Note That: $\text{sinc}(x) = \dfrac{\sin(\pi x)}{\pi x}$

Question 00

Find and plot the Fourier transform representation, $F(\omega)$, of the rectangular signal $f(t)$ shown in Fig. 1.

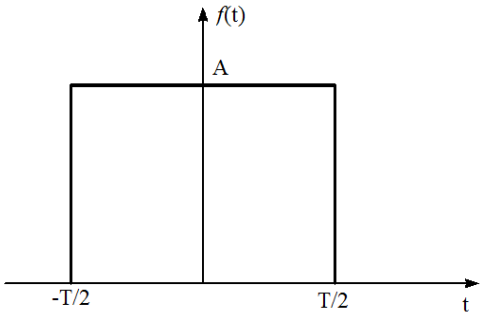
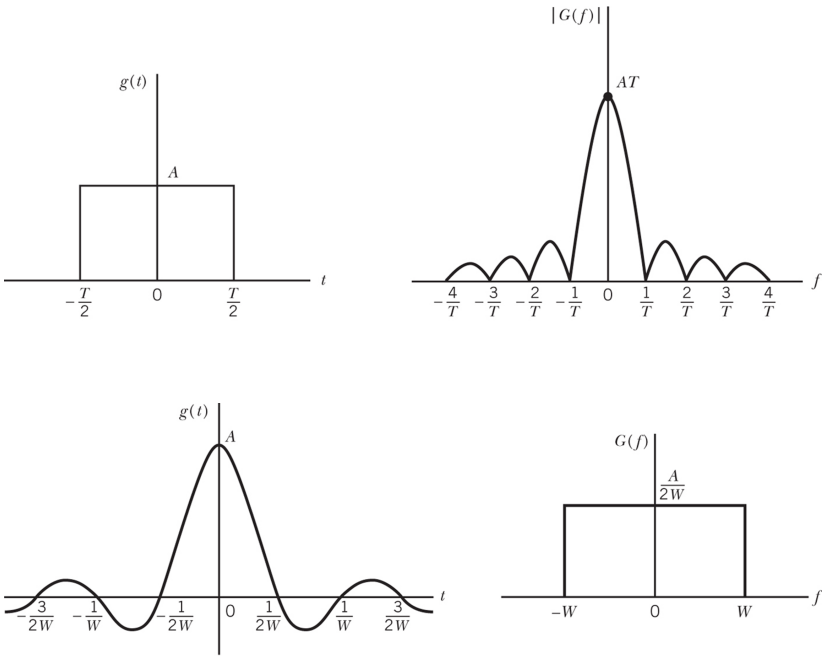


Fig. 1

Hence, find the Fourier transform of $g(t) = f(t) * f(t)$. Plot $g(t)$ and $G(\omega)$.

Question 0

Identify the Fourier transform pairs shown in the figures below.



Question 1

Two signal, $g_1(t)$ and $g_2(t)$, whose Fourier transform is shown in Fig. 2.
Determine the Nyquist interval ans the Nyquist rate for the following signals:

1) $g_1(t)$
2) $g_2(t)$
3) $g_1(t) * g_2(t)$
4) $g_1(t) \times g_2(t)$
5) $[g_1(t)]^2$
6) $[g_2(t)]^m$

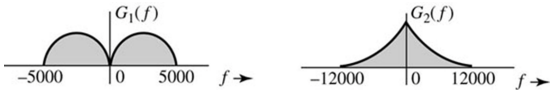


Fig. 2

Question 2

Determine the Nyquist rate for the following signals:

1) $\text{sinc}(100t)$
2) $\text{sinc}^2(100t)$
3) $\text{sinc}(100t) + \text{sinc}(200t)$
4) $\text{sinc}(100t) \times \text{sinc}(500t)$

Question 3

A signal $m(t)$ is defined as

$$m(t) = \text{sinc}^2(5t)$$

The signal $m(t)$ is sampled using uniformly spaced impulses at a rate of (a) 5 Hz, (b) 10 Hz, (c) 20 Hz. For each of the three cases:

1) Sketch the sampled signal
2) Sketch the spectrum of the sampled signal
3) Can you recover the signal $m(t)$ from the sampled signal
4) If the sampled signal is passed through an ideal low-pass filter of bandwidth 5 Hz, sketch the spectrum of the output signal, and write its expression in time-domain.

Question 4

Given a signal $m(t)$ defined as

$$m(t) = 4\cos(200\pi t) + 2\cos(400\pi t)$$

- Sketch the magnitude of the Fourier transform of $m(t)$, i.e. $|M(j\omega)|$
- What is the Nyquist rate for this signal?
- Assuming the the signal $m(t)$ is sampled at a frequency of 300 Hz, sketch the magnitude of the sampled signal in frequency domain.
- If the sampled signal is passed through a LPF with cutoff frequency 250 Hz, write an expression of the signal $y(t)$ at the output of the LPF
- Can the original signal $x(t)$ be recovered from $y(t)$? Why?

Question 5

Assuming a baseband signal defined as

$$m(t) = \frac{\sin^2(30\pi t)}{\pi t^2}$$

1) What is the minimum sampling rate for this signal to avoid aliasing?
2) Draw the frequency domain representation of the signal if it is sampled with 75% of the Nyquist rate
3) If the LPF available at the receiver is not ideal, i.e. it does not have sharp edges. Instead, the available LPF has a trapezoidal shape, where the flat portion of the filter has a gain of 1, and the filter's slope is $2/W$, where W is the cutoff of the flat portion of the filter in rad/s.
   Note: Part 3) is independent of part 2).
   - Design and draw (with complete labels) the frequency response of the LPF required to reconstruct $m(t)$ from its samples.
   - What is the minimum sampling rate that can be used in this case?