Series Resonance

Dr. Mohamed Refky Amin
Electronics and Electrical Communications Engineering Department (EECE)
Cairo University
elc.n112.eng@gmail.com
http://scholar.cu.edu.eg/refky/
OUTLINE

• Course contents, References, Course Plan
• Resonance Circuits
• Series Resonance
References

• **Textbook:**

• **References:**
Important topics to be revised

• Analyzing circuits with D.C. and A.C. input signals.
• Using loop and node analysis.
• Calculating input impedances and admittances of networks.
• Calculations of maximum power transfer to a load.
• Dealing with independent and dependent sources.
Course Plan

• **Instructor:** Dr. Mohamed Refky Amin
• **TA:** Eng. Tarek Khedr
• **Grading:**
  – 20% Labs (4 labs, 5% each, no labs will be repeated under any circumstances).
  – 10% Lab Exam.
  – 20 % Midterm
  – 10% Quizzes
  – 40% Final Exam.
• **Office Hours:**
  - Sunday  12:00 AM-1:00 PM
  - Monday  1:00 PM-2:00 PM
  - Wednesday  1:00 PM-2:00 PM
# Course Contents

<table>
<thead>
<tr>
<th>Week</th>
<th>Topic</th>
<th>Lab</th>
<th>Quiz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Resonance Circuits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Passive Filters</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>Quiz 1</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>Lab 1-HW</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>Lab 2-SW</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Midterm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Fourier and Harmonic Analysis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Application of Laplace transforms to Circuit Analysis</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td>Quiz 2</td>
</tr>
<tr>
<td>11</td>
<td>Two-port Networks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td>Lab 3-HW</td>
<td>Quiz 3</td>
</tr>
</tbody>
</table>

Dr. Mohamed Refky
Resonance Circuits

In physics, resonance is the tendency of a system to oscillate with greater amplitude at some frequencies than at others.

Frequency at which the response amplitude is a maximum is known as the system's resonant frequency, or resonance frequency.
Resonance Circuits

For electric circuit to resonate, the circuit must have L, and C elements.

\[ j\omega L \quad R \quad \frac{1}{j\omega c} = \frac{-j}{\omega c} \]

Resonance is a condition in an RLC circuit in which the capacitive and inductive reactances are equal in magnitude, thereby resulting in a purely resistive impedance.
Resonance Circuits

Series resonance

Parallel resonance

|I| \quad \omega_s

|V| \quad \omega_p

Dr. Mohamed Refky
Resonance Circuits

Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective.

They are used in many applications such as selecting the desired stations in radio and TV receivers.
Resonance Circuits

Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective.

They are used in many applications such as selecting the desired stations in radio and TV receivers.
Resonance Circuits

Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective.

They are used in many applications such as selecting the desired stations in radio and TV receivers.
Resonance Circuits

Resonant circuits (series or parallel) are useful for constructing filters, as their transfer functions can be highly frequency selective.

They are used in many applications such as selecting the desired stations in radio and TV receivers.
Series Resonance

\[ X_C = \frac{1}{\omega C} \]

\[ Z_C = \frac{1}{j\omega C} = -jX_C \]

\[ X_L = \omega L \]

\[ Z_L = j\omega L = jX_L \]

\[ Z_T = R + Z_C + Z_L \]

\[ = R - jX_C + jX_L \]
Series Resonance

\[ Z_T = R + j(X_L - X_C) \]

For resonance, \( Z_T \) must be real

\[ X_L - X_C = 0 \]
\[ \omega_s L - \frac{1}{\omega_s C} = 0 \]
\[ \omega_s L = \frac{1}{\omega_s C} \]
\[ \omega_s^2 = \frac{1}{LC} \]

\[ \omega_s = \frac{1}{\sqrt{LC}} \quad , \quad f_s = \frac{1}{2\pi \sqrt{LC}} \]
Series Resonance

At Resonance

Let \( V_s = V_m \angle 0 \)

\[
I_s = \frac{V_s}{R} = \frac{V_m}{R} \angle 0^\circ
\]

\[
V_C = I_s(-jX_C) = \frac{V_m X_C}{R} \angle -90^\circ
\]

\[
V_L = I_s(jX_L) = \frac{V_m X_L}{R} \angle 90^\circ
\]

\[
V_R = I_s R = \frac{V_m R}{R} \angle 0^\circ = V_m \angle 0
\]

\[V_L + V_C = 0\]
Series Resonance

At Resonance

\[ S = P + jQ \]

\[ P = I_s^2 R \]

\[ Q = I_s^2 X_L - I_s^2 X_C = 0 \]

\[ PF = \cos \theta = \frac{P}{S} = 1 \]

Power Factor

Dr. Mohamed Refky
Series Resonance

General Expression for the Current

\[ I = \frac{V_s}{Z_T} = |I| \angle I \]

\[ \angle I = \angle V_s - \angle Z_T \]

\[ \angle Z_T = \tan^{-1} \left( \frac{\omega L - \frac{1}{\omega C}}{R} \right) \]

\[ \angle I = 0 - \tan^{-1} \left( \frac{\omega L - \frac{1}{\omega C}}{R} \right) \]

\[ Z_T = R + j \left( \omega L - \frac{1}{\omega C} \right) \]

\[ V_s = V_m \angle 0 \]
Series Resonance

General Expression for the Current

\[ |I| = \frac{|V_s|}{|Z(\omega)|} \]

\[ = \frac{V_m}{|R + j(\omega L - \frac{1}{\omega C})|} \]

\[ = \frac{V_m}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \]

\[ Z_T = R + j\left(\omega L - \frac{1}{\omega C}\right) \]

\[ V_s = V_m \angle 0 \]

Dr. Mohamed Refky
Series Resonance

The Bandwidth ($BW$)

Bandwidth ($BW$) is the band of frequencies around resonance in which the current is equal to or greater than $1/\sqrt{2}$ of its value at resonance.

$$BW = \omega_2 - \omega_1$$

$\omega_1$ and $\omega_2$ are called the cut-off frequencies or the 3-dB or the half-power points.
Series Resonance

The Bandwidth \((BW)\)

At \(\omega_s\)

\[
P_{\omega_s} = I_s^2 R
\]

At \(\omega_1\) and \(\omega_2\)

\[
P_{\omega_1} = P_{\omega_2} = \left( \frac{I_s}{\sqrt{2}} \right)^2 R
\]

\[
= \frac{1}{2} \times [I_s^2 R] = \frac{1}{2} \times P_{\omega_s}
\]

\[
P_{\omega_1} = P_{\omega_2} = \frac{1}{2} P_{\omega_s}
\]

Half-power points
Series Resonance

The Bandwidth ($BW$)

What is $dB$?

The decibel ($dB$) is a logarithmic unit used to express the ratio between two values of a physical quantity, in our case power or voltage or current. One of these quantities is often a reference value.

For Power

$$P_{dB} = 10 \log(|P/P_r|)$$

For Voltage and current

$$V_{dB} = 20 \log(|V/V_r|)$$
$$I_{dB} = 20 \log(|I/I_r|)$$
Series Resonance

The Bandwidth \((BW)\)

\[
P_{dB} = 10 \log \left( \frac{P_{\omega_1}}{P_{\omega_s}} \right) \\
= 10 \log \left( \frac{1}{2} \right) \\
= -3 \text{dB}
\]
Series Resonance

The Bandwidth ($BW$)

$$|I| = \frac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$= \frac{I_s}{\sqrt{2}} = \frac{V_m}{\sqrt{2R}}$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2R}$$

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$
Series Resonance

The Bandwidth \((BW)\)

\[
\left( \omega L - \frac{1}{\omega C} \right)^2 = R^2
\]

\[
\omega L - \frac{1}{\omega C} = \pm R
\]

\[
\omega^2 LC - 1 = \pm \omega CR
\]

\[
\omega^2 - \frac{1}{LC} = \pm \omega \frac{R}{L}
\]

\[
\omega^2 + \omega \frac{R}{L} - \frac{1}{LC} = 0
\]
Series Resonance

The Bandwidth \((BW)\)

\[
1 \omega^2 + \frac{R}{L} \omega \left( -\frac{1}{LC} \right) = 0
\]

\[
\omega = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
\omega = \pm \frac{R}{2L} \pm \frac{\sqrt{\left( \frac{R}{L} \right)^2 - 4 \left( -\frac{1}{LC} \right)}}{2}
\]

\[
\omega_{1,2} = \pm \frac{R}{2L} + \sqrt{\left( \frac{R}{2L} \right)^2 + \left( \frac{1}{LC} \right)}
\]

The negative sign before the square root is refused.
Series Resonance

The Bandwidth ($BW$)

$$\omega_{1,2} = \pm \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

**Hint:** $(a + b)(a - b) = a^2 - b^2$

Arithmetic mean (Average)

$$\omega_s = \frac{\omega_1 + \omega_2}{2}$$

Geometric mean

$$\omega_s = \sqrt{\omega_1 \times \omega_2}$$

In general, $\omega_s$ is the **geometric** mean of $\omega_1$ and $\omega_2$ and not the arithmetic mean

Dr. Mohamed Refky
Series Resonance

The Bandwidth ($BW$)

$$\omega_{1,2} = \pm \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)^2}$$

$$BW = \omega_2 - \omega_1$$

$$BW = \omega_2 - \omega_1 = \frac{R}{L}$$ in rad/s
Series Resonance

The Bandwidth ($BW$)

$$\omega_{1,2} = \pm \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)} |I|$$

$$\left(\frac{BW}{2}\right)^2 \ll \omega_s^2$$

For high selective circuit where $BW \ll \omega_s$

$$\omega_{1,2} = \pm \frac{R}{2L} + \sqrt{\left(\frac{1}{LC}\right)}$$

$$= \omega_s \pm \frac{BW}{2}$$

$$BW = \omega_2 - \omega_1 = \frac{R}{L}$$
Series Resonance

The Quality factor \( (Q_s) \)

The ratio of the reactive power of either the coil or the capacitor to the average power of the resistance at resonance.

\[
Q_s = \frac{\text{Reactive Power}}{\text{Active power}} \bigg|_{\text{resonance}}
\]

\[
= \frac{I_s^2 X_L}{I_s^2 R} = \frac{I_s^2 X_C}{I_s^2 R} = \frac{X_L}{R} = \frac{X_C}{R}
\]

\[
Q_s = \frac{X_L}{R} = \frac{X_C}{R}
\]
Series Resonance

The Quality factor \( (Q_s) \)

\[
Q_s = \frac{\omega_s L}{R} = \frac{1}{\omega_s CR}
\]

\[
BW = \frac{R}{L} = \frac{\omega_s}{Q_s}
\]

\[
Q_s = \frac{\omega_s}{BW}
\]

The quality factor of a resonant circuit is a measurement of the selectivity of the circuit. The circuit is considered high selective when \( Q_s \geq 10 \).
Series Resonance

The Quality factor \((Q_s)\)

- Voltages at resonance in terms of quality factor

\[
V_L = jI_sX_L = j \frac{V_s}{R} X_L = Q_s V_s \angle 90^\circ
\]

\[
V_C = -jI_sX_C = -j \frac{V_s}{R} X_C = Q_s V_s \angle -90^\circ
\]

- Impedance in terms of quality factor

\[
Z = R + j \left( \omega L - \frac{1}{\omega C} \right) = R \left[ 1 + j \frac{1}{R} \left( \omega L - \frac{1}{\omega C} \right) \right]
\]

\[
= R \left[ 1 + j \frac{\omega L}{R} \left( 1 - \frac{1}{\omega^2 LC} \right) \right]
\]
Series Resonance

The Quality factor ($Q_s$)

\[ Z = R \left[ 1 + j \frac{\omega L}{R} \frac{\omega_s}{\omega_s} \left( 1 - \frac{\omega_s^2}{\omega^2} \right) \right] \]

\[ = R \left[ 1 + j \frac{\omega_s L}{R} \left( \frac{\omega}{\omega_s} - \frac{\omega_s}{\omega} \right) \right] \]

\[ = R[1 + jQ_s \Omega] \]

$\Omega$ is called detuning factor
Series Resonance

The Quality factor \( (Q_s) \)

- Current in terms of quality factor

\[
I = \frac{V_s}{R[1 + jQ_s\Omega]} = \frac{I_s}{[1 + jQ_s\Omega]}
\]

\[
= \frac{I_s}{\sqrt{1 + (Q_s\Omega)^2}} \angle - \tan^{-1}(Q_s\Omega)
\]

Dr. Mohamed Refky
Series Resonance

The Quality factor \( Q_s \)

- Fractional detuning factor

\[
\delta = \frac{\omega - \omega_s}{\omega_s} = \frac{\omega}{\omega_s} - 1
\]

\[
\frac{\omega}{\omega_s} = \delta + 1,
\]

\[
\Omega = \frac{\omega}{\omega_s} - \frac{\omega_s}{\omega} = (\delta + 1) - \left( \frac{1}{\delta + 1} \right)
\]

\[
= \frac{\delta(\delta + 2)}{\delta + 1}
\]
Series Resonance

The Quality factor \((Q_s)\)

For **Narrowband** (near resonance) operation \((\delta \leq 5\%)\)

\[
\Omega = \frac{\delta(\delta + 2)}{\delta + 1} = 2\delta
\]

\[
Z = R[1 + j\Omega Q_s] \rightarrow Z = R[1 + j2\delta Q_s]
\]
## Series Resonance

<table>
<thead>
<tr>
<th></th>
<th>$Q_s &lt; 10$</th>
<th>$Q_s \geq 10$</th>
<th>$\delta \leq 5%$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Resonance at</strong></td>
<td>$X_L = X_C$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>$\frac{1}{\sqrt{LC}}$</td>
<td>$\omega_s \pm \frac{BW}{2}$</td>
<td></td>
</tr>
<tr>
<td>$\omega_{1,2}$</td>
<td>$\omega_s \left[ \pm \frac{1}{2Q_s} + \sqrt{\left( \frac{1}{2Q_s} \right)^2 + 1} \right]$</td>
<td>$\omega_s \pm \frac{BW}{2}$</td>
<td></td>
</tr>
<tr>
<td><strong>$BW$</strong></td>
<td>$\frac{R}{L} = \frac{\omega_s}{Q_s}$ in rad/s</td>
<td>$\frac{f_s}{Q_s}$ in Hz</td>
<td></td>
</tr>
<tr>
<td>$I$</td>
<td>$I_s = \frac{I_s}{\sqrt{1 + (Q_s\Omega)^2}} \angle -\tan^{-1}(Q_s\Omega)$</td>
<td></td>
<td>Same expr. but $\Omega = 2\delta$</td>
</tr>
<tr>
<td>$Z$</td>
<td>$Z = R[1 + jQ_s\Omega]$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Omega$</td>
<td>$\frac{\omega}{\omega_s} - \frac{\omega_s}{\omega}$</td>
<td></td>
<td>$2\delta$</td>
</tr>
</tbody>
</table>

Dr. Mohamed Refky
Series Resonance

Example (1)

For the resonant circuit shown, find $I_S$, $V_R$, $V_L$ and $V_C$. Also, calculate $Q_s$ of the circuit. If the resonance frequency is 5000 Hz, calculate the $BW$ and the power dissipated in the circuit at half-power points.

\[ R = 2\Omega \quad X_L = 10\Omega \]

\[ V_S = 10\angle 0^\circ \]

\[ + V_R - \quad + V_L - \quad + V_C \quad X_C = 10\Omega \]
Series Resonance

Example (2)

The $BW$ of a series resonant circuit is 400 Hz. If the resonance frequency is 4000 Hz, calculate $Q_s$. If $R = 10\Omega$, calculate the value of $X_L$ at resonance, $L$ and $C$. 
Series Resonance

Example (3)

A series RLC circuit has a series resonance frequency of 12000 Hz, \( R = 5\Omega \), \( X_L \) at resonance = 300\( \Omega \). Find the \( BW \) and the cut-off frequencies.
**Series Resonance**

**Example (4)**

The magnitude of the current in a series resonance circuit varies with frequency according to the curve shown in the figure:

a) Determine $Q_s$ and the $BW$.

b) For $C = 0.1 \mu F$, determine $L$, $R$ for the series resonant circuit.

c) If the applied source $V_S = 10 \angle 0^\circ$, calculate the current at the cut off frequencies.
Series Resonance

Example (5)

A series resonance is designed using a 10 mH inductor whose resistance is 75Ω. The center frequency of the circuit is 25 KHz.

a) What value of capacitance should be used?

b) If the BW of the filter must not be greater than 2500 Hz, what is the maximum value of the resistance across which the output can be obtained?

c) When the maximum resistance found in part (b) is used, what is the maximum output voltage of the circuit ($V_o$) if $V_s = 12V$?