300.00	0.49	189.80	0.23	201.02	13.78	7.10	6.00	2.47	6.63	5.73	10.00	8.00	**47.90**
33	332.65	107.55	17.43	162.04	165.31	241.84	5.41	200.00	0.42	171.43	0.47	245.92	11.33	5.47	2.45	9.82	8.88	6.28	5.71	9.14	**12.53**
34	271.43	101.02	4.57	188.98	200.00	201.02	4.55	226.53	0.44	195.92	0.14	222.45	8.27	12.00	3.19	8.90	13.37	8.46	8.14	10.43	**9.57**
35	228.57	99.39	2.73	182.86	177.55	192.86	4.80	289.80	0.23	104.08	0.30	247.96	8.88	5.27	4.66	6.88	6.43	2.28	10.29	7.00	**41.91**
36	277.55	98.98	8.61	191.43	191.84	156.12	6.39	246.94	0.52	146.94	0.12	243.88	14.18	8.33	4.17	4.49	8.67	3.73	11.29	10.71	**37.25**
37	148.98	99.80	6.78	153.47	169.39	227.55	5.53	293.88	0.50	128.57	0.22	200.00	5.41	10.98	2.94	1.18	5.61	8.27	11.86	7.43	**62.48**
38	173.47	102.65	13.39	158.37	173.47	231.63	5.65	218.37	0.22	197.96	0.48	235.71	13.57	3.43	2.58	6.33	5.20	5.91	6.71	11.14	**20.56**
39	253.06	91.22	11.55	195.10	197.96	182.65	2.71	273.47	0.38	116.33	0.39	239.80	7.04	4.45	2.70	3.76	9.49	7.73	12.00	11.43	**23.26**
40	142.86	98.16	5.31	174.29	155.10	221.43	2.59	220.41	0.37	126.53	0.11	220.41	7.24	3.22	0.62	9.45	14.39	1.37	7.86	6.00	**217.17**
41	393.88	93.27	14.12	186.53	172.45	174.49	5.04	234.69	0.31	169.39	0.33	225.51	8.06	8.53	2.82	1.92	13.16	8.82	11.43	9.86	**2.42**
42	124.49	103.88	15.96	143.67	157.14	154.08	2.47	297.96	0.31	157.14	0.32	224.49	9.69	4.04	3.68	3.94	12.14	2.46	5.43	8.43	**77.31**
43	197.96	90.41	12.29	168.16	176.53	172.45	3.57	275.51	0.51	118.37	0.21	227.55	6.84	10.78	5.02	1.37	11.53	3.55	8.43	10.57	**77.59**
44	387.76	92.04	4.20	169.39	171.43	164.29	6.02	261.22	0.26	136.73	0.10	223.47	8.47	9.35	3.31	2.10	13.57	3.19	10.14	6.14	**66.24**
45	375.51	94.08	3.47	155.92	195.92	168.37	1.98	281.63	0.46	122.45	0.34	237.76	7.65	2.61	5.76	3.02	13.78	0.46	10.43	12.00	**46.28**
46	185.71	90.00	18.16	192.65	181.63	150.00	3.82	216.33	0.45	200.00	0.46	206.12	11.94	11.59	0.99	9.63	9.08	9.00	7.43	8.29	**23.90**
47	106.12	92.86	10.82	179.18	174.49	162.24	3.33	202.04	0.35	173.47	0.43	202.04	14.59	9.55	0.74	1.55	5.41	7.37	5.14	7.71	**67.41**
48	216.33	96.53	13.76	177.96	168.37	194.90	1.12	265.31	0.40	108.16	0.36	215.31	9.08	7.51	3.56	5.59	6.02	5.37	10.86	8.86	**118.74**
49	161.22	106.33	14.49	200.00	170.41	233.67	7.00	232.65	0.56	140.82	0.42	234.69	6.02	6.29	5.51	2.29	13.98	1.19	8.86	7.14	**23.40**
50	295.92	101.84	14.86	166.94	152.04	245.92	4.92	277.55	0.34	185.71	0.38	248.98	11.12	2.20	5.14	8.71	12.35	6.46	9.43	6.57	**29.71**
51	250.00	95.00	15.00	150.00	160.00	220.00	3.00	280.00	0.50	150.00	0.25	240.00	10.00	7.00	3.00	6.50	10.00	4.00	9.00	7.00	**38.46**

Table 11: Sobol sequence samples and corresponding calculated objective function

Exp Number	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	Objective Fun
1.00	100.00	90.00	2.00	140.00	150.00	150.00	1.00	200.00	0.20	100.00	0.10	200.00	5.00	2.00	0.01	1.00	5.00	0.10	5.00	5.00	**1124.26**
2.00	250.00	100.00	11.00	170.00	175.00	200.00	4.00	250.00	0.40	150.00	0.30	225.00	10.00	7.00	3.01	5.50	10.00	4.55	8.50	8.50	**13.56**
3.00	175.00	105.00	6.50	185.00	162.50	225.00	2.50	275.00	0.50	125.00	0.40	212.50	12.50	4.50	4.50	3.25	7.50	6.77	6.75	10.25	**59.33**
4.00	325.00	95.00	15.50	155.00	187.50	175.00	5.50	225.00	0.30	175.00	0.20	237.50	7.50	9.50	1.51	7.75	12.50	2.33	10.25	6.75	**14.99**
5.00	137.50	102.50	17.75	192.50	181.25	162.50	3.25	237.50	0.55	162.50	0.35	243.75	13.75	3.25	2.26	4.38	13.75	5.66	5.88	5.88	**35.03**
6.00	287.50	92.50	8.75	162.50	156.25	212.50	6.25	287.50	0.35	112.50	0.15	218.75	8.75	8.25	5.25	8.88	8.75	1.21	9.38	9.38	**51.90**
7.00	212.50	97.50	13.25	147.50	193.75	237.50	1.75	262.50	0.25	187.50	0.25	231.25	6.25	5.75	3.75	2.13	11.25	3.44	7.63	11.13	**38.74**
8.00	362.50	107.50	4.25	177.50	168.75	187.50	4.75	212.50	0.45	137.50	0.45	206.25	11.25	10.75	0.76	6.63	6.25	7.89	11.13	7.63	**6.41**
9.00	118.75	108.75	14.38	166.25	190.63	156.25	3.63	256.25	0.52	168.75	0.13	209.38	9.38	7.63	1.88	8.31	13.13	6.22	6.31	11.56	**70.00**
10.00	268.75	98.75	5.38	196.25	165.63	206.25	6.63	206.25	0.32	118.75	0.32	234.38	14.37	2.63	4.88	3.81	8.13	1.77	9.81	8.06	**33.54**
11.00	193.75	93.75	18.87	181.25	178.13	231.25	2.13	231.25	0.23	193.75	0.42	221.88	11.88	10.13	3.38	6.06	10.62	3.99	8.06	6.31	**70.76**
12.00	343.75	103.75	9.88	151.25	153.13	181.25	5.13	281.25	0.42	143.75	0.23	246.88	6.88	5.13	0.38	1.56	5.63	8.44	11.56	9.81	**40.16**
13.00	156.25	96.25	7.63	173.75	171.88	168.75	1.38	293.75	0.28	106.25	0.38	240.63	10.62	8.88	1.13	7.19	6.88	0.66	5.44	10.69	**154.71**
14.00	306.25	106.25	16.63	143.75	196.88	218.75	4.38	243.75	0.48	156.25	0.18	215.63	5.63	3.88	4.13	2.69	11.88	5.11	8.94	7.19	**22.65**
15.00	231.25	101.25	3.13	158.75	159.38	243.75	2.88	218.75	0.58	131.25	0.28	228.13	8.13	11.38	5.63	9.44	9.38	7.33	7.19	5.44	**47.96**
16.00	381.25	91.25	12.13	188.75	184.38	193.75	5.88	268.75	0.38	181.25	0.48	203.13	13.13	13.13	2.63	4.94	14.37	2.88	10.69	8.94	**41.33**
17.00	109.38	100.63	9.31	179.38	195.31	209.38	5.69	228.13	0.24	184.38	0.29	245.31	11.56	9.19	3.57	4.09	12.81	2.05	7.84	8.72	**24.29**
18.00	259.38	90.63	18.31	149.38	170.31	159.38	2.69	278.13	0.44	134.38	0.49	220.31	6.56	4.19	0.57	8.59	7.81	6.50	11.34	5.22	**21.13**
19.00	184.38	95.63	4.81	164.38	182.81	184.38	4.19	253.13	0.54	159.38	0.39	232.81	9.06	11.69	2.07	1.84	10.31	8.72	6.09	6.97	**18.67**
20.00	334.38	105.63	13.81	194.38	157.81	234.38	1.19	203.13	0.34	109.38	0.19	207.81	14.06	6.69	5.06	6.34	5.31	4.27	9.59	10.47	**217.24**
21.00	146.88	93.13	11.56	156.88	164.06	221.88	4.94	215.63	0.59	146.88	0.44	201.56	7.81	7.94	5.81	2.97	6.56	5.38	6.97	9.59	**11.52**
22.00	296.88	103.13	2.56	186.88	189.06	171.88	1.94	265.63	0.39	196.88	0.24	226.56	12.81	2.94	2.82	7.47	11.56	0.93	10.47	6.09	**49.69**
23.00	221.88	108.13	16.06	171.88	151.56	196.88	6.44	290.63	0.29	121.88	0.14	214.06	10.31	10.44	1.32	5.22	9.06	3.16	5.22	7.84	**105.95**
24.00	371.88	98.13	7.06	141.88	176.56	246.88	3.44	240.63	0.49	171.88	0.34	239.06	5.31	5.44	4.32	9.72	14.06	7.61	8.72	11.34	**15.81**
25.00	128.13	99.38	17.19	190.63	160.94	203.13	5.31	284.38	0.51	140.63	0.26	242.19	13.44	3.56	4.69	5.78	5.94	4.83	7.41	8.28	**52.69**
26.00	278.13	109.38	8.19	160.63	185.94	153.13	2.31	234.38	0.31	190.63	0.46	217.19	8.44	8.56	1.69	1.28	10.94	0.38	10.91	11.78	**47.46**
27.00	203.13	104.38	12.69	145.63	173.44	178.13	6.81	209.38	0.21	115.63	0.36	229.69	5.94	6.06	0.20	8.03	8.44	2.60	5.66	10.03	**146.01**
28.00	353.13	94.38	3.69	175.63	198.44	228.13	3.81	259.38	0.41	165.63	0.16	204.69	10.94	11.06	3.19	3.53	13.44	7.05	9.16	6.53	**23.81**
29.00	165.63	106.88	5.94	153.13	179.69	215.63	6.06	271.88	0.26	178.13	0.41	210.94	7.19	2.31	3.94	9.16	14.69	1.49	8.28	7.41	**28.19**
30.00	315.63	96.88	14.94	183.13	154.69	165.63	3.06	221.88	0.46	128.13	0.21	235.94	12.19	7.31	0.95	4.66	9.69	5.94	11.78	10.91	**59.93**
31.00	240.63	91.88	10.44	198.13	192.19	190.63	4.56	246.88	0.56	153.13	0.11	223.44	14.69	4.81	2.44	6.91	12.19	8.17	6.53	9.16	**48.57**
32.00	390.63	101.88	19.44	168.13	167.19	240.63	1.56	296.88	0.36	103.13	0.31	248.44	9.69	9.81	5.44	2.41	7.19	3.72	10.03	5.66	**108.58**
33.00	104.69	105.94	19.16	159.69	169.53	182.81	1.84	207.81	0.49	129.69	0.48	205.47	8.91	7.78	3.10	1.98	5.78	1.63	9.27	11.89	**114.59**
34.00	254.69	95.94	10.16	189.69	194.53	232.81	4.84	257.81	0.29	179.69	0.28	230.47	13.91	2.78	0.10	6.48	10.78	6.08	5.77	8.39	**9.29**
35.00	179.69	90.94	14.66	174.69	157.03	207.81	3.34	282.81	0.39	104.69	0.18	217.97	11.41	10.28	1.60	4.23	8.28	8.30	11.02	6.64	**132.34**
36.00	329.69	100.94	5.66	144.69	182.03	157.81	6.34	232.81	0.59	154.69	0.38	242.97	6.41	5.28	4.60	8.73	13.28	3.85	7.52	10.14	**29.16**
37.00	142.19	98.44	3.41	182.19	188.28	195.31	2.59	245.31	0.34	192.19	0.23	249.22	10.16	9.03	5.34	5.36	14.53	4.97	10.14	11.02	**48.76**
38.00	292.19	108.44	12.41	152.19	163.28	245.31	5.59	295.31	0.54	142.19	0.43	224.22	5.16	4.03	2.35	9.86	9.53	0.52	6.64	7.52	**45.85**
39.00	217.19	103.44	7.91	167.19	175.78	220.31	1.09	270.31	0.44	167.19	0.33	236.72	7.66	11.53	0.85	3.11	12.03	2.74	11.89	5.77	**40.93**
40.00	367.19	93.44	16.91	197.19	150.78	170.31	4.09	220.31	0.24	117.19	0.13	211.72	12.66	6.53	3.85	7.61	7.03	7.19	8.39	9.27	**102.32**
41.00	123.44	94.69	6.78	148.44	185.16	176.56	2.97	251.56	0.37	198.44	0.46	208.59	5.78	2.16	4.97	8.17	12.66	5.52	9.70	5.33	**32.13**
42.00	273.44	104.69	15.78	178.44	160.16	226.56	5.97	201.56	0.57	148.44	0.26	233.59	10.78	7.16	1.98	3.67	7.66	1.07	6.20	8.83	**18.54**
43.00	198.44	109.69	2.28	193.44	197.66	201.56	1.47	226.56	0.47	173.44	0.16	221.09	13.28	4.66	0.48	5.92	10.16	3.30	11.45	10.58	**81.20**
44.00	348.44	99.69	11.28	163.44	172.66	151.56	4.47	276.56	0.27	123.44	0.36	246.09	8.28	9.66	3.47	1.42	5.16	7.75	7.95	7.08	**34.72**
45.00	160.94	102.19	13.53	185.94	153.91	189.06	2.22	289.06	0.42	135.94	0.21	239.84	14.53	3.41	4.22	7.05	6.41	2.19	8.83	6.20	**105.59**
46.00	310.94	92.19	4.53	155.94	178.91	239.06	5.22	239.06	0.22	185.94	0.41	214.84	9.53	8.41	1.23	2.55	11.41	6.64	5.33	9.70	**4.56**
47.00	235.94	97.19	18.03	140.94	166.41	214.06	3.72	214.06	0.32	110.94	0.31	227.34	7.03	5.91	2.72	9.30	8.91	8.86	10.58	11.45	**87.93**
48.00	385.94	107.19	9.03	170.94	191.41	164.06	6.72	264.06	0.52	160.94	0.11	202.34	12.03	10.91	5.72	4.80	13.91	4.41	7.08	7.95	**48.57**
49.00	114.06	95.31	12.97	199.06	177.34	229.69	6.53	235.94	0.46	157.81	0.32	247.66	12.97	3.72	0.67	3.39	13.59	0.80	11.67	8.17	**28.21**
50.00	264.06	105.31	3.97	169.06	152.34	179.69	3.53	285.94	0.26	107.81	0.12	222.66	7.97	8.72	3.66	7.89	8.59	5.25	8.17	11.67	**108.92**
51.00	250.00	95.00	15.00	150.00	160.00	220.00	3.00	280.00	0.50	150.00	0.25	240.00	10.00	7.00	3.00	6.50	10.00	4.00	9.00	7.00	**38.46**

Table 12: Halton sequence samples and corresponding calculated objective function

Experiment Numer	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	Objective Fun
1.00	250.00	96.67	5.60	148.57	154.55	157.69	1.35	205.26	0.22	103.45	0.11	201.35	5.24	2.23	0.14	1.17	5.17	0.25	5.10	5.10	**707.72**
2.00	175.00	103.33	9.20	157.14	159.09	165.38	1.71	210.53	0.23	106.90	0.13	202.70	5.49	2.47	0.26	1.34	5.34	0.39	5.21	5.20	**604.48**
3.00	325.00	92.22	12.80	165.71	163.64	173.08	2.06	215.79	0.25	110.34	0.14	204.05	5.73	2.70	0.39	1.51	5.51	0.54	5.31	5.30	**380.92**
4.00	137.50	98.89	16.40	174.29	168.18	180.77	2.41	221.05	0.27	113.79	0.15	205.41	5.98	2.93	0.52	1.68	5.68	0.68	5.42	5.39	**389.91**
5.00	287.50	105.56	2.72	182.86	172.73	188.46	2.76	226.32	0.29	117.24	0.16	206.76	6.22	3.16	0.65	1.85	5.85	0.83	5.52	5.49	**222.13**
6.00	212.50	94.44	6.32	191.43	177.27	196.15	3.12	231.58	0.30	120.69	0.18	208.11	6.46	3.40	0.77	2.02	6.02	0.98	5.63	5.59	**183.80**
7.00	362.50	101.11	9.92	141.22	181.82	203.85	3.47	236.84	0.32	124.14	0.19	209.46	6.71	3.63	0.90	2.19	6.19	1.12	5.73	5.69	**117.01**
8.00	118.75	107.78	13.52	149.80	186.36	211.54	3.82	242.11	0.34	127.59	0.20	210.81	6.95	3.86	1.03	2.36	6.36	1.27	5.84	5.79	**153.09**
9.00	268.75	90.74	17.12	158.37	190.91	219.23	4.18	247.37	0.36	131.03	0.22	212.16	7.20	4.09	1.16	2.53	6.53	1.41	5.94	5.89	**68.16**
10.00	193.75	97.41	3.44	166.94	195.45	226.92	4.53	252.63	0.37	134.48	0.23	213.51	7.44	4.33	1.28	2.70	6.69	1.56	6.04	5.99	**63.05**
11.00	343.75	104.07	7.04	175.51	150.41	234.62	4.88	257.89	0.39	137.93	0.24	214.86	7.68	4.56	1.41	2.87	6.86	1.70	6.15	6.08	**44.17**
12.00	156.25	92.96	10.64	184.08	154.96	242.31	5.24	263.16	0.41	141.38	0.25	216.22	7.93	4.79	1.54	3.04	7.03	1.85	6.25	6.18	**56.81**
13.00	306.25	99.63	14.24	192.65	159.50	150.59	5.59	268.42	0.43	144.83	0.27	217.57	8.17	5.02	1.67	3.21	7.20	2.00	6.36	6.28	**31.51**
14.00	231.25	106.30	17.84	142.45	164.05	158.28	5.94	273.68	0.44	148.28	0.28	218.92	8.41	5.26	1.79	3.38	7.37	2.14	6.46	6.38	**39.76**
15.00	381.25	95.19	4.16	151.02	168.60	165.98	6.29	278.95	0.46	151.72	0.29	220.27	8.66	5.49	1.92	3.55	7.54	2.29	6.57	6.48	**35.03**
16.00	109.38	101.85	7.76	159.59	173.14	173.67	6.65	284.21	0.48	155.17	0.31	221.62	8.90	5.72	2.05	3.72	7.71	2.43	6.67	6.58	**43.41**
17.00	259.38	108.52	11.36	168.16	177.69	181.36	1.02	289.47	0.50	158.62	0.32	222.97	9.15	5.95	2.18	3.89	7.88	2.58	6.78	6.68	**59.25**
18.00	184.38	91.48	14.96	176.73	182.23	189.05	1.37	294.74	0.51	162.07	0.33	224.32	9.39	6.19	2.30	4.06	8.05	2.73	6.88	6.77	**54.34**
19.00	334.38	98.15	18.56	185.31	186.78	196.75	1.73	200.28	0.53	165.52	0.35	225.68	9.63	6.42	2.43	4.23	8.22	2.87	6.99	6.87	**39.53**
20.00	146.88	104.81	4.88	193.88	191.32	204.44	2.08	205.54	0.55	168.97	0.36	227.03	9.88	6.65	2.56	4.40	8.39	3.02	7.09	6.97	**41.42**
21.00	296.88	93.70	8.48	143.67	195.87	212.13	2.43	210.80	0.57	172.41	0.37	228.38	10.12	6.88	2.69	4.57	8.56	3.16	7.19	7.07	**20.37**
22.00	221.88	100.37	12.08	152.24	150.83	219.82	2.79	216.07	0.58	175.86	0.38	229.73	10.37	7.12	2.81	4.74	8.73	3.31	7.30	7.17	**24.32**
23.00	371.88	107.04	15.68	160.82	155.37	227.51	3.14	221.33	0.20	179.31	0.40	231.08	10.61	7.35	2.94	4.91	8.90	3.46	7.40	7.27	**40.09**
24.00	128.13	95.93	19.28	169.39	159.92	235.21	3.49	226.59	0.22	182.76	0.41	232.43	10.85	7.58	3.07	5.08	9.07	3.60	7.51	7.37	**54.42**
25.00	278.13	102.59	2.14	177.96	164.46	242.90	3.84	231.86	0.24	186.21	0.42	233.78	11.10	7.81	3.20	5.25	9.24	3.75	7.61	7.46	**16.12**
26.00	203.13	109.26	5.74	186.53	169.01	151.18	4.20	237.12	0.25	189.66	0.44	235.14	11.34	8.05	3.32	5.42	9.41	3.89	7.72	7.56	**18.99**
27.00	353.13	90.25	9.34	195.10	173.55	158.88	4.55	242.38	0.27	193.10	0.45	236.49	11.59	8.28	3.45	5.58	9.58	4.04	7.82	7.66	**16.99**
28.00	165.63	96.91	12.94	144.90	178.10	166.57	4.90	247.65	0.29	196.55	0.46	237.84	11.83	8.51	3.58	5.75	9.75	4.19	7.93	7.76	**15.98**
29.00	315.63	103.58	16.54	153.47	182.64	174.26	5.26	252.91	0.31	100.12	0.47	239.19	12.07	8.74	3.71	5.92	9.92	4.33	8.03	7.86	**19.45**
30.00	240.63	92.47	2.86	162.04	187.19	181.95	5.61	258.17	0.32	103.57	0.49	240.54	12.32	8.98	3.83	6.09	10.08	4.48	8.13	7.96	**19.38**
31.00	390.63	99.14	6.46	170.61	191.74	189.64	5.96	263.43	0.34	107.02	0.10	241.89	12.56	9.21	3.96	6.26	10.25	4.62	8.24	8.06	**70.89**
32.00	104.69	105.80	10.06	179.18	196.28	197.34	6.31	268.70	0.36	110.46	0.11	243.24	12.80	9.44	4.09	6.43	10.42	4.77	8.34	8.15	**128.99**
33.00	254.69	94.69	13.66	187.76	151.24	205.03	6.67	273.96	0.37	113.91	0.13	244.59	13.05	9.67	4.22	6.60	10.59	4.91	8.45	8.25	**72.66**
34.00	179.69	101.36	17.26	196.33	155.79	212.72	1.04	279.22	0.39	117.36	0.14	245.95	13.29	9.91	4.34	6.77	10.76	5.06	8.55	8.35	**186.06**
35.00	329.69	108.02	3.58	146.12	160.33	220.41	1.39	284.49	0.41	120.81	0.15	247.30	13.54	10.14	4.47	6.94	10.93	5.21	8.66	8.45	**117.61**
36.00	142.19	90.99	7.18	154.69	164.88	228.11	1.75	289.75	0.43	124.26	0.16	248.65	13.78	10.37	4.60	7.11	11.10	5.35	8.76	8.55	**147.54**
37.00	292.19	97.65	10.78	163.27	169.42	235.80	2.10	295.01	0.44	127.71	0.18	200.04	14.02	10.60	4.73	7.28	11.27	5.50	8.87	8.65	**86.94**
38.00	217.19	104.32	14.38	171.84	173.97	243.49	2.45	200.55	0.46	131.15	0.19	201.39	14.27	10.84	4.85	7.45	11.44	5.64	8.97	8.75	**122.12**
39.00	367.19	93.21	17.98	180.41	178.51	151.78	2.81	205.82	0.48	134.60	0.20	202.74	14.51	11.07	4.98	7.62	11.61	5.79	9.07	8.85	**74.38**
40.00	123.44	99.88	4.30	188.98	183.06	159.47	3.16	211.08	0.50	138.05	0.22	204.09	14.76	11.30	5.11	7.79	11.78	5.94	9.18	8.94	**119.88**
41.00	273.44	106.54	7.90	197.55	187.60	167.16	3.51	216.34	0.51	141.50	0.23	205.44	5.01	11.53	5.24	7.96	11.95	6.08	9.28	9.04	**23.86**
42.00	198.44	95.43	11.50	147.35	192.15	174.85	3.87	221.61	0.53	144.95	0.24	206.79	5.25	11.77	5.36	8.13	12.12	6.23	9.39	9.14	**30.50**
43.00	348.44	102.10	15.10	155.92	196.69	182.54	4.22	226.87	0.55	148.39	0.26	208.14	5.49	2.01	5.49	8.30	12.29	6.37	9.49	9.24	**11.45**
44.00	160.94	108.77	18.70	164.49	151.65	190.24	4.57	232.13	0.57	151.84	0.27	209.50	5.74	2.24	5.62	8.47	12.46	6.52	9.60	9.34	**41.64**
45.00	310.94	91.73	5.02	173.06	156.20	197.93	4.92	237.40	0.58	155.29	0.28	210.85	5.98	2.47	5.75	8.64	12.63	6.67	9.70	9.44	**22.66**
46.00	235.94	98.40	8.62	181.63	160.74	205.62	5.28	242.66	0.20	158.74	0.29	212.20	6.23	2.70	5.87	8.81	12.80	6.81	9.81	9.54	**45.58**
47.00	385.94	105.06	12.22	190.20	165.29	213.31	5.63	247.92	0.22	162.19	0.31	213.55	6.47	2.94	0.01	8.98	12.97	6.96	9.91	9.63	**23.84**
48.00	114.06	93.95	15.82	198.78	169.83	221.01	5.98	253.19	0.24	165.64	0.32	214.90	6.71	3.17	0.14	9.15	13.14	7.10	10.01	9.73	**78.22**
49.00	264.06	100.62	19.42	140.17	174.38	228.70	6.34	258.45	0.25	169.08	0.33	216.25	6.96	3.40	0.27	9.32	13.31	7.25	10.12	9.83	**30.71**
50.00	189.06	107.28	2.29	148.75	178.93	236.39	6.69	263.71	0.27	172.53	0.35	217.60	7.20	3.63	0.40	9.49	13.47	7.40	10.22	9.93	**37.50**
51.00	250.00	95.00	15.00	150.00	160.00	220.00	3.00	280.00	0.50	150.00	0.25	240.00	10.00	7.00	3.00	6.50	10.00	4.00	9.00	7.00	**38.46**

Results

The three studied experimental design techniques are applied on two different scale material balance test problems and the results are presented in this section. Artificial neural network models are built and minimized using genetic algorithm. The optimization algorithm, genetic algorithm, is run five times to take more representative performance indicators. The average value of the performance indicators out of the five runs is used for our comparison.

The Single Tank Material Balance Test Problem

Table 13 presents the values of the error indicator obtained in case of the multiple tanks material balance test problem. For comparison purposes, the obtained results are plotted in **Fig. 5.** As shown in Fig. 5, Sobol and Halton sequences have significantly lower value of the average error than Latin hypercube method. Given that the same algorithm was used for proxy model generation and the same algorithm was used for optimization, the experimental design algorithm is responsible for the superior results of Sobol and Halton techniques over Latin hypercube.

Table 13: Single tank material balance test problem results

DOE	Estimated Solution			Error Indicator			
	X1	X2	X3	X1, %	X2, %	X3, %	Average, %
Latin Hybercube	246.7	115	14	16.99	26.56	49.88	31.14
Sobol Sequence	243	150.2	9.85	15.23	4.11	5.12	8.15
Halton Sequence	240	173.4	7.73	13.83	10.75	17.52	14.03

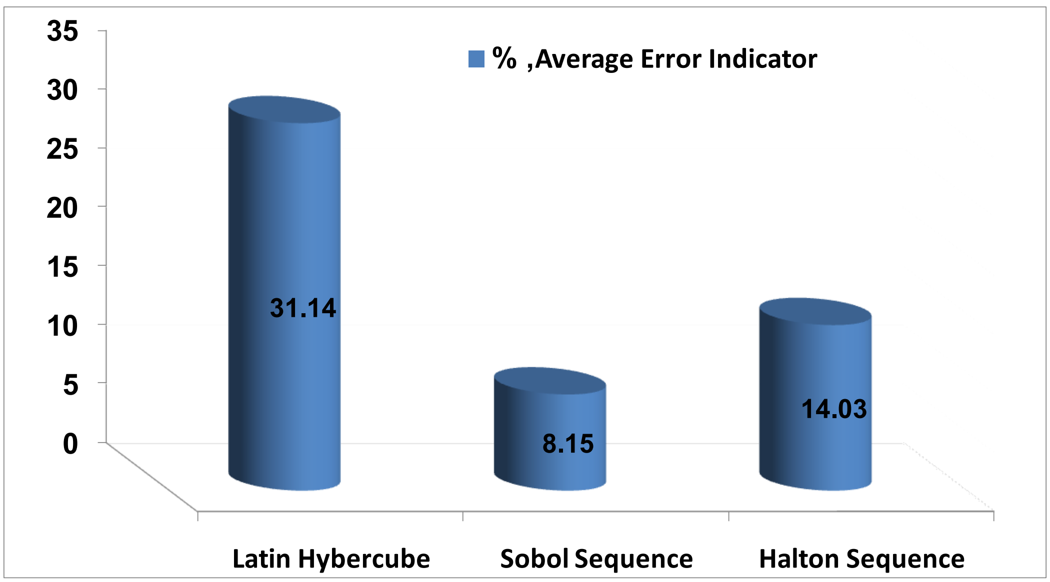

Fig. 5: Single tank material balance test problem results comparison

The Multiple Tanks Material Balance Test Problem

Table 14 summarizes the values of the error indicator obtained in case of the multiple tanks material balance test problem. For comparison purposes, the obtained results are plotted in **Figs. 6.** As shown in Fig. 6 and similarly to the single tanks material balance problem, Sobol and Halton sequences have lower values of the average error than Latin hypercube method.

Table 14: Multiple tanks material balance test problem results

DOE	Error Indicator																				
	X1, %	X2, %	X3, %	X4, %	X5, %	X6, %	X7, %	X8 %	X9, %	X10, %	X11, %	X12, %	X13, %	X14, %	X15, %	X16, %	X17, %	X18, %	X19, %	X20, %	Average, %
LHC	29.82	9.51	99.44	16.87	16.54	24.99	38.06	19.88	18.87	11.09	8.93	6.35	84.57	19.10	17.19	229.06	66.23	26.98	94.38	44.43	44.12
Sobol Sequence	2.08	8.56	96.39	15.47	8.85	1.98	18.14	12.97	29.86	7.93	27.05	6.09	65.66	5.80	7.73	200.14	33.07	15.77	34.29	5.83	30.18
Halton Sequence	30.41	9.50	99.73	17.58	16.62	24.96	39.03	19.93	90.75	44.40	23.73	8.26	34.43	70.82	19.98	54.99	65.91	27.53	20.92	31.69	37.56

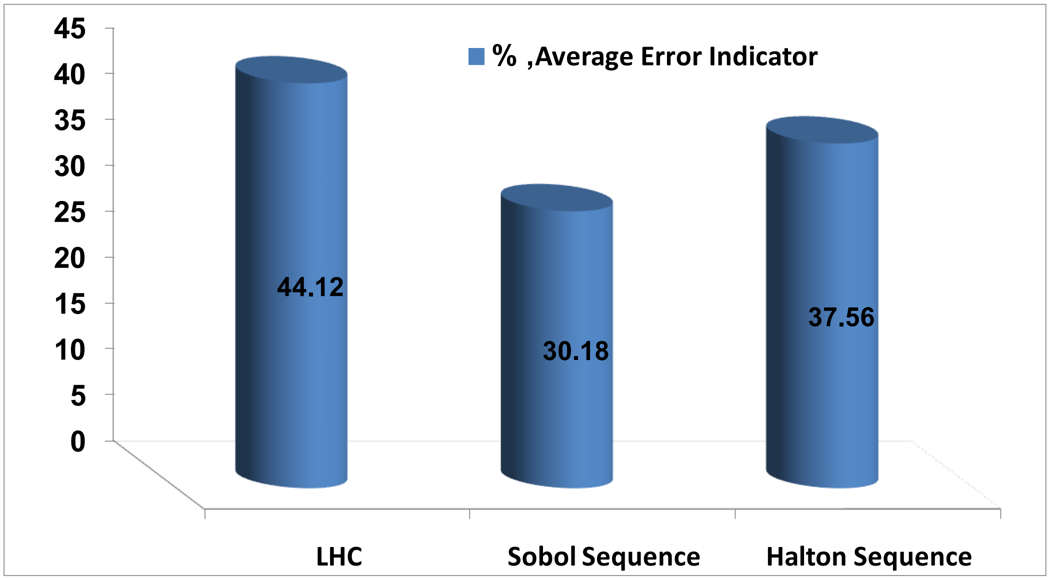

Fig. 6: Multiple tanks material balance test problem results comparison

Conclusions

Based on the results of comparing recent experimental design techniques with more widely used experimental design techniques in assisted history matching, we can make the following conclusions:

- Both Sobol and Halton sequences experimental design techniques are remarkably superior to the most widely used sampling technique, Latin hypercube.

- Sobol and Halton sequences sampling technique give solutions closer to the exact solution, and consequently improve the assisted history matching process.

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