Question 1

A Random Process defined as

$$x(t) = A\cos(\omega_c t + \theta)$$

where A and ω_c are constant, while θ is a random variable $\theta \sim \mathcal{U}[-\pi, \pi]$. Find:

1) The ensemble average of $x(t)$
2) The ACF of $x(t)$
3) The time average of $x(t)$
4) The time ACF of $x(t)$
5) The PSD of $x(t)$

Question 2

A Random Process defined as

$$x(t) = A\cos(\omega_c t + \theta)$$

where θ and ω_c are constant, while A is a random variable $A \sim \mathcal{U}[-1, 1]$. Find:

1) The ensemble average of $x(t)$
2) The ACF of $x(t)$
3) The time average of $x(t)$
4) The time ACF of $x(t)$

Question 3

A Random Process defined as

$$x(t) = A\cos(\omega_c t + \theta)$$

where θ and A are constant, while ω_c is a random variable $\omega_c \sim \mathcal{U}[0, 1000]$. Find:

1) The ensemble average of $x(t)$
2) The ACF of $x(t)$
3) The time average of $x(t)$
4) The time ACF of $x(t)$

Question 4

A Random Process defined as

$$x(t) = At + B$$

where A is constant, while $B \sim \mathcal{U}[-3, 4]$. Find:

1) The ensemble average of $x(t)$
2) The ACF of $x(t)$

Question 5

A Random Process $x(t)$ is defined in relation to another Random Process $m(t)$ as

$$x(t) = m(t)\cos(\omega_c t + \theta)$$

where ω_c is constant, θ is a random variable, independent of $m(t)$ and has a uniform distribution $\mathcal{U}[0, 2\pi]$. Find:

1) The ACF of $x(t)$ in terms of the ACF of $m(t)$
2) The PSD of $x(t)$ in terms of the ACF of $m(t)$

Question 6

The power spectral density of a random process $x(t)$ is defined as

$$S_x(f) = \begin{cases} A, & |f| \le f_0 \\ 0, & \text{otherwise} \end{cases}$$

1) Determine and sketch the autocorrelation function of $x(t)$
2) If the Random process passes by the channel $H(f)$ defined as

$$H(f) = \begin{cases} \beta, & |f| \le f_h, \quad f_h < f_0 \\ 0, & \text{otherwise} \end{cases}$$

   What is the PSD and the ACF of the output Random process?

Question 7

A Random Process defined as

$$x(t) = A\cos(\omega_c t)$$

where ω_c is constant, while A is a Gaussian random variable with zero mean and variance σ^2. Find:

1) The ensemble average of $x(t)$
2) The ACF of $x(t)$
3) The time average of $x(t)$
4) The time ACF of $x(t)$

This process is then passed through an ideal integrator whose output is

$$y(t) = \int_0^t x(\tau)d\tau$$

1) Find the PDF of $y(t)$ at any particular time t_1.
2) Is $y(t)$ stationary?
3) Is $y(t)$ ergodic?