MODELLING OF MISCIBLE DISPLACEMENT BY SOLVING CONVECTIVE-DIFFUSION EQUATION

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ABSTRACT

This paper presents a one-dimensional model to predict the behavior of miscible-type processes using the convective-diffusion equation. This equation was solved by analytical solution for semi-infinite porous medium and by implicit and explicit numerical techniques. The numerical solution was based on a finite linear porous medium. Solutions by numerical techniques are compared with the analytical solution. It is noticed that the implicit method was more stable than the explicit technique. Also, this work shows a simplified approach for modeling the problem of miscible-type displacement process.

INTRODUCTION

The problem of finding a suitable numerical solutions by finite difference approximation for the convective-diffusion equation which describes the process by which one miscible fluid displaces another in porous medium has been of interest of some time. Peaceman and Rachford (1) applied standard finite methods developed for transient flow problems. Price et al (2) presented a se-
cond order correct spatial discretization that was non-
oscillatory for any mesh spacing. Garder et al.\(^{(3)}\) recog-
nized that problems described by convective-diffusion
equation behaved more like hyperbolic than parabolic
differential equations. Stone et al.\(^{(4)}\) and Perrine
et al.\(^{(5)}\) have proposed other solution techniques for con-
vection-diffusion-type equations; however, each of these
has a serious drawback. Numerical solutions of higher
order accuracy for convective-diffusion equations have
been presented by Price et al.\(^{(6)}\) and Launbach\(^{(7)}\).

This investigation presents a simplified solution
to predict the behavior of miscible-type process des-
cribed by the convective-diffusion type-equation. Nu-
merical techniques as well as analytical method to solv-
ing the equation were used. The numerical solutions are
based on the finite difference approximations.

FORMULATION OF THE MODEL

Basically, this investigation considers a single
phase, one-dimensional oil reservoir, where oil is dis-
placed by a surfactant solution completely miscible with
oil.

The convective-diffusion equation arises from the
conservation of mass equations in miscible type process.
The equation describing the process by which one miscible
fluid displaces another fluid in a linear homogeneous
porous medium can be written as:

\[
D \frac{\delta^2 c(x,t)}{\delta x^2} - \frac{\delta c(x,t)}{\delta x} = \frac{\delta c(x,t)}{\delta t} \quad \quad (1)
\]

\[0 < x < L \quad \text{and} \quad t > 0\]
Where:
\[ c = \text{concentration of liquid} \]
\[ x = \text{length of porous medium} \]
\[ t = \text{time} \]
\[ D = \text{diffusivity coefficient} \]
\[ V = \text{velocity of flow}. \]

With the following boundary and initial conditions:
\[ c(x, t) = 0 \quad \text{for} \quad x > 0 \quad \text{and} \quad t = 0 \quad \ldots \ldots \ldots \ldots (2) \]
\[ c(x, t) = c_0 \quad \text{for} \quad x = 0 \quad \text{and} \quad t > 0 \quad \ldots \ldots \ldots \ldots (3) \]
\[ \lim \limits_{x \to \infty} c(x, t) = 0 \quad \text{for} \quad t > 0 \quad \ldots \ldots \ldots \ldots (4) \]

**ANALYTICAL SOLUTION**

Kirkham and Powers (8) presented the analytical solution of equation 1 for semi-infinite porous medium as follows:

\[ c(x, t) = c_0 \left( \text{erfc} \frac{x - vt}{\sqrt{4Dt}} + e^{-\frac{vx}{2\sqrt{Dt}}} \text{erfc} \frac{x + vt}{2\sqrt{Dt}} \right) \]

\[ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5) \]

**NUMERICAL SOLUTION**

The convective-diffusion equation 1 is solved by two finite difference methods:

**Explicit Method:**

The finite difference approximation of equation 1 can be written as:

\[ D \frac{C_i^{n+1} - 2C_i^n + C_i^{n-1}}{\Delta x^2} - \nu \frac{C_{i+1}^{n+1} - C_{i-1}^{n+1}}{2 \Delta x} = \frac{C_{i+1}^{n+1} - C_{i}^{n+1}}{\Delta t} \quad \ldots \ldots \ldots \ldots (6) \]

Multiply through by \( \Delta t \) and define:

\[ \gamma = \frac{D \Delta t}{(\Delta x)^2} \quad \text{and} \quad \alpha = \frac{\nu \Delta t}{2(\Delta x)} \quad \ldots \ldots \ldots \ldots \ldots (7) \]
\[ c^{n+1}_1 = (\gamma - \alpha) c^n_{1+1} + (1 - 2\gamma) c^n_1 + (\gamma + \alpha) c^n_{1-1} \quad \ldots \quad (8) \]

This equation is an explicit formula for the dependent variable at time level \((n+1)\) expressed in terms of known values of the right hand side of the equation.

**Implicit Method of Solution:**

We can set up a finite difference scheme to solve for all concentrations, \(c_i\), values in equation 1 as follows:

\[
\begin{align*}
\frac{c^{n+1}_{i+1} - 2c^{n+1}_i + c^{n+1}_{i-1}}{(\Delta x)^2} &- \frac{c^{n+1}_{i+1} - c^{n+1}_i}{\Delta t} = \frac{c^n_i}{2(\Delta x)} \\
&= \frac{c^n_i}{\Delta t} \quad \ldots \quad (9)
\end{align*}
\]

by using the definitions shown in equations 7, then equation 9 becomes:

\[
(\gamma - \alpha) c^{n+1}_{i+1} - (2 \gamma + 1) c^{n+1}_i + (\gamma + \alpha) c^{n+1}_{i-1} = c^n_i \quad \ldots \quad (10)
\]

This equation has all unknown concentrations at all new time level \((n+1)\), and will be with general type:

\[
a_i c_{i+1} - b_i c_i + c_i c_{i+1} = d_i \quad \ldots \quad (11)
\]

Where the coefficients \(a_i\), \(b_i\) and \(c_i\) are related to the geometry of the system and its physical properties and \(d_i\) contains known terms. This set of simultaneous equations can be written in matrix notation:

\[
\begin{pmatrix}
a & c \\ A & C & = & d
\end{pmatrix} \quad \ldots \quad (12)
\]

This matrix is called tridiagonal matrix and this system can be solved for the liquid concentration by using Thomas algorithm, which is a modified form of Gaussian elimination.
COMPUTER PROGRAMMING AND DISCUSSION

Consider the block-centered grid system shown in Fig. 1 for one-dimension in the positive x-direction. The convective-diffusion equation 1 with the boundary and initial conditions represented by equations 2, 3 and 4 was solved for the system shown in Fig. 1. When a surfactant solution is displacing oil from a porous medium, Fig. 2 shows the variation of concentration of surfactant solution with length of the porous medium. Analytical and numerical solutions are compared. Fig. 2 indicates that the solution by the implicit scheme is better than the solution by the explicit scheme. However, a significant improvement in the numerical treatment of the convective-diffusion equation was achieved by using the explicit solution. The differences appear in Fig. 2 between the analytical and the numerical solution may be due to either the grid size that used or the number of blocks. It was found previously that grid size and type have an effect on the numerical solutions\(^{(8,9)}\).

In addition, one main difference between the numerical and analytical solution is that the numerical one was solved for a finite distance while the analytical solution obtained for semi-infinite porous medium. The difficulty in representing the convective-diffusion equation numerically, arises due to the hyperbolic character assumed as the Peclet number becomes high. Sometimes the numerical method possesses some oscillatory behavior in the vicinity of large gradients in the dependent
variable when convection is strongly predominant. This is also, particularly true for large time steps. Hence, the numerical solution for this simplified problem exhibits the most important numerical difficulties associated with the more general problem: oscillation and numerical dispersion.

CONCLUSIONS

Based on this study the following conclusions can be derived:

1. A simplified approach for modeling of the miscible displacement type process have been developed.

2. Convective-diffusion equation have been solved by different analytical and numerical methods, and used to predict the behavior of miscible-type processes.

3. Implicit numerical technique showed a powerful tool in solving the convective-diffusion equation.

REFERENCES


6. Price, H.S.; Cavendish, J.C. and Varago, R.S.: 

![Fig. 1 Linear 1-D Model](image1)

![Fig. 2 Concentration Vs Distance](image2)


