A New Approach for Accurate Prediction of Loading in Gas Wells Under Different Flowing Conditions

M.A. Nosseir, SPE, Suez Oil Co., and **T.A. Darwich,** SPE, **M.H. Sayyouh,** SPE, and **M. El Sallaly,** SPE, Cairo U.

Summary

Several authors have introduced various mathematical equations to calculate the critical flow rate necessary to keep gas wells unloaded. The most widely used equation is that of Turner *et al.*[1] However, Turner's equation required empirical adjustment with different ranges of data which made the application rather questionable. In this paper we present a new approach for calculating the critical flow rate necessary to keep gas wells unloaded. This approach still adopts Turner's basic concepts, but considers different flow conditions that result in different flow regimes. Hence it explains the previous discrepancies of Turner's equation (the droplet model) with different data ranges, and presents a new set of equations that eliminates the need for empirical adjustment and better matches actual data records.

Introduction

The gas well loading phenomenon is one of the most serious problems that reduces, and eventually cuts, production in gas wells. This phenomenon occurs as a result of liquid accumulation: either water and/or condensate in the well bore. Over time, these liquids cause additional hydrostatic backpressure on the reservoir which results in a continual reduction of the available transport energy. The well therefore starts slugging which gives an even larger chance of liquid accumulation that completely overcomes the reservoir pressure and causes the well to die. **Fig. 1** illustrates the development of the loading phenomenon in a gas well.

Typical solutions were to unload the well artificially, either mechanically (using pumps) or with gas lift (kicking with nitrogen through coiled tubing). However, in addition to the expense and loss of production, artificial lift solutions remain temporary and the well is subject to reloading again.

Therefore, thought was directed toward developing some solutions that enable the well to continuously unload itself without the aid of external help (unloading operations). Numerous theories have offered methods by which to predict and control the onset of loadup. Turner *et al.*'s method for predicting when gas well loadup will occur is most widely used. Turner *et al.* developed two physical models for transporting fluids up vertical conduits: the liquid droplet and the liquid film models. A comparison of these two models with field data led to the conclusion that the onset of loadup could be predicted adequately with the droplet model, but that a 20% adjustment of the equation upward was necessary. This upward adjustment improved the match and was empirically recognized by other researchers working on the same subject. Lescarboura,[2] adopting the same empirical adjustment, presented a computerized version of the droplet model to predict critical gas flow rates for continuous liquid removal from the wellbore. Later, Coleman[3] *et al.* stated they obtained a good match with their actual field records using the droplet model without any adjustment. They found that, in practice the critical flow rate required to keep low pressure gas wells unloaded can be predicted adequately with the liquid droplet model without the 20% upward adjustment. This in turn raises questions of the droplet model limitations and when to apply an adjustment. And how much? In this paper we focus on changes in flow regimes and their impact on gas well loading. Hence it presents a better explanation of the loading phenomenon under different flowing conditions.

Analytical Approach: Concepts and Development

The analytical approach dealing with the gas well loading phenomenon is mainly based on the force balance concept.

There are two major forces acting upon a droplet of liquid falling in a gas stream: gravitional force pulling the droplet downward and gas stream force tending to drag the droplet upward. **Fig. 2** shows a schematic of the forces acting on a liquid droplet.[1]

Analytical Equations For Different Flow Conditions (Historical Review)

Laminar Flow Regime. In 1851 Stokes introduced his equation for calculating the critical terminal velocity when the relative motion between the particle and fluid is laminar, i.e., for values of $N_{Re}<1$.

For the same laminar flow region, Hadamard[5] and Rybczynski[6] in 1911 and 1912, respectively, developed independently the same type of equation. However, their equation can be considered a modification of Stokes' law. Stokes assumed smooth, hard, spherical particles which cannot be exactly the case with gas bubbles or liquid droplets. This is because circulation within a fluid drop lessens the velocity adjacent the drop, decreases the energy dissipation, and results in a higher fall or rise velocity for a fluid drop compared to a solid particle.[4] That is why the Hadamard and Rybczynski equation is initially Stokes' law multiplied by a factor that is always greater than unity $(3\mu+3\mu_p)/(2\mu+3\mu_p)$.

In 1962, Levich[7] and others working on the same subject compared the above two equations of Stokes and Hadamard and found by experimental data that Stokes' law applies better to small impure fluid particles (which is probably the case in gas wells) while for larger particle sizes, or small particles of exceptionally pure fluid, the Hadamard and Rybczynski equation applies better. Levich explained this by the fact that, in ordinary systems, trace impurities may have some surfactant properties that inhibit internal circulation within the particle, and cause it to behave essentially as a solid particle, i.e., in accordance with Stokes' law. Larger particles, on the other hand, are not so greatly affected, and even when surfactants are present the behavior is close to that given by the Hadamard and Rybczynski equation.[4]

Transition Flow Regime. The transition, or gradually developing turbulence region, was described by Allen[8] in 1900. He introduced an equation for the range of $1<N_{Re}<1{,}000$.

Turbulent Flow Regime. The third region represents fully developed turbulence. It extends from N_{Re} of about 1,000 to 200,000 where the drag coefficient is fairly constant at 0.44 as shown in **Fig. 3**. Hinze,[9] in 1955, solved the problem of determining the largest drop diameter. He showed that liquid droplets moving relative to a gas stream are subjected to forces that try to shatter them while the surface tension of the liquid tends to hold the drop together. He determined that it is the balance between two pressures, the velocity pressure, $v_g^2 p/g_c$, and the surface tension pres-

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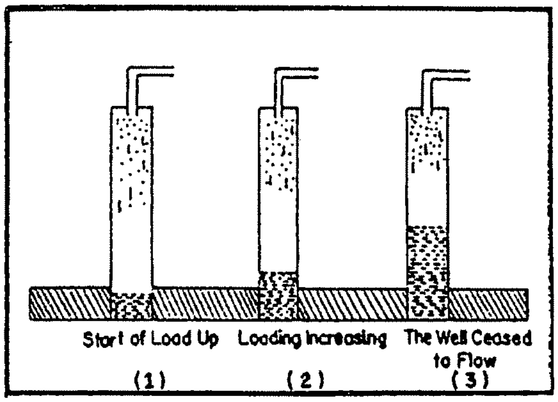

Fig. 1–Development of loading phenomenon in gas wells.

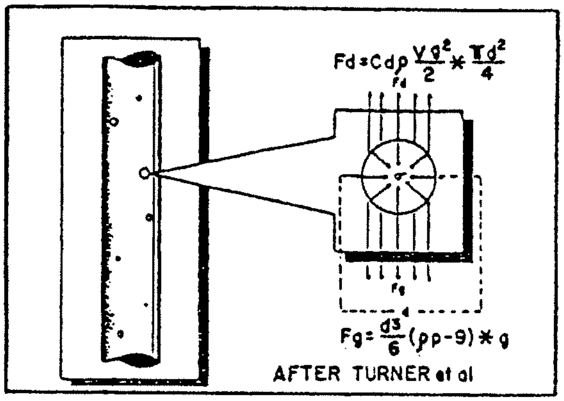

Fig. 2–Forces acting on a liquid drop falling in a gas stream.

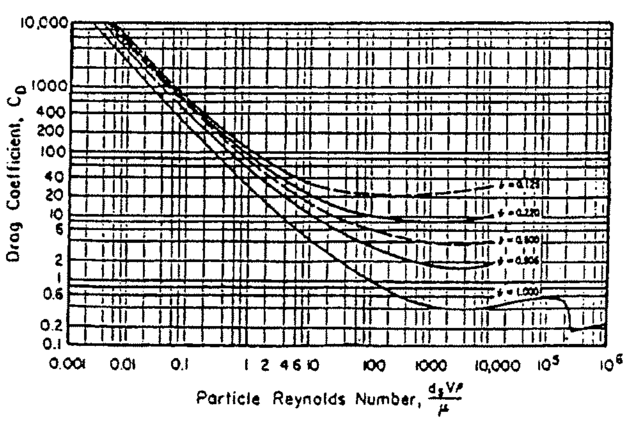

Fig. 3–Dependence of drag coefficient on the Reynolds number (from Ref. 10).

TABLE 1–TURNER ACTUAL DATA VS. DROPLET MODEL ESTIMATES

Test Rate Mscf/D	TIE* Mscf/D	TAE** Mscf/D	Reynolds No. Dimensionless	Well Status
568	264	306	185,288	Near loadup
442	361	419	144,123	Near loadup
417	503	583	136,046	Near loadup
711	570	661	232,256	Near loadup
774	672	779	253,004	Near loadup
1,117	808	936	363,993	Questionable
1,607	827	958	537,806	Near loadup
2,065	941	1,091	629,920	Near loadup
1,312	948	1,099	424,009	Loaded
1,791	956	1,108	584,465	Loaded
1,247	990	1,148	403,886	Loaded
1,524	997	1,156	1,132,168	Loaded
2,489	999	1,158	802,032	Questionable
1,356	1,033	1,197	436,834	Loaded
1,365	1,224	1,419	429,465	Loaded
2,911	1,380	1,600	949,716	Questionable
2,547	1,384	1,604	830,715	Loaded
2,500	1,389	1,610	816,257	Questionable
2,259	1,400	1,623	737,355	Loaded
1,811	1,411	1,635	589,461	Loaded
3,177	1,427	1,654	1,032,814	Questionable
2,541	1,432	1,660	829,065	Loaded
800	1,489	1,726	265,714	Loaded
4,887	1,675	1,941	1,591,363	Questionable
3,467	1,688	1,956	1,132,168	Loaded
499	1,797	2,083	158,720	Loaded
400	1,884	2,184	124,000	Loaded
4,093	2,066	2,395	1,336,366	Questionable
3,258	2,071	2,401	1,050,634	Questionable
2,606	2,081	2,412	851,986	Loaded
3,548	2,789	3,233	1,153,200	Questionable
470	2,802	3,248	156,240	Loaded
3,006	2,831	3,281	985,800	Loaded
4,989	2,944	3,412	1,635,992	Questionable
4,121	3,117	3,613	1,338,385	Loaded
6,397	3,241	3,757	2,080,543	Questionable
5,495	3,250	3,767	1,778,018	Loaded
4,842	4,135	4,793	1,573,029	Loaded
4,436	4,245	4,920	1,433,529	Loaded
3,855	4,353	5,045	1,250,186	Loaded
5,728	4,394	5,093	1,841,400	Loaded
8,395	4,796	5,559	2,735,582	Questionable
7,096	4,814	5,580	2,306,187	Loaded
6,353	4,867	5,641	2,074,867	Loaded
8,054	4,893	5,671	2,625,774	Questionable
9,036	5,092	5,902	2,952,405	Questionable
3,890	5,110	5,923	1,270,823	Loaded
8,185	5,117	5,931	2,670,712	Loaded
8,204	5,189	6,015	2,647,843	Questionable
6,699	5,247	6,082	2,168,229	Loaded
2,780	5,337	6,186	894,394	Loaded
4,295	5,486	6,359	1,402,971	Loaded
1,639	5,493	6,367	540,144	Loaded

*TIE: Turner initial equation.
**TAE: Turner adjusted equation.

sure described by σ/d_p, that determines the maximum size a drop can attain. The ratio of these two pressures is the Weber number (N_{We}) which is defined as $N_{We} = v_g^2 d_{pp} / \sigma g_c$.

Turner *et al.* Approach (The Droplet Model)

Turner *et al.* developed their equation known as the droplet model. They assumed a constant turbulent flow regime prevailed in the gas wells, that is, always in the range of $10^4 < N_{Re} < 2 \times 10^5$.

Verifying Turner's Assumptions

To verify the drop model, we had to examine Turner's initial assumptions. This includes the dominance of the turbulent flow regime in a certain range. This is very important in determining the shape of the drag coefficient which then determines the critical velocity equation. Therefore a statistical survey was carried out on his data and on the Exxon group's data.

TABLE 2–EXXON ACTUAL DATA VS. DROPLET MODEL ESTIMATES

Test Rate Mscf/D	Reynolds No. Dimensionless	TIE* Mscf/D	Well Status
726	302,517	874	Well loadup
660	275,016	744	Well loadup
585	243,764	737	Well loadup
468	195,011	618	Well loadup
573	238,764	691	Well loadup
593	247,097	619	Well loadup
617	257,098	619	Well loadup
250	104,173	412	Well loadup
607	252,931	580	Well loadup
600	250,014	575	Well loadup
635	264,598	586	Well loadup
583	242,930	563	Well loadup
649	270,432	628	Well loadup
647	269,599	1,031	Well loadup
612	63,754	821	Well loadup
952	99,172	962	Well loadup
430	44,794	520	Well loadup
396	41,252	494	Well loadup
164	17,084	410	Well loadup
329	34,273	323	Well loadup
267	27,814	356	Well loadup
640	66,670	983	Well loadup
615	64,066	780	Well loadup
1,072	111,673	1,174	Well loadup
276	28,752	395	Well loadup
366	49,565	348	Well loadup
324	43,272	311	Well loadup
90	4,833	484	Well loadup
220	14,897	389	Well loadup
355	39,858	478	Well loadup
338	42,252	341	Well loadup
471	41,270	508	Well loadup
372	14,353	553	Well loadup
518	44,075	590	Well loadup
330	35,699	562	Well loadup
511	69,462	460	Well loadup
558	87,192	461	Well loadup
493	67,278	491	Well loadup
627	46,399	676	Well loadup
518	36,874	542	Well loadup
358	71,414	349	Well loadup
885	29,268	924	Well loadup
712	82,262	638	Well loadup
408	68,004	438	Well loadup
666	8,581	924	Well loadup
648	137,185	630	Well loadup
564	66,857	608	Well loadup
781	25,966	782	Well loadup
755	132,832	764	Well loadup
620	60,673	610	Well loadup
430	132,554	335	Well loadup
397	164,725	372	Well loadup

*TIE: Turner initial equation.

Table 1 shows Reynolds number calculations for Turner's data. It is quite clear that most of the data points fall in the highly turbulent region where (N_{Re}) exceeds a value of 200,000. In this region, where (N_{Re}) extends from 2×10^5 to 10^6 the drag coefficient acquires a value of 0.2 as shown in Fig. 3.

On the other hand, when calculating the Reynolds number for the Exxon data, **Table 2**, it appears that most of the data fall in the range of Turner's assumption for $10^4<N_{\mathrm{Re}}<2\times10^5$ with its corresponding drag coefficient of 0.44, and that is why Turner's initial equation gave a good match with this set of data without the need for empirical adjustment. **Fig. 4** illustrates this good match.

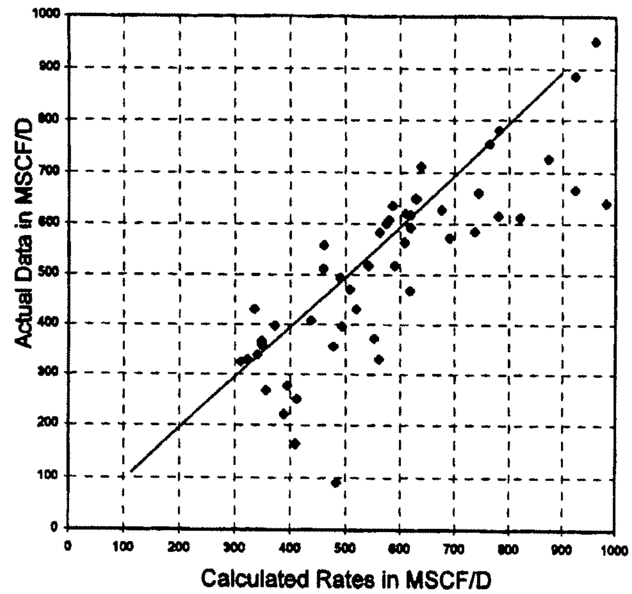

Fig. 4–Exxon research group data (March 1991).

New Analytical Models (Authors' View)

The wide range of pressures, temperatures, and flow rates encountered in gas wells that result in different flow regimes is not necessarily confined to the range assumed by Turner, i.e., $10^4<N_{\mathrm{Re}}<2\times10^5$. By applying the same theory of a hard, smooth, spherical droplet of liquid, two analytical droplet models were developed in this work: one for the transition, and the other for the highly turbulent flow regime. Starting from Allen's equation for the transition and Newton's law for the highly turbulent, and applying the Hinze equation to determine the largest drop diameter, two equations similar to the initial droplet model were derived to cover different flow conditions (derived in the Appendix).

Transition Flow Regime

In low flow rate systems, a transition flow regime might occur. In such a case Allen's equation for the critical gas velocity will be our starting point and will lead to the following equation:

$$v_g = 14.6\sigma^{0.35}(\rho_p-\rho)^{0.21}/\mu^{0.134}\rho^{0.426}. \quad (1)$$

Highly Turbulent Flow Regime

In high flow rate systems, by substituting with the corresponding value of the drag coefficient in this regime, i.e., (0.2), we arrive at the following equation:

$$v_g = 21.3\sigma^{0.25}(\rho_p-\rho)^{0.25}/\rho^{0.5}. \quad (2)$$

Results

The above equation is very similar to Turner's equation after adjustment. It is 5% higher in the equation coefficient which even improves the degree of matching.

This clearly reveals the ambiguity associated with Turner's equation regarding its match to different ranges of actual data with and without adjustment. It explains why the equation needed a 20% adjustment upward to match Turner's actual data records, while the same equation matched the Exxon's group actual data without any adjustment. This was basically because of ignoring flow regime considerations, which directly affect the shape of the drag coefficient and hence the critical velocity equation. Applying the equation of a certain range of turbulent flow under all conditions was therefore misleading, and led to the previous discrepancies of the model when matched with different sets of actual data.

TABLE 3–TURNER DATA VS. ESTIMATES OF THE NEW APPROACH

Series No.	Test Rate Mscf/D	TIE* Mscf/D	TAE** Mscf/D	Reynolds No. Dimensionless	New Eq. Mscf/D	Well Status
1	568	264	306	185,288	264	Near load up
2	442	361	419	144,123	361	Near load up
3	417	503	583	136,046	503	Near load up
4	711	570	661	232,256	690	Near load up
5	774	672	779	253,004	813	Near load up
6	1,117	808	936	363,993	977	Questionable
7	1,607	827	958	537,806	1,000	Near load up
8	2,065	941	1,091	629,920	1,139	Near load up
9	1,312	948	1,099	424,009	1,147	Loaded
10	1,791	956	1,108	584,465	1,157	Loaded
11	1,247	990	1,148	403,886	1,199	Loaded
12	1,524	997	1,156	1,132,168	1,207	Loaded
13	2,489	999	1,158	802,032	1,209	Questionable
14	1,356	1,033	1,197	436,834	1,250	Loaded
15	1,365	1,224	1,419	429,465	1,482	Loaded
16	2,911	1,380	1,600	949,716	1,671	Questionable
17	2,547	1,384	1,604	830,715	1,675	Loaded
18	2,500	1,389	1,610	816,257	1,681	Questionable
19	2,259	1,400	1,623	737,355	1,695	Loaded
20	1,811	1,411	1,635	589,461	1,707	Loaded
21	3,177	1,427	1,654	1,032,814	1,727	Questionable
22	2,541	1,432	1,660	829,065	1,733	Loaded
23	800	1,489	1,726	265,714	1,802	Loaded
24	4,887	1,675	1,941	1,591,363	2,027	Questionable
25	3,467	1,688	1,956	1,132,168	2,042	Loaded
26	499	1,797	2,083	158,720	1,797	Loaded
27	400	1,884	2,184	124,000	1,884	Loaded
28	4,093	2,066	2,395	1,336,366	2,501	Questionable
29	3,258	2,071	2,401	1,050,634	2,507	Questionable
30	2,606	2,081	2,412	851,986	2,518	Loaded
31	3,548	2,789	3,233	1,153,200	3,376	Questionable
32	470	2,802	3,248	156,240	2,802	Loaded
33	3,006	2,831	3,281	985,800	3,426	Loaded
34	4,989	2,944	3,412	1,635,992	3,563	Questionable
35	4,121	3,117	3,613	1,338,385	3,772	Loaded
36	6,397	3,241	3,757	2,080,543	3,923	Questionable
37	5,495	3,250	3,767	1,778,018	3,933	Loaded
38	4,842	4,135	4,793	1,573,029	5,004	Loaded
39	4,436	4,245	4,920	1,433,529	5,137	Loaded
40	3,855	4,353	5,045	1,250,186	5,268	Loaded
41	5,728	4,394	5,093	1,841,400	5,318	Loaded
42	8,395	4,796	5,559	2,735,582	5,804	Questionable
43	7,096	4,814	5,580	2,306,187	5,826	Loaded
44	6,353	4,867	5,641	2,074,867	5,890	Loaded
45	8,054	4,893	5,671	2,625,774	5,921	Questionable
46	9,036	5,092	5,902	2,952,405	6,162	Questionable
47	3,890	5,110	5,923	1,270,823	6,184	Loaded
48	8,185	5,117	5,931	2,670,712	6,193	Loaded
49	8,204	5,189	6,015	2,647,843	6,280	Questionable
50	6,699	5,247	6,082	2,168,229	6,350	Loaded
51	2,780	5,337	6,186	894,394	6,459	Loaded
52	4,295	5,486	6,359	1,402,971	6,640	Loaded
53	1,639	5,493	6,367	540,144	6,648	Loaded

*TIE: Turner initial equation.
**TAE: Turner adjusted equation.

Turner's data fell mainly in the highly turbulent flow region. Therefore, it obeys Eq. 2 which is very close to the equation Turner arrived at with the empirical adjustment he made. The Exxon data fell mainly in the range of turbulent flow originally assumed by Turner, and therefore matched his initial equation without any need for adjustment.

Upon comparing Turner data with the estimates of his initial equation (Table 1), it appears that the calculated estimates were far too low from the actual figures as Turner originally stated. The absolute cumulative error in this case was 23.5% calculated as follows:

$$\mathrm{Err}_{cum}\% = (\mathrm{Sum}_{act} - \mathrm{Sum}_{calc}/\mathrm{Sum}_{act}) \times 100.$$

After making the 20% upward adjustment, the quality of the

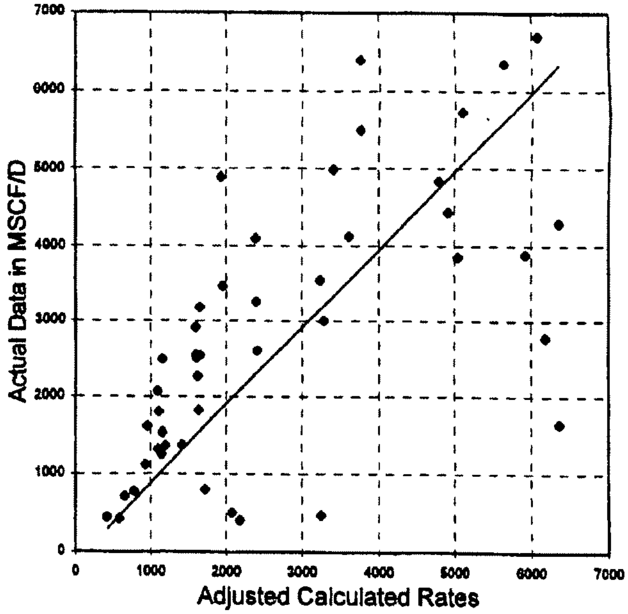

Fig. 5–Turner data (November 1969).

match improved as can be seen from **Fig. 5**, and the absolute cumulative error was significantly decreased to 11.4%.

However, after applying the new approach and using the appropriate equation for each flow regime, as shown in **Table 3**, the absolute cumulative error was further improved and came down to 8.3%. **Fig. 6** illustrates Turner data vs. calculated rates using the new approach. It shows further improvement in the match quality and on an analytical basis which explains previous discrepancies.

Conclusions

The wide variation in the prevailing flow conditions in gas wells makes it very difficult to assume a constant flow regime for all wells.

The new approach serves to provide a better physical explanation and understanding of the loading phenomenon concerning flow regime changes, and hence converts the droplet model empirical equation into a generalized analytical approach.

The main idea behind the droplet model is very valid, however, the previous discrepancies of the model with actual data were because flow regime considerations were ignored.

Upon calculating the critical flow rate for a gas well, care should be given to the prevailing flow conditions so as to apply the appropriate equation for each case.

There is the possibility of having more than one flow regime in a well, depending on where the calculations are being made (at the well head or at the sand face). It is therefore recommended that calculations are carried out at the wellhead pressure, because it is the point at which the gas slippage, and hence the gas velocity, is at its maximum value. Using the maximum gas velocity will insure a maximum critical flow rate to unload the well. It is also recommended to consider water as the loading phase. Making calculations for removing water drops (denser phase) will certainly guarantee removing the lighter phase (oil and/or condensate).

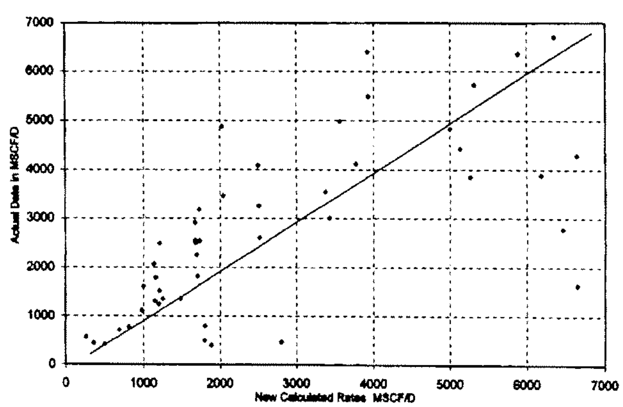

Fig. 6–Turner data new approach (March 1997).

Nomenclature

A = flow area of conduit, ft^2
C_d = drag coefficient (dimensionless)
d_p = diameter of liquid drop, ft
d_m = maximum diameter of liquid drop, ft
g_c = gravitional constant 32.17 lbm-ft/lbf-sec^2
g = local accerlartaion of gravity, ft/sec^2
N_{Re} = Reynolds' number (dimensionless)
N_{We} = Weber's number (dimensionless)
F_g = gravity force, lbf/ft^2
F_d = drag force, lbf/ft^2
v_g = gas velocity, ft/second
p_p = density of liquid drop, lbm/ft^3
p = density of gas, lbm/ft^3
σ = interfacial tension, dynes/cm
μ = gas viscosity, lbm/ft/sec
μ_p = liquid viscosity, lbm/ft/sec
N_{NS} = Nosseir numerical constant
$\mathrm{Err}_{cum}\%$ = cumulative error percent
Sum_{act} = summation of actual rates
Sum_{calc} = summation of calculated rates

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Appendix

Following is a derivation of the critical velocity equation under the transition flow regime.

Starting from Allen's Equation

$$v_g = 0.2[g(p_p - p)/p]^{0.72} \times d_p^{1.18}/(\mu/p)^{0.45}.$$

Hinze Equation

$$d_m = 30 g_c / v_g^2 p.$$

Substituting for d_p by the d_m term,

$$\nu_g = 0.2(30\partial_c/\nu_g^2 p)^{1.18}/(\mu/p)^{0.45}\times[g(p_p - p)/p]^{0.72}$$

$$= N_{\mathrm{NS}}[\{g(p_p - p)/p\}^{0.72}\times\partial^{1.18}/p^{1.18}\nu_g^{2.36}/(\mu/p)^{0.45},$$

where

$$N_{\mathrm{NS}} = 0.2(32.17)^{0.72}(30)^{1.18}(32.17)^{1.18} = 8094.5011,$$

$$\nu_g^{3.36} = N_{\mathrm{NS}}[\{(p_p - p)/p\}^{0.72}\times\partial^{1.18}/p^{1.18}/(\mu/p)^{0.45}$$

$$= N_{\mathrm{NS}}^{1/3.36}[\{(p_p - p)/p\}^{0.21}\times\partial^{0.35}/p^{0.35}(\mu/p)^{0.134},$$

therefore,

$$\nu_g = 14.6\partial^{0.35}(p_p - p)^{0.21}/\mu^{0.134}p^{0.426}.$$

Highly Turbulent Model

The following is a derivation of the highly turbulent flow regime equation.

Starting from the gravitional and drag force equations, i.e., Eqs. 1 and 2, and substituting for the maximum diameter by Hinze equation we get

$$\nu_g = 1.154[g(p_p - p)/pC_d]^{0.5}(30\partial g_c/\nu_g^2 p)^{0.5}.$$

Substituting the value of C_d by 0.2, and reducing the equation we obtain

$$\nu_g = 454.67(p_p - p)^{0.5}\partial^{0.5}/p\nu_g.$$

Solving for ν_g we obtain

$$\nu_g = 21.3\partial^{0.25}(p_p - p)^{0.25}/p^{0.5}.$$

Mohamed Nossier is a senior reservoir engineer working for Suez Oil Co. He holds BS and MS degrees in petroleum engineering from Cairo U. **Tarek Darwich** is Petroleum Engineering Technical Manager of North Africa and Middle East Business Development at Sipetrol, U.K. He was an associate professor of petroleum engineering at Cairo U. Darwich holds BS and MS degrees from Cairo U. and a PhD degree from Imperial Collete, U. of London. **Mohamed H. Sayyouh** is a professor of petroleum reservoir engineering at Cairo U. He holds BS and MS degrees from Cairo U. and a PhD degree in petroleum and gas engineering from Pennsylvania State U. **M. El Sallaly** is a professor of refining, Cairo U. He holds BS, MS, and PhD degrees from Cairo U.