Goal programming: a new tool for optimization in petroleum reservoir history matching

M. H. Sayyouh
Department of Petroleum Engineering, PO Box 800, Riyadh University, Riyadh, Saudi Arabia
(Received February 1981)

This paper considers the new field of goal programming under a pre-emptive priority structure. The goal programming model presented was constructed as a practical optimization technique applied to the history matching of actual petroleum reservoir performance. The results showed that this technique is a powerful tool for optimizing the differences between the actual and predicted performance of those reservoirs. Therefore, any numerical petroleum reservoir simulator can be employed for studying actual cases of oilfields.

Introduction

The use of petroleum reservoir simulators to predict field performance is receiving increasing consideration for field applications. Considerable effort has been devoted recently to the development of various techniques in order to minimize the differences between the actual field performance and the predicted performance obtained from numerical reservoir simulation.[1-3] The goal programming model provides a reasonable structure under which the traditional, single objective tools (such as linear and nonlinear programming) may be viewed simply as special cases. The goal programming approach represents a substantial improvement in the modelling and analysis of multiobjective problems. Interest in this approach has increased significantly in the past five years, as has its actual implementation.[4-8]

The objective of the present study was to apply the goal programming model (as an optimization technique) to minimizing the differences between the actual and simulated behaviour of a gas reservoir. A single phase (gas), two-dimensional dry gas reservoir simulator was designed in particular to study the validity of simulating the performance of the reservoir by using the optimization technique presented in this paper. The novel nature of this study, along with its applications to the prediction of actual performance make it important.

Goal programming model

In general, goal programming aims to handle optimization problems where more than one objective is involved. In this model there is no difference between objective function and constraints. They are all considered as objectives. The goal programming model can be presented as follows:

Find

$$\vec{x} = x_1, x_2, \ldots, x_j, \ldots, X_J$$

so as to minimize:

$$\vec{a} = \{e_1(\vec{n}, \vec{p}), e_2(\vec{n}, \vec{p}_1), \ldots, e_m(\vec{n}, \vec{p}), \ldots, e_M(\vec{n}, \vec{p})\}$$

Such that:

$$g_i(\vec{x}) + n_i - P_i = b_i \qquad \text{for all } i = 1, 2, \ldots, m$$

and $\vec{x}, \vec{n}, \vec{p} \geqslant 0$

where:

x_j = jth decision variable
$\vec{a}$ = achievement function, a row vector measure of attainment of objectives or constraints at each priority level
$e_m(\vec{n}, \vec{p})$ = function of deviation variables associated with objectives or constraints at priority level m
M = total number of priority level in model
b_i = right-hand-side constant for goal i
$g_i(\vec{x})$ = left-hand-side of linear or nonlinear goal i

Under such a formulation, given any type of goal, it is necessary to minimize the nonachievement of that goal by minimizing specific deviation variables. The general procedure to construct a sequential goal programming code is shown as an algorithm in *Figure 1*.

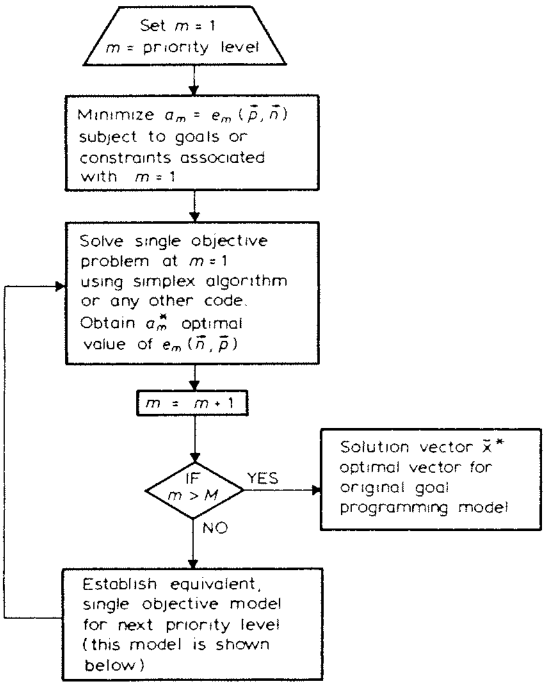
Applications to petroleum reservoir simulation

Reservoir description

The present work considers a single phase (gas), two-dimensional petroleum reservoir simulator which was

0307-904X/81/040223-04/$02.00

developed to simulate production and pressure behaviour. The map presented in *Figure 2* corresponds to the reservoir simulated. Data for the reservoir studied are given in *Table 1*. The two-dimensional description of the reservoir was implemented using the block grid shown in *Figure 3*. The block thicknesses were assigned in accordance with the isopach map of the reservoir (see *Figure 3*). Porosity and permeability distributions were considered as the average



Next priority level is given by:

minimize $\bar{a}_m = e_m(\vec{n}, \vec{p})$ subject to $g_t(\vec{x}) + n_t - P_t = b_t$

$e_s(\vec{n}, \vec{p}) = a_s^*, \vec{x}, \vec{n}, \vec{p} \geqslant 0$

where: $s = 1, 2, \ldots, m-1$; t = sibscript associated with goals included in priority level $1, 2, \ldots, m$

Figure 1 Goal programming model algorithm

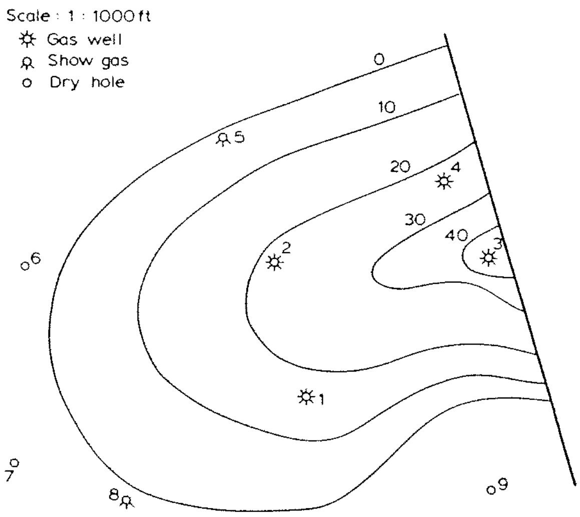


Figure 2 Reservoir isopach map

values given in *Table 1*. The reservoir is divided into four zones each of which contains a well.

History matching

In a reservoir simulation, it is usual to use well pressures or gas-oil ratios and/or water-oil ratios as the basis for the match. In this study, two years of well pressure behaviour were simulated on the basis of the data discussed previously. During the matching of the reservoir performance, some parameters are assumed to be known while others are varied to achieve a match. The parameters varied in the present work were permeability and porosity, because of their importance in production behaviour.

The objective of the history-matching procedure presented here is to minimize the error between the set of actual and predicted performance data by using the goal programming technique. The formulation used is shown in *Table 2*. The following procedure was employed:

(1) Assume initial values for the decision variables within a range of variation, i.e.:

$$k_i^l \leqslant k_i \leqslant k_i^u \quad \text{and} \quad \phi_i^l \leqslant \phi_i \leqslant \phi_i^u$$

where, u and l denote to the upper and lower limits of

Table 1 Reservoir data

Specific gravity of gas	= 0.63
Average reservoir temperature	= 80° F
Initial reservoir pressure	= 1600 psia
Connate water saturation	= 32%
Reservoir porosity	≈ 12%
Absolute reservoir permeability	≈ 15 md
Number of wells used in simulation	= 4 wells

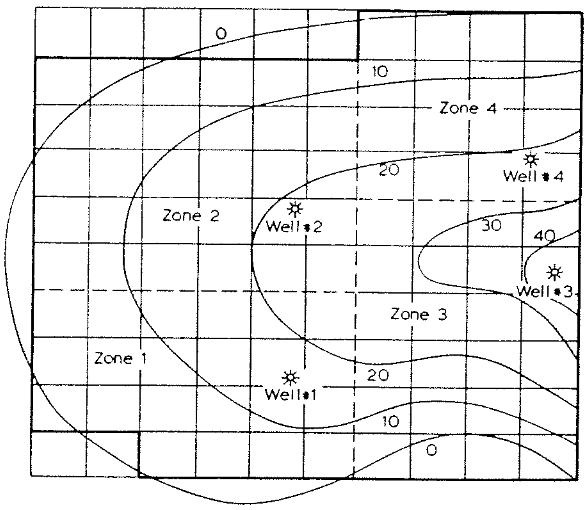


Figure 3 Two-dimensional description and thickness distribution used in simulation

Table 2 Goal programming formulation used in history matching

Goal or constraint type	Processed goal or constraint	Deviation variables to be minimized
$g_i(\bar{x}) \leqslant b_i$	$g_i(\bar{x}) + n_i - P_i = b_i$	P_i
$g_i(\bar{x}) \geqslant b_i$	$g_i(\bar{x}) + n_i - P_i = b_i$	n_i
$g_i(\bar{x}) = b_i$	$g_i(\bar{x}) + n_i - P_i = b_i$	$P_i + n_i$

All $P_i, n_i \geqslant 0$, $i = 1, 2, \ldots$

variation, and k, ϕ are the reservoir permeability and porosity respectively.

(2) Generate random generator FK and FP as follows:

$$FK = 1 + RN\frac{k_i^u - k_i^l}{k_i^l} \quad \text{and} \quad FP = 1 + RN\frac{\phi_i^u - \phi_i^l}{\phi_i^l}$$

where:

$$k_i = RN(k_i^u - k_i^l) + k_i^l \qquad 0 \leqslant RN \leqslant 1$$

and

$$\phi_i = RN(\phi_i^u - \phi_i^l) + \phi_i^l \qquad i = 1, 2, \ldots$$

Here, FK and FP are our goals.

(3) Carry out simulations.

(4) Calculate the error

$$E_i = P_i^{\text{actual}} - P_i^{\text{predicted}}$$

where: P_i^{actual} is the actual reservoir pressure, and $P_i^{\text{predicted}}$ is the simulated reservoir pressure.

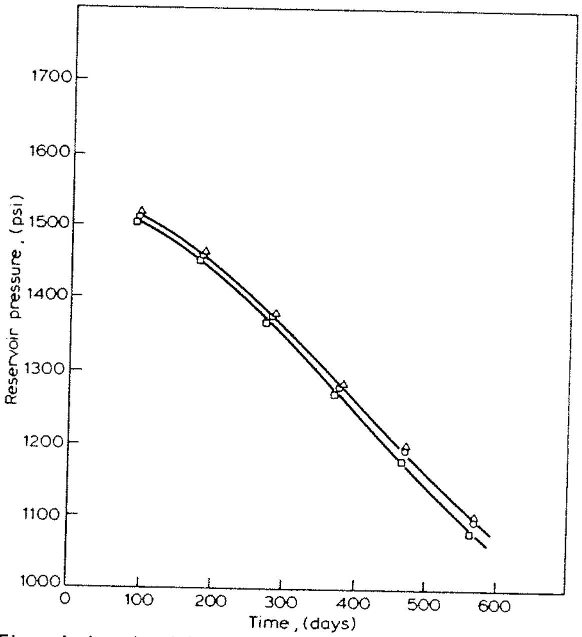


Figure 4 Actual and simulated pressure behaviour of well 1. (○), actual pressure; (□), simulated pressure using linear programming; (△), simulated pressure using goal programming

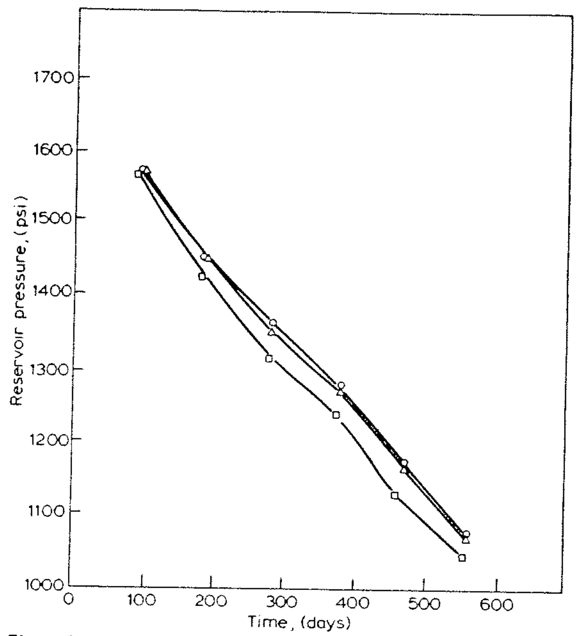


Figure 6 Actual and simulated pressure behaviour of well 3. (Key as in *Figure 4*)

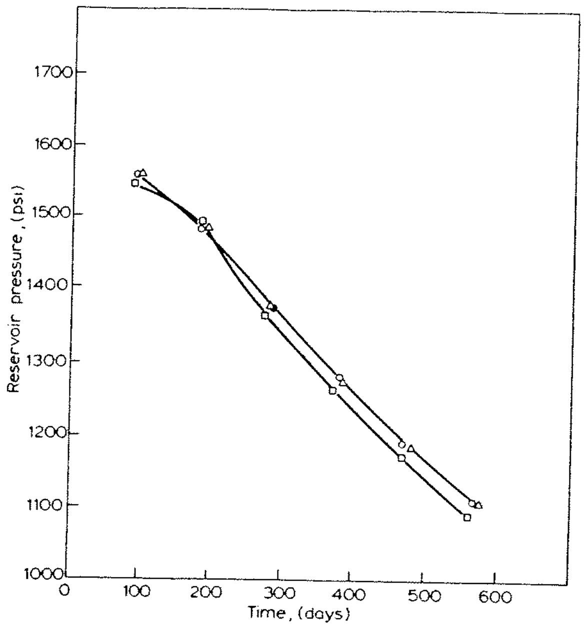


Figure 5 Actual and simulated pressure behaviour of well 2. (Key as in *Figure 4*)

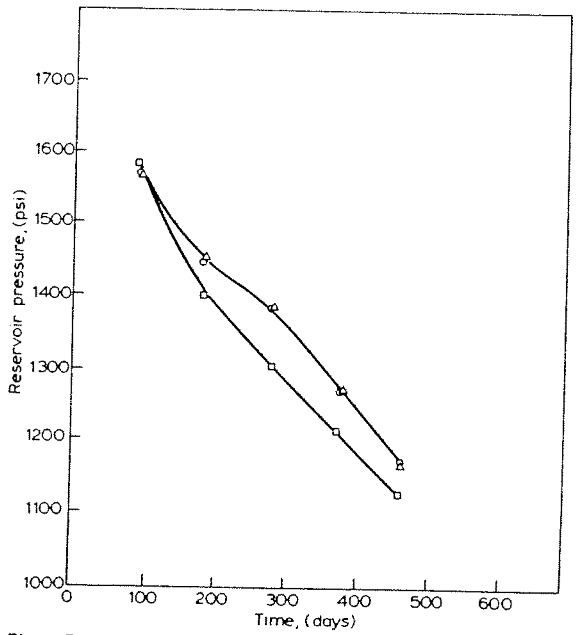


Figure 7 Actual and simulated pressure behaviour of well 4. (Key as in *Figure 4*)

(5) Express error as a function of the decision variables and then minimize this objective function:

$$E = \sum_{1}^{i} [F^i(x_1, x_2, \ldots, x_n)]^2$$

subject to the goals:

$$FK^l \leqslant (FK)_i \leqslant FK^u \quad \text{and} \quad FP^l \leqslant (FP)_i \leqslant FP^u$$

(6) Apply the goal programming algorithm to obtain the optimal solution of the true reservoir permeability and porosity which gives the desired match.

The actual and simulated pressure behaviours for the four wells are shown in *Figures 4, 5, 6* and *7*. All these figures show that the simulated pressure history is in excellent agreement with the actual behaviour using the goal programming approach. On the other hand, these results show that the simulator used can be employed for studying actual cases of oil fields. This is only achieved by using the optimization technique used in this study.

Conclusions

The goal programming model is a new, practical optimization technique applied to history matching between field and calculated performance by numerical simulation.

Any set of reservoir description parameters (such as porosity and/or permeability) and performance data (such as pressure) can be used to match history using goal programming approach.

The applications of a gas reservoir simulator demonstrates the utility of this method. Therefore, any other reservoir simulator can be employed for studying actual cases.

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