Undergraduate Program
Signals and Systems (ELC 225B)
Computer Engineering Department
Spring 2018
Problem Set no. 3

Question 1 *(Fourier Series)*

Determine the Fourier series coefficients for each of the periodic discrete-time signals given below. Plot the magnitude and phase of each set of coefficients a_k

1) $x[n]$ is periodic with period 6 and $x[n] = (1/2)^n$; for $-2 < n < 3$
2) $x[n] = \sin(2\pi n/3)\cos(\pi n/2)$

Question 2 *(Fourier Series)*

Given the Fourier series coefficients of a signal that is periodic with period $N = 8$, determine the signal $x[n]$

$$a_k = \cos(k\pi/4) + \sin(3k\pi/4)$$

Question 3 *(Fourier Transform)*

Compute the Fourier transform of each of the following signals:

1) $x[n] = \delta[4 - 2n]$
2) $x[n] = a^n \sin(\Omega_0 n)u[n]$, where $|a| < 1$

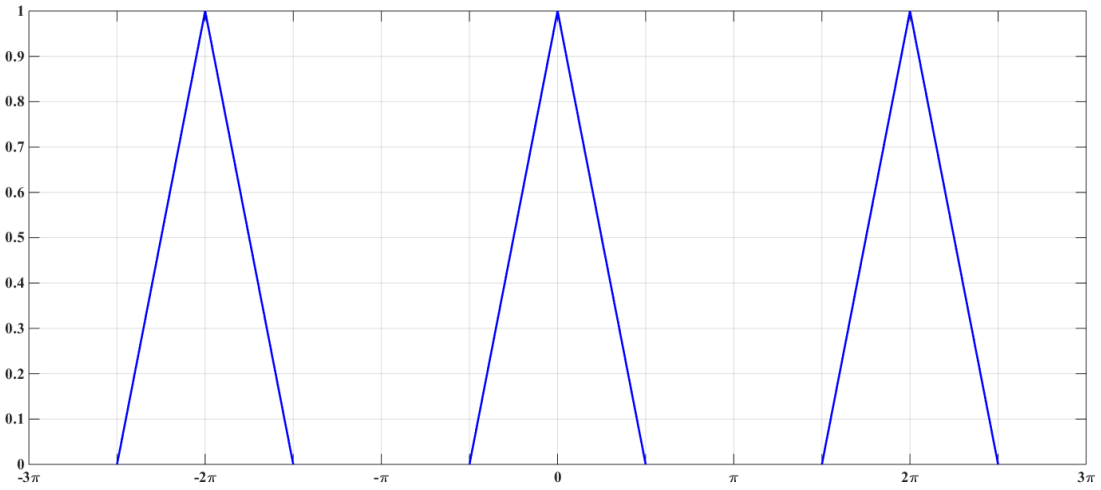
Question 4 *(Fourier Transform)*

The following is Fourier transform of discrete-time signal. Determine the signal corresponding to that transform:

$$X(\Omega) = \cos^2(\Omega)$$

Question 5 *(Properties)*

Let $x[n]$ be a discrete-time sequence with Fourier transform $X(\Omega)$, as shown in the figure below. Sketch the Fourier transform of $y[n] = x[n]\cos(\pi n)$



Question 6 *(Properties)*

Consider a discrete-time signal $h[n]$, with Fourier transform

$$H(\Omega) = \begin{cases} 1 & \pi - \Omega_c \leq |\Omega| \leq \pi \\ 0 & |\Omega| < \pi - \Omega_c \end{cases}$$

1) Determine a function $g[n]$ such that $h[n] = \dfrac{\sin[\Omega_c n]}{\pi n} g[n]$
2) As Ω_c is increased, does $h[n]$ get more concentrated or less concentrated about the origin?

Question 7 *(Fourier Series)*

A discrete-time periodic signal $x[n]$ is real-valued and has a fundamental period $N = 5$. The non-zero Fourier Series coefficients for $x[n]$ are

$$a_0 = 1, a_2 = a^*{}_2 = e^{j\pi/4}, a_4 = a^*_{-4} = 2e^{j\pi/3}$$

Express $x[n]$ in the form

$$x[n] = A_0 + \sum_{k=1}^{\infty} A_k \sin(\Omega_k n + \phi_k)$$

Question 8 *(Properties)*

Let $x[n]$ be a real and odd periodic signal with period $N = 7$ and Fourier series coefficients a_k. Given that

$$a_{15} = j, a_{16} = 2j, a_{17} = 3j$$

Determine the values of a_0, a_{-1}, a_{-2} and a_{-3}

Question 9 *(Properties)*

For two periodic sequences $x[n]$ and $y[n]$, with period $N = 4$ and Fourier series coefficients a_k and b_k such that

$$a_0 = a_3 = 0.5a_1 = 0.5a_2 = 1 \text{ and } b_0 = b_1 = b_2 = b_3 = 1$$

Determine the Fourier series coefficients of $z[n] = x[n]y[n]$

Question 10 *(Properties)*

Given the following facts about a particular signal $x[n]$ with Fourier transform $X(\Omega)$, find $x[n]$

1) $x[n] = 0$ for $n > 0$
2) $x[0] > 0$
3) $\Im\{X(\Omega)\} = \sin(\Omega) - \sin(2\Omega)$
4) $\dfrac{1}{2\pi} \displaystyle\int_{-\infty}^{\infty} |X(\Omega)|^2 d\Omega = 3$