Lecture 5 - Frequency Response

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Outline of this Lecture

- Introduction
- High Frequency Response
  - High Frequency Model of the MOSFET
  - Miller's Theorem
- Common Source Configuration
- Common Gate Configuration
- Cascode Amplifier
- Differential Pair

Introduction

Bode Plot

The general rules to draw the bode plot are:

- For the magnitude curve: the slope of the magnitude plot changes by $20dB/decade$ at every zero frequency and by $-20dB/decade$ at every pole frequency.

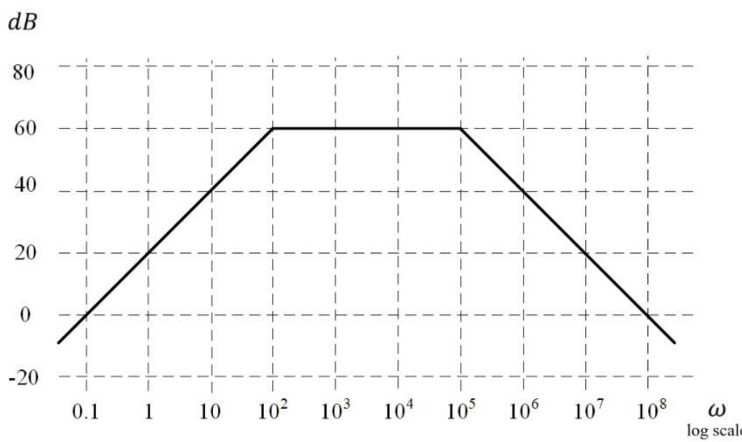
- For the phase curve: the slope of the phase plot changes by $45^o/decade$ starting a decade before every zero frequency and by $-45^o/decade$ starting a decade before every pole frequency. The effect of each zero or pole ends one decade after its frequency.

Introduction

Bode Plot

The shown figure illustrates the magnitude Bode plot of the following transfer function

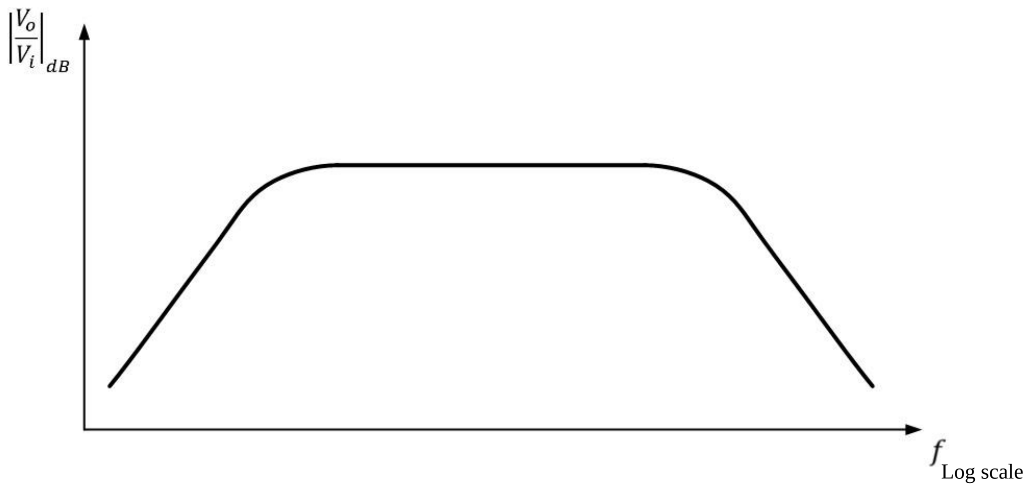
$$A(s) = \frac{10^8 s}{(10^2 + s)(10^5 + s)}$$



Introduction

Transfer Function of an Amplifier

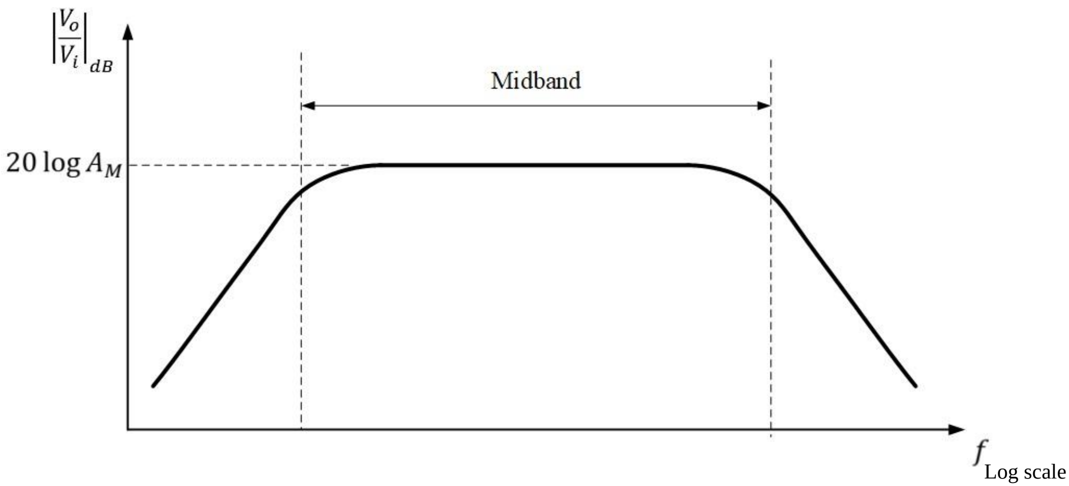
Up to now the gain of the amplifier is assumed to be constant and independent of the frequency of the input signal



Introduction

Midband

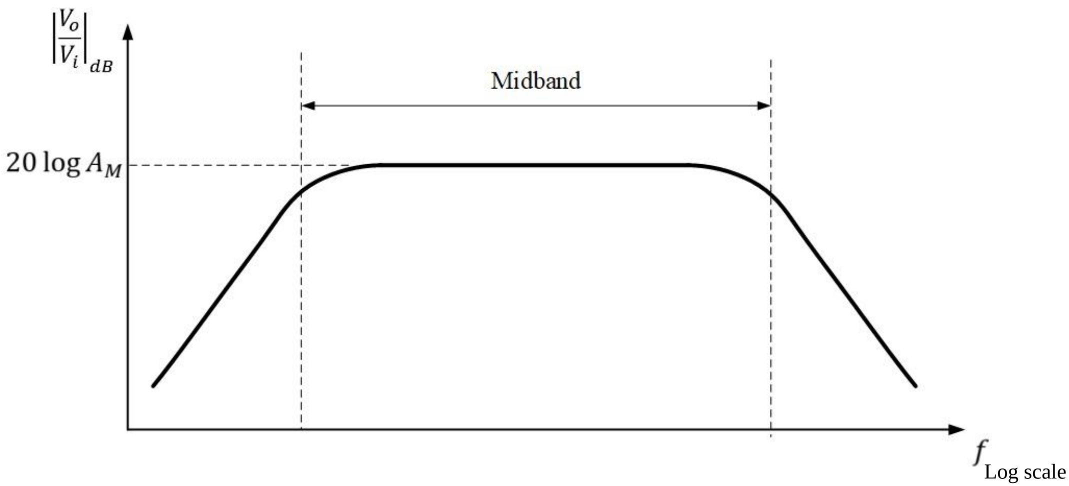
There is a wide frequency range over which the gain remains almost constant (A_M). This range is called the middle-frequency band or **midband**.



Introduction

Midband

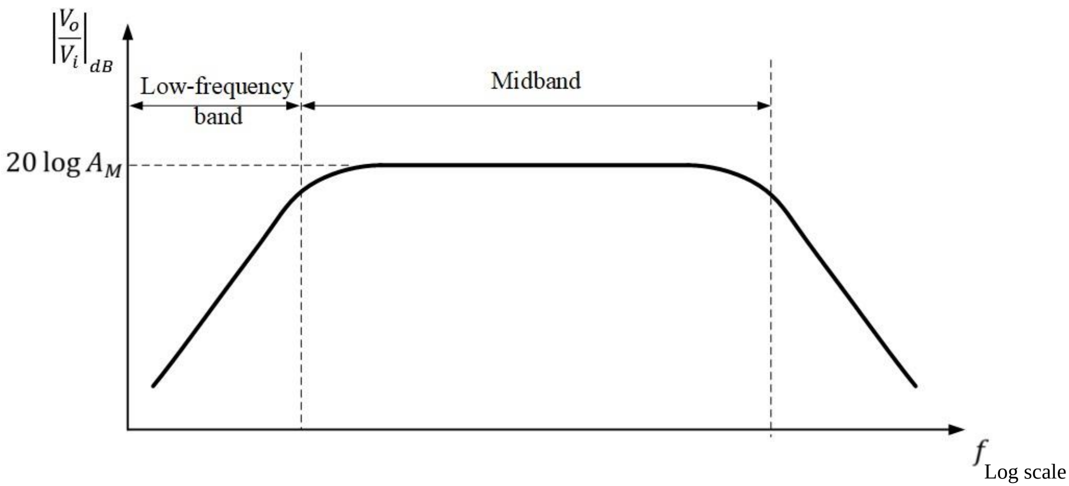
The amplifier is designed so that its midband coincides with the frequency spectrum of the signals it is required to amplify. Otherwise, the amplifier would **distort** the input signal.



Introduction

Low-Frequency Band

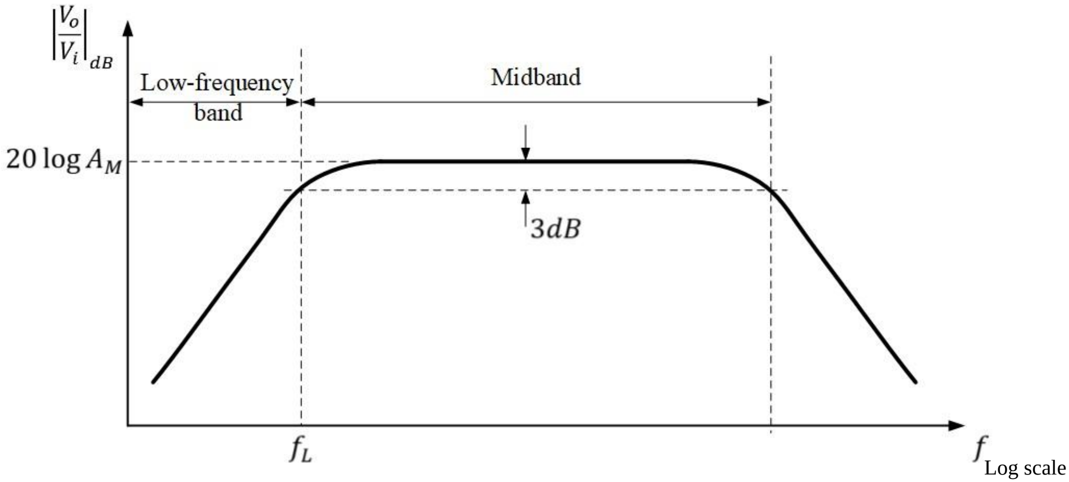
At low frequencies, the magnitude of the amplifier gain falls off due to that the **coupling and bypass capacitors** no longer have low impedances.



Introduction

Low-Frequency Band

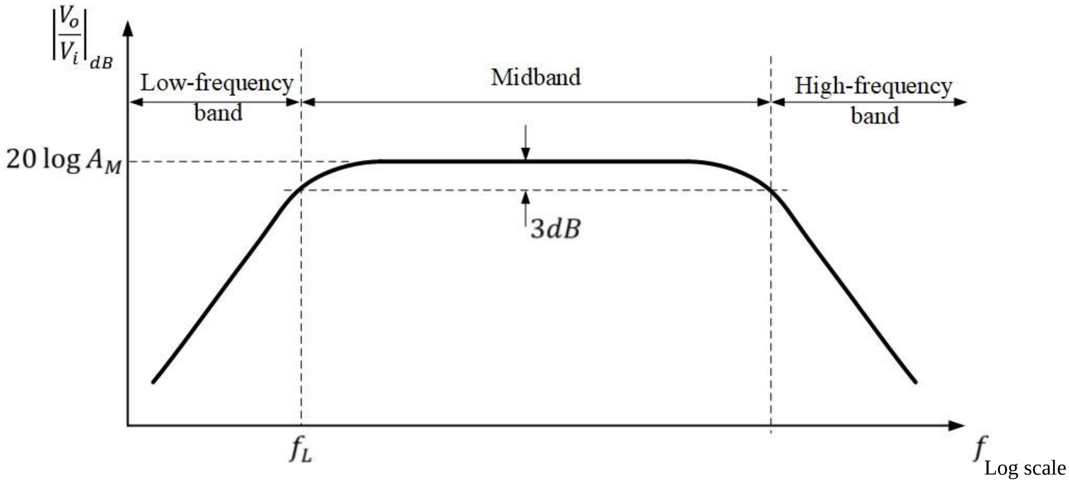
The frequency f_L is the lower end of the midband. f_L is defined as the frequency at which the gain drops by 3dB below its value in midband.



Introduction

<u>High-Frequency Band</u>

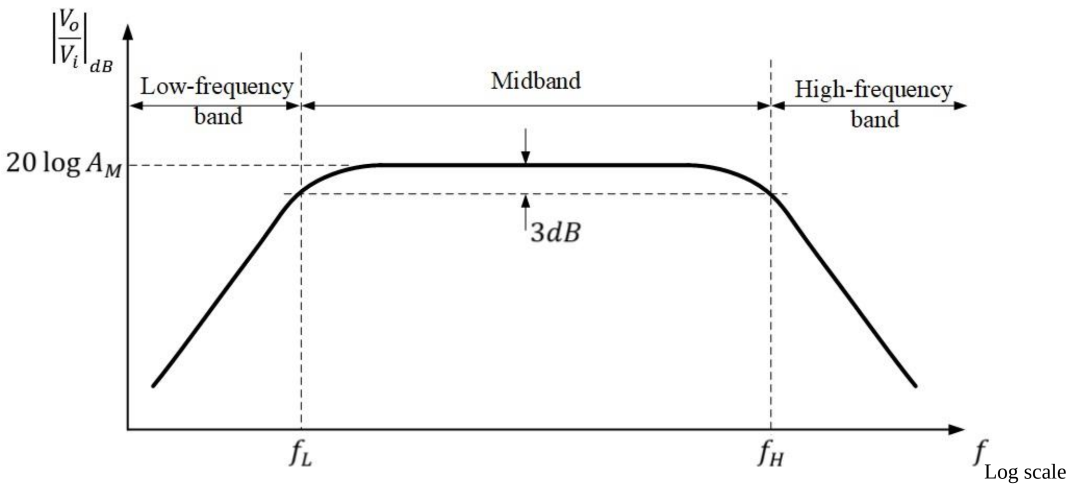
At high frequencies, the magnitude of the amplifier gain falls off due to that **internal capacitance** in the BJT and in the MOSFET no longer have high impedances.



Introduction

High-Frequency Band

The frequency f_H is the higher end of the midband. f_H is defined as the frequency at which the gain drops by 3dB below its value in midband.

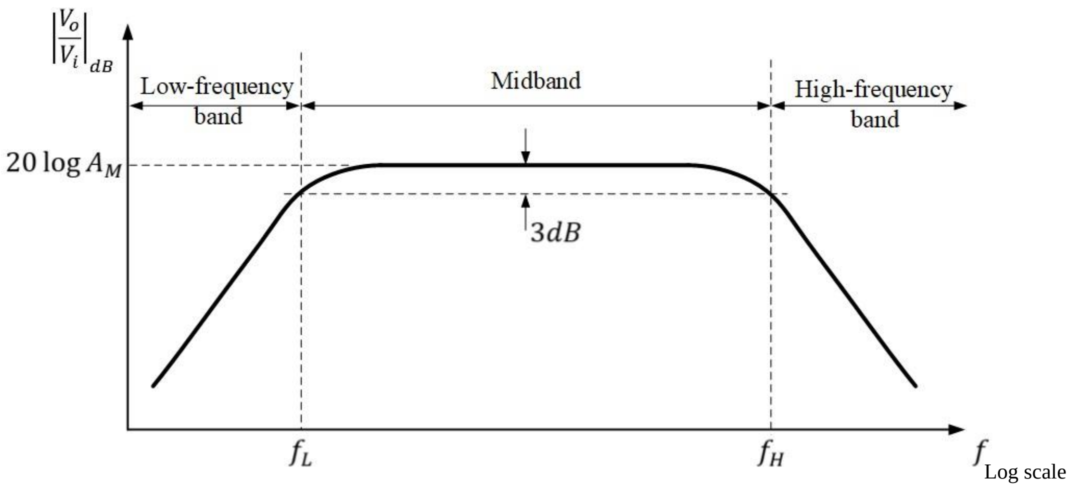


Introduction

Bandwidth

The amplifier **bandwidth** (BW) is defined by f_L and f_H

$$BW = f_H - f_L$$

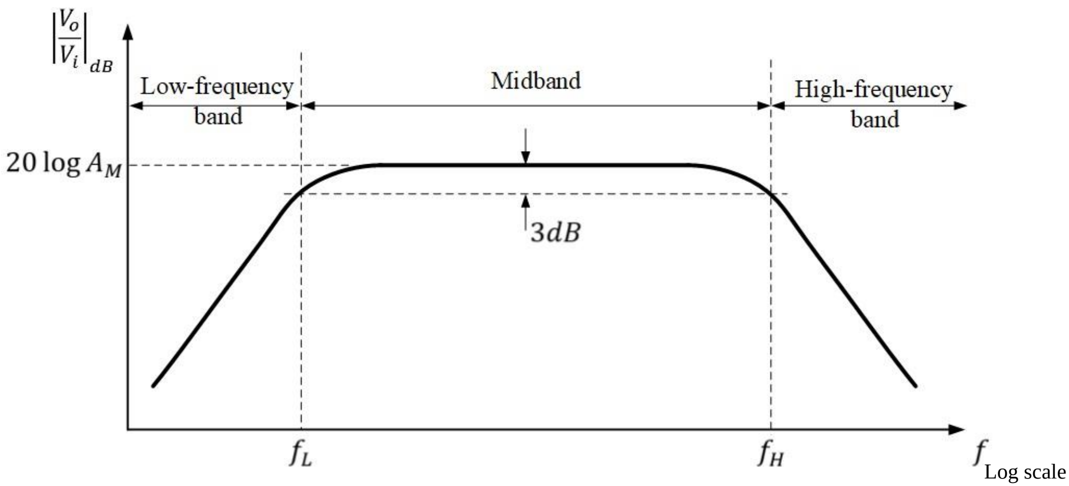


Introduction

Gain Bandwidth Product

The gain bandwidth product is a figure-of-merit for the amplifier and is defined as

$$GB = |A_M| \times BW$$

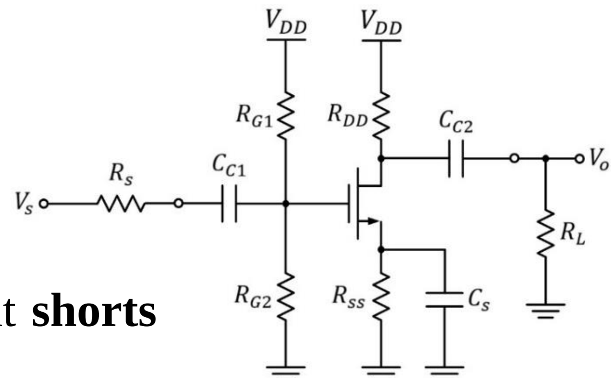


Introduction

Low Frequency Response

In low frequency band, the gain falls off due to the effect of the **coupling** and **bypass** capacitors

Coupling capacitor is usually used to couple **only** the AC signal from one circuit to another.



Bypass capacitor is a capacitor that **shorts** the AC signals to the ground

Bypass capacitor removes any AC noise that present on a DC signal. This result in a much **cleaner** DC signal.

Introduction

Low Frequency Response

The value of the coupling and bypass capacitors are usually **high**

At midband and high frequencies, these large capacitances have negligibly small impedances and can be assumed to be **short circuits**.

In low frequency band, the impedances of these capacitances are big and cannot be negligible

	Low frequency band	Midband	High frequency band
$\frac{1}{\omega C_c}$	$Z_{C_c} \neq 0$	$Z_{C_c} \simeq 0$	$Z_{C_c} \simeq 0$

Introduction

High Frequency Response

In high frequency band, the gain falls off due to the effect of the internal (parasitic) capacitance inside the transistors.

The parasitic capacitors are usually **very small** (in the range of fF) and can be neglected in low and midband frequencies by assuming it to be **open circuits**. However, at high frequency its impedance are not negligible.

	Low frequency band	Midband	High frequency band
$\frac{1}{\omega C_p}$	$Z_{c_p} \simeq \infty$	$Z_{c_p} \simeq \infty$	$Z_{c_p} \neq \infty$

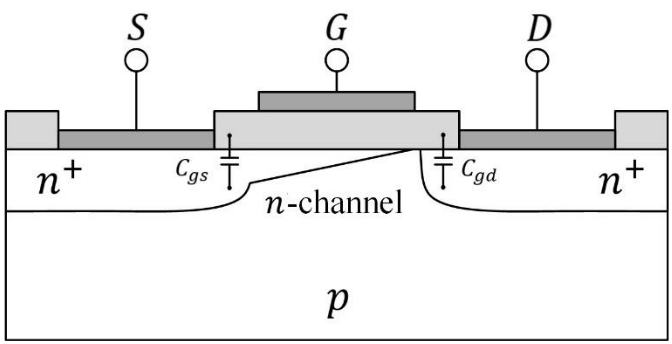
Frequency Response

High Frequency Response

High Frequency Model of the MOSFET

The MOSFET has four internal capacitances:

C_{gs} and C_{gd}, result from the gate-capacitance effect



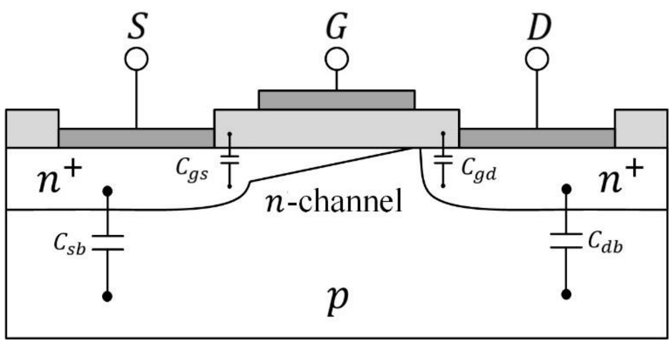
Frequency Response

High Frequency Response

High Frequency Model of the MOSFET

The MOSFET has four internal capacitances:

C_{sb} and C_{db}, are the depletion capacitances of the pn junctions formed by the source region and the substrate, and the drain region and the substrate, respectively.



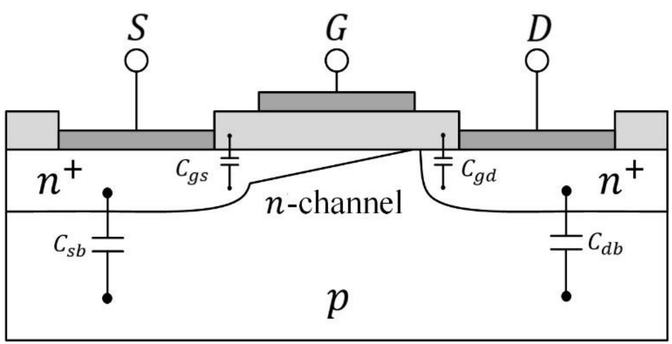
Frequency Response

High Frequency Response

High Frequency Model of the MOSFET

The polysilicon gate forms a capacitor with the channel region, with the oxide layer serving as the capacitor dielectric.

The gate capacitance per unit gate area is denoted C_{ox}.



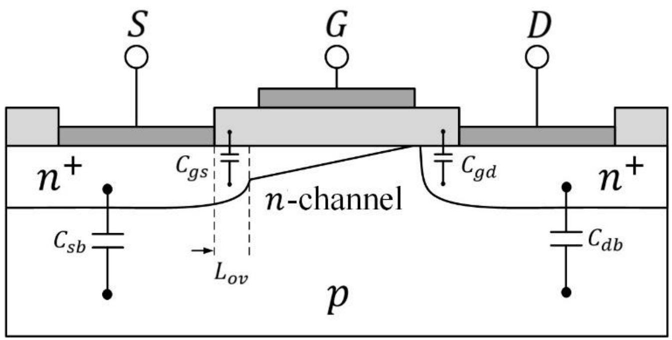
Frequency Response

High Frequency Response

High Frequency Model of the MOSFET

When the channel is pinched off, the gate capacitance is given by $2/3\,(W \times L)C_{ox}$ and exist completely at the source side.

There is a small capacitance results from the overlap of the gate with the source region. This capacitance is given by $W \times L_{ov}\ C_{ox}$



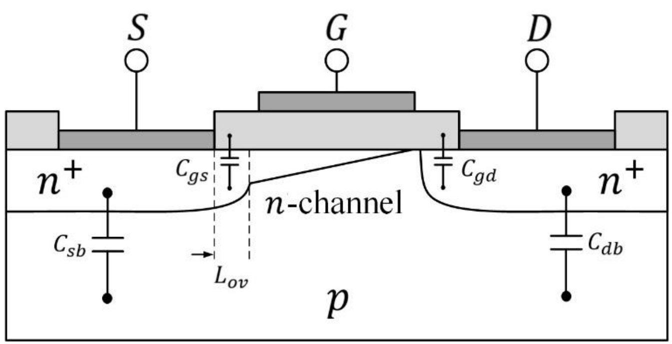
Frequency Response

High Frequency Response

High Frequency Model of the MOSFET

The total capacitance between the gate and the source is given by

$$C_{gs} = 2/3\,(W \times L)C_{ox} + W \times L_{ov}\,C_{ox}$$



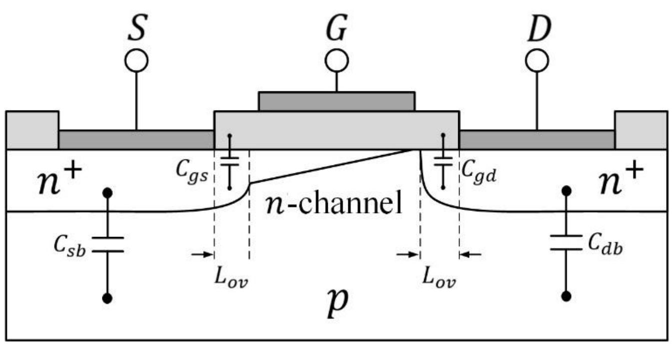
Frequency Response

High Frequency Response

High Frequency Model of the MOSFET

Because the channel pinch-off at the drain end, the capacitance between the gate and the source consists entirely of the overlap component

$$C_{gd} = W \times L_{ov}\, C_{ox}$$



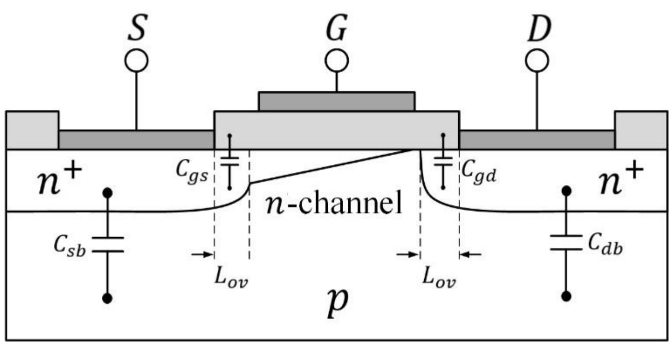
Frequency Response

High Frequency Response

High Frequency Model of the MOSFET

The capacitances of the reverse-biased *pn* junctions formed between the source and the *p*-type substrate is given by

$$C_{sb} = \frac{C_{sb0}}{\sqrt{1 + \frac{V_{SB}}{V_0}}}$$



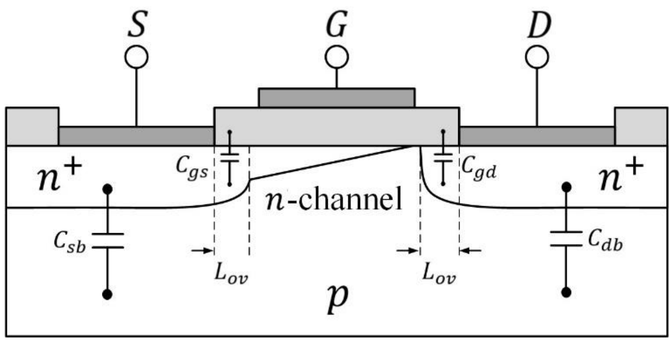
Frequency Response

High Frequency Response

High Frequency Model of the MOSFET

The capacitances of the reverse-biased pn junctions formed between the drain and the p-type substrate is given by

$$C_{db} = \frac{C_{db0}}{\sqrt{1 + \frac{V_{DB}}{V_0}}}$$

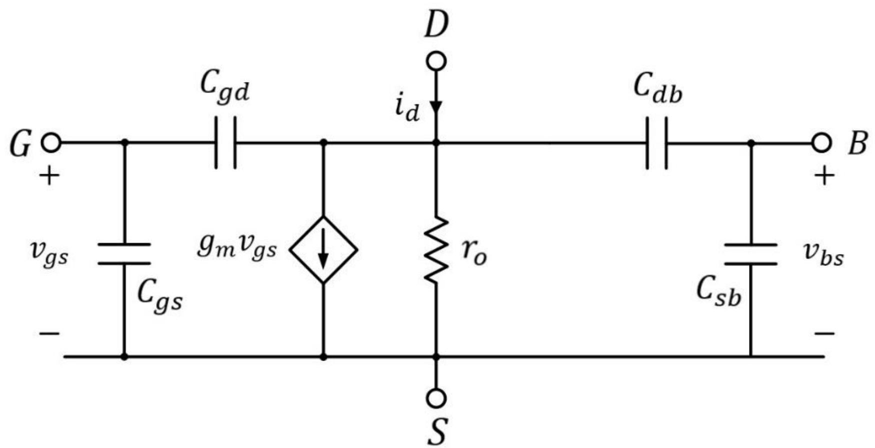


Frequency Response

High Frequency Response

High Frequency Model of the MOSFET

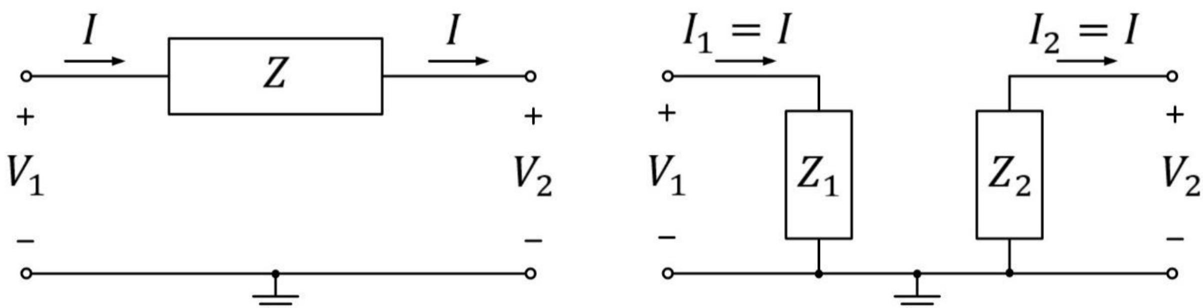
To preform high frequency analysis, the small-signal model of the MOSFET is modified to include the parasitic capacitances.



High Frequency Response

Miller's Theorem

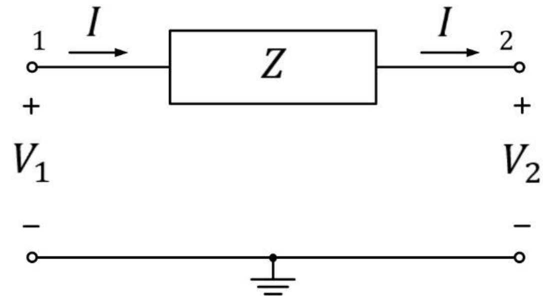
Miller's Theorem is a technique for replacing a **floating** impedance by two equivalent **grounded** impedances.



Frequency Response

High Frequency Response

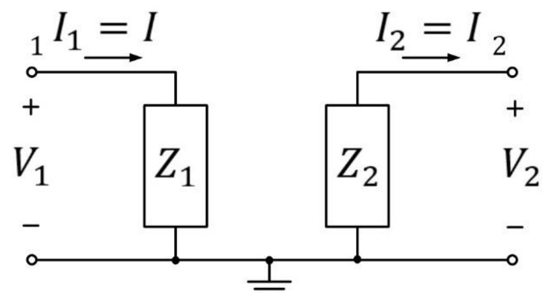
Miller's Theorem



Nodes 1 and 2 are also connected to other parts of the circuit

It is assumed that the voltage at node 2 is related to that at node 1 by

$$V_2 = KV_1$$



From the top circuit

$$I = \frac{V_1 - V_2}{Z}$$

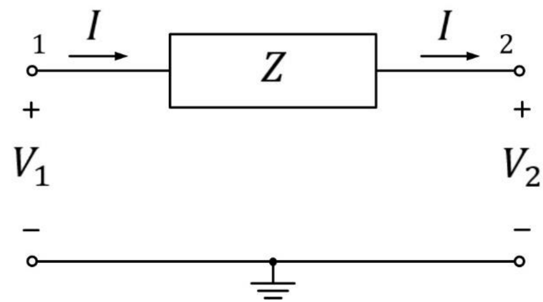
Frequency Response

High Frequency Response

Miller's Theorem

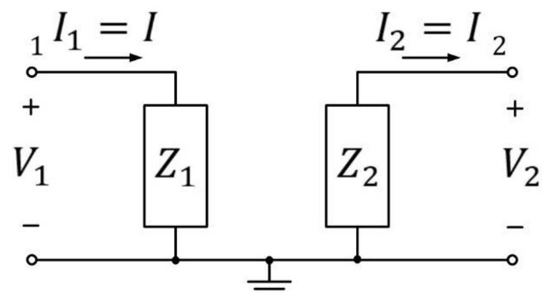
From the bottom circuit

$$I_1 = \frac{V_1}{Z_1}, \qquad I_2 = -\frac{V_2}{Z_2}$$



For the two circuits to be equivalent

$I_1 = I$	$I_2 = I$
$\frac{V_1}{Z_1} = \frac{V_1 - V_2}{Z}$	$-\frac{V_2}{Z_2} = \frac{V_1 - V_2}{Z}$



Frequency Response

High Frequency Response

Miller's Theorem

$$\frac{V_1}{Z_1} = \frac{V_1 - V_2}{Z}$$

$$Z_1 = Z\frac{V_1}{V_1 - V_2}$$

$$= Z\frac{1}{1 - V_2/V_1}$$

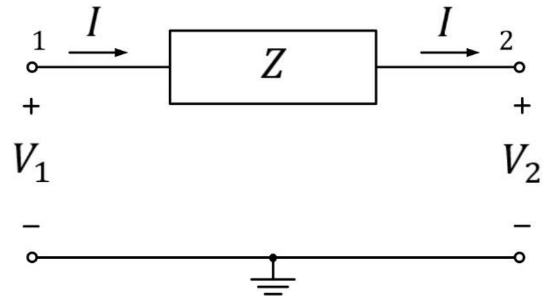
$$= Z\frac{1}{1 - K}$$

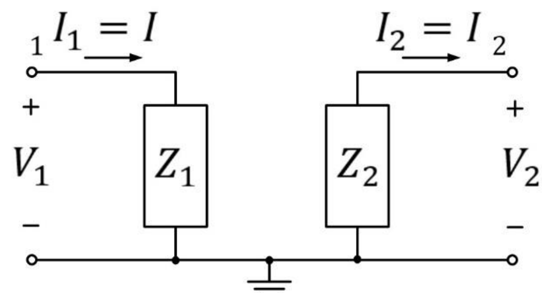
$$-\frac{V_2}{Z_2} = \frac{V_1 - V_2}{Z}$$

$$Z_2 = Z\frac{V_2}{V_2 - V_1}$$

$$= Z\frac{1}{1 - V_1/V_2}$$

$$= Z\frac{1}{1 - 1/K}$$

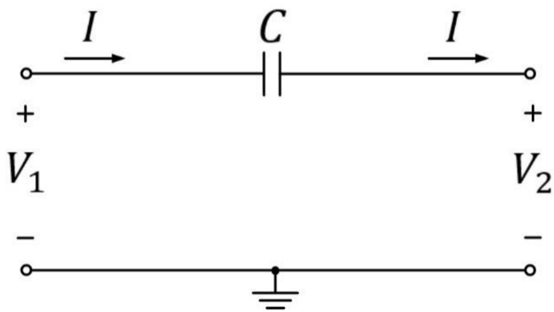


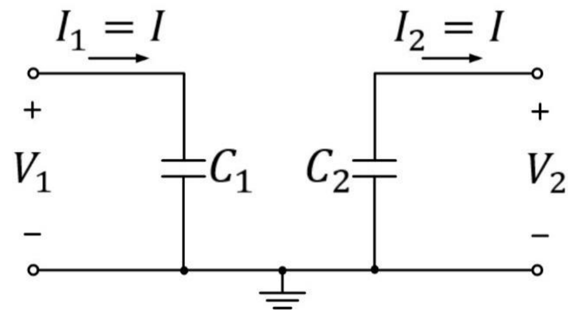


Frequency Response

High Frequency Response

Miller's Theorem





$$Z_1 = Z\frac{1}{1-K}$$

$$\frac{1}{j\omega C_1} = \frac{1}{j\omega C(1-K)}$$

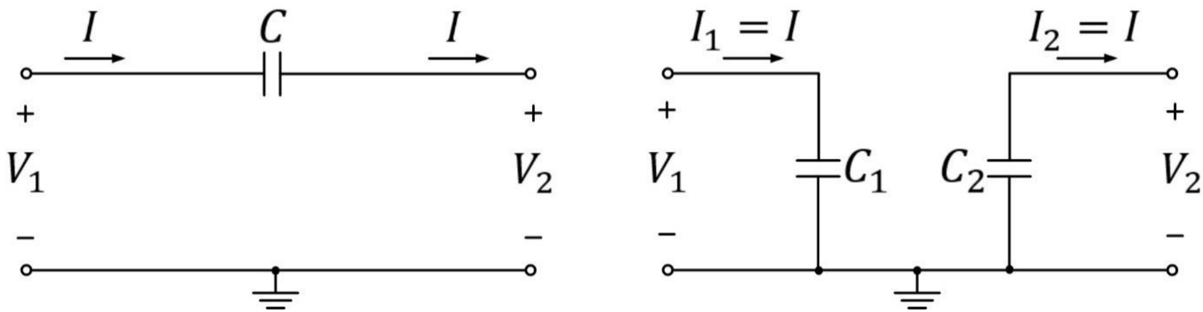
$$Z_2 = Z\frac{1}{1-1/K}$$

$$\frac{1}{j\omega C_2} = \frac{1}{j\omega C(1-1/K)}$$

Frequency Response

High Frequency Response

Miller's Theorem



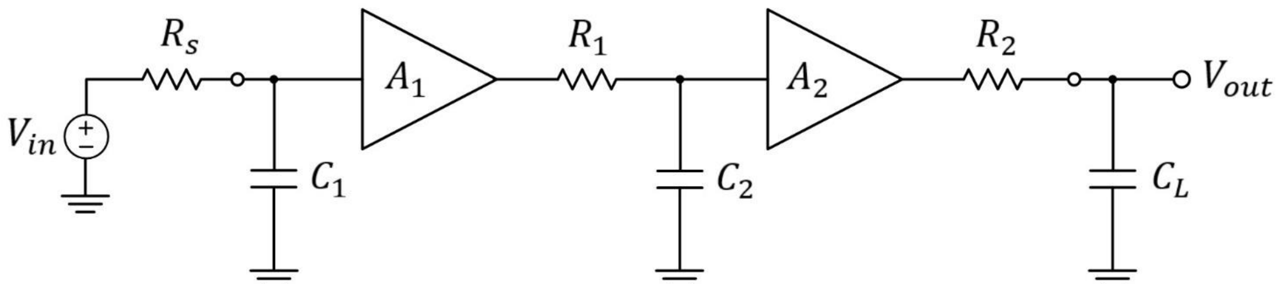
$$C_1 = C(1 - K)$$

$$C_2 = C(1 - 1/K)$$

Frequency Response

High Frequency Response

Amplifier Poles Estimation



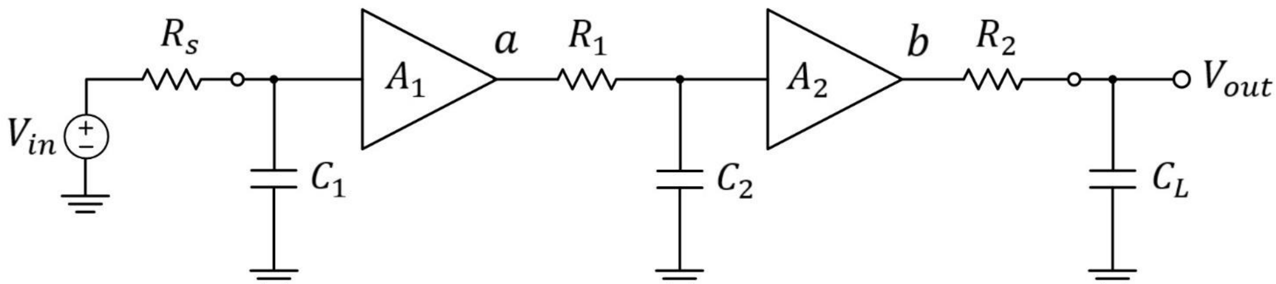
For the two cascade amplifiers shown above:

- R_i represents the output resistance of amplifier i
- C_i represents the input capacitance of amplifier i
- A_i represents the Midband gain of the amplifier i

Frequency Response

High Frequency Response

Amplifier Poles Estimation



The overall transfer function is given by

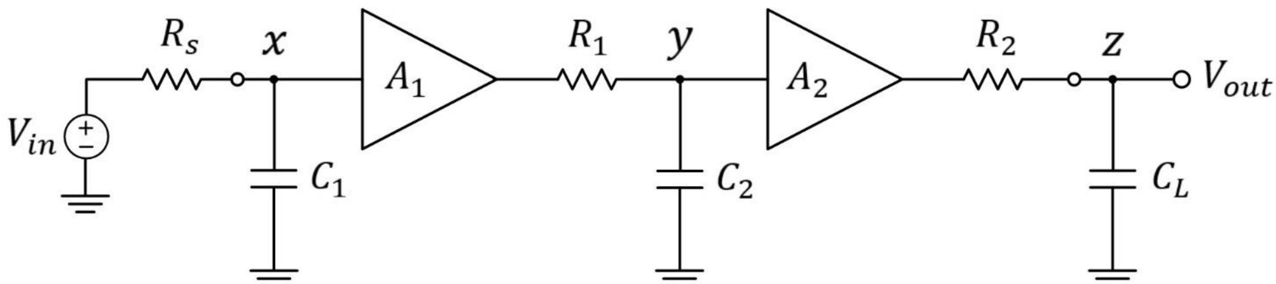
$$\frac{V_{out}}{V_{in}} = \frac{V_a}{V_{in}}\frac{V_b}{V_a}\frac{V_{out}}{V_b} = \frac{A_1}{1+sR_sC_1}\,\frac{A_2}{1+sR_1C_2}\,\frac{1}{1+sR_2C_L}$$

The overall transfer function has three poles

Frequency Response

High Frequency Response

Amplifier Poles Estimation

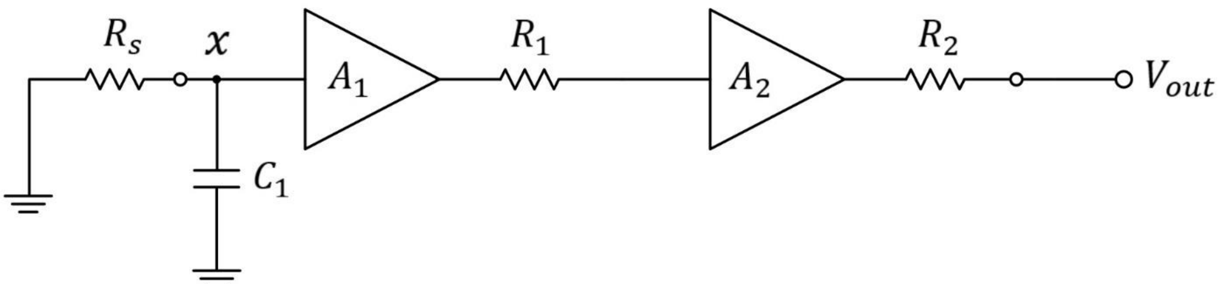


Each pole can be determine directly by calculating the reciprocal of the multiplication of the **total capacitance** seen from the node to **ground** by the **total resistance** seen from the node to **ground** after **shutting down** the input.

Frequency Response

High Frequency Response

Amplifier Poles Estimation



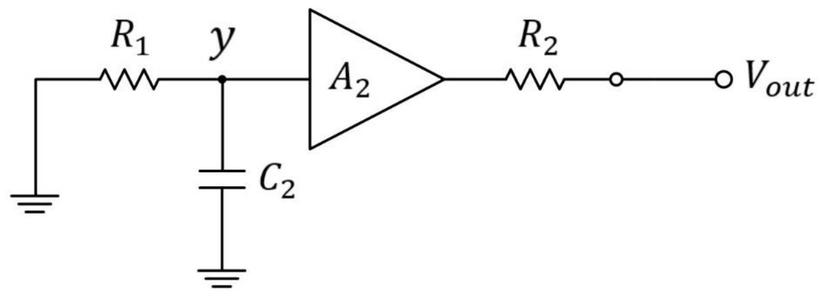
While calculating the pole at certain node, replace the capacitors at the other nodes by **open circuit**

x	$C_x = C_1$	$R_x = R_s$	$\omega_{px} = 1/[R_x C_x]$ $= 1/[R_s C_1]$

Frequency Response

High Frequency Response

Amplifier Poles Estimation



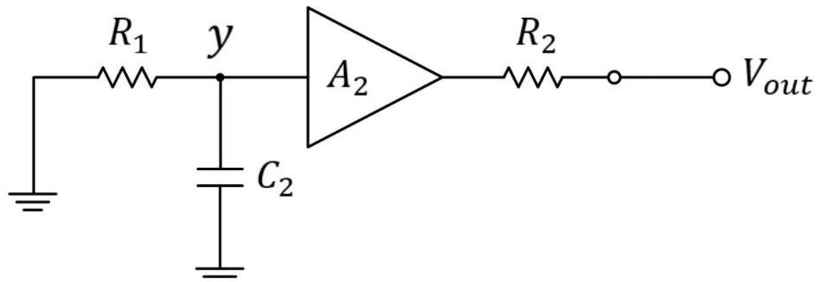
While calculating the pole at certain node, replace the capacitors at the other nodes by **open circuit**

y	$C_y = C_2$	$R_y = R_1$	$\omega_{py} = 1/[R_y C_y] = 1/[R_1 C_2]$

Frequency Response

High Frequency Response

Amplifier Poles Estimation



While calculating the pole at certain node, replace the capacitors at the other nodes by **open circuit**

z	$C_z = C_L$	$R_z = R_2$	$\omega_{pz} = 1/[R_z C_z]$ $= 1/[R_2 C_L]$

Frequency Response

High Frequency Response

Amplifier Poles Estimation

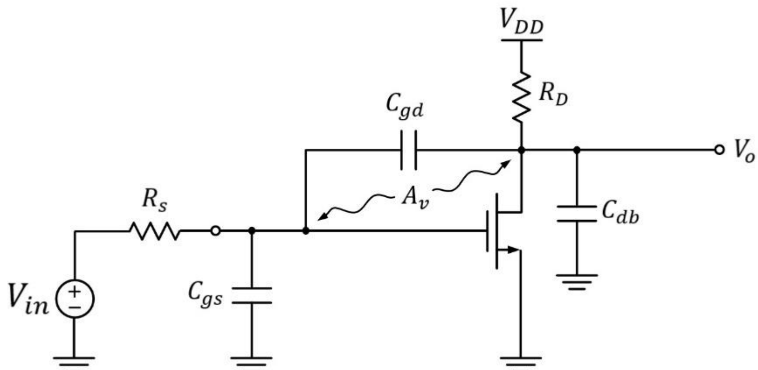
The association of one pole with each node in the amplifier circuit provides an **intuitive** approach to **estimate** the poles of its transfer function

- Determine the total equivalent capacitance from the node of interest to ground C_{node}
- Determine the total small-signal resistance from the node of interest to ground R_{node}
- The pole frequency is given by $\omega_{node} = 1/[C_{node}R_{node}]$

While calculating the pole at certain node, replace the capacitors at the other nodes by **open circuit**

Frequency Response

Common Source Configuration

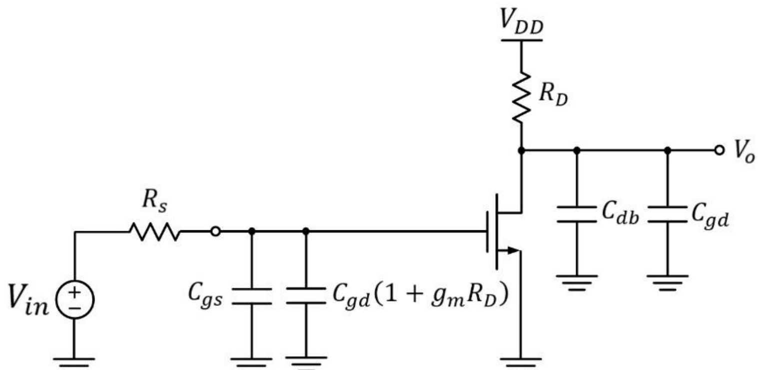


To simplify the analysis, Miller's **approximation** is used to convert C_{gd} in two grounded capacitors

In Miller's approximation, the **Midband gain** is used instead of the high frequency gain

Frequency Response

Common Source Configuration



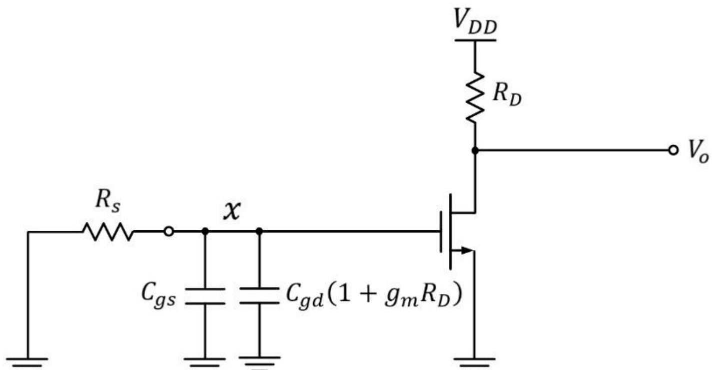
To simplify the analysis, Miller's **approximation** is used to convert C_{gd} in two grounded capacitors

$$C_1 = (1 - A_v)C_{gd}$$
$$= (1 + g_m R_D)C_{gd}$$

$$C_2 = (1 - 1/A_v)C_{gd}$$
$$= (1 + 1/[g_m R_D])C_{gd} \simeq C_{gd}$$

Frequency Response

Common Source Configuration



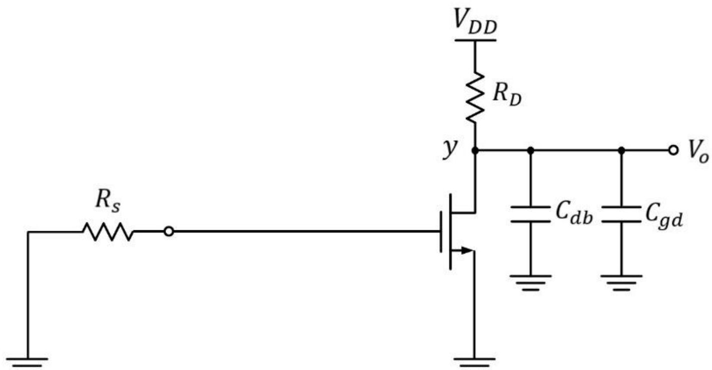
For the input node x

$$C_x = C_{gs} + C_{gd}(1 + g_m R_D) \qquad\qquad R_x = R_s$$

$$\omega_{px} = \frac{1}{R_s\left[C_{gs} + C_{gd}(1 + g_m R_D)\right]}$$

Frequency Response

Common Source Configuration



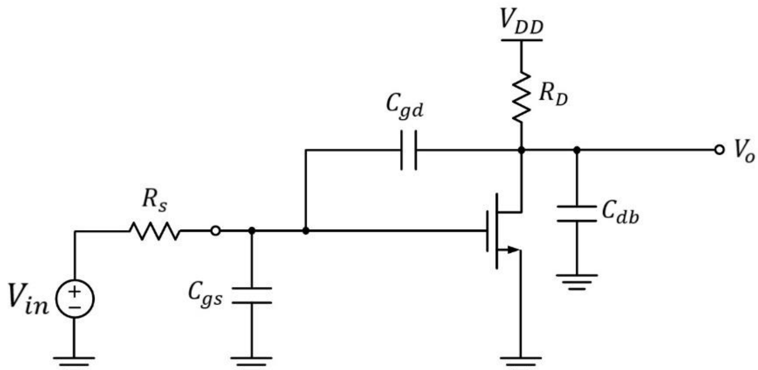
For the output node y

$$C_y = C_{db} + C_{gd} \qquad\qquad R_y = R_D$$

$$\omega_{py} = \frac{1}{R_D\left[C_{db} + C_{gd}\right]}$$

Frequency Response

Common Source Configuration



By knowing the two poles, the transfer function can be written as

$$A(s) = \frac{A_M}{(1 + s/\omega_{px})(1 + s/\omega_{py})}$$

$$= \frac{-g_m R_D}{[1/(\omega_{px}\omega_{py})]s^2 + [(1/\omega_{px}) + (1/\omega_{py})]s + 1}$$

Frequency Response

Common Source Configuration

$$A(s) = \frac{-g_m R_D}{[R_s R_D \mathbb{C}_1]s^2 + \left[R_s C_{gs} + R_s(1 + g_m R_D)C_{gd} + R_D\left(C_{db} + C_{gd}\right)\right]s + 1}$$

$$\mathbb{C}_1 = C_{gs}\left[C_{db} + C_{gd}\right] + (1 + g_m R_D)C_{gd}\left[C_{db} + C_{gd}\right]$$

The direct analysis (without any approximation) results in the following transfer function

$$A(s) = \frac{\left(C_{gd}s - g_m\right)R_D}{[R_s R_D \mathbb{C}_2]s^2 + \left[R_s C_{gs} + R_s(1 + g_m R_D)C_{gd} + R_D\left(C_{db} + C_{gd}\right)\right]s + 1}$$

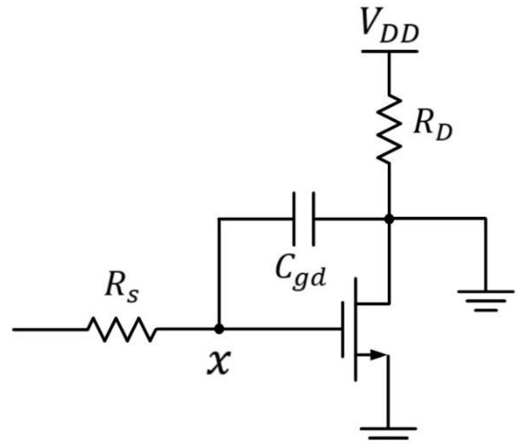
$$\mathbb{C}_2 = C_{gs}\left[C_{db} + C_{gd}\right] + C_{gd}C_{db}$$

Frequency Response

Common Source Configuration

The quick method gives a very good estimation of the poles of the transfer function. However, it does not count for the zeros.

The zero arises from the feedthrough path that conducts the input signal to the output through C_{gd} at very high frequencies.



The zero can be computed by **shorting** the output to ground

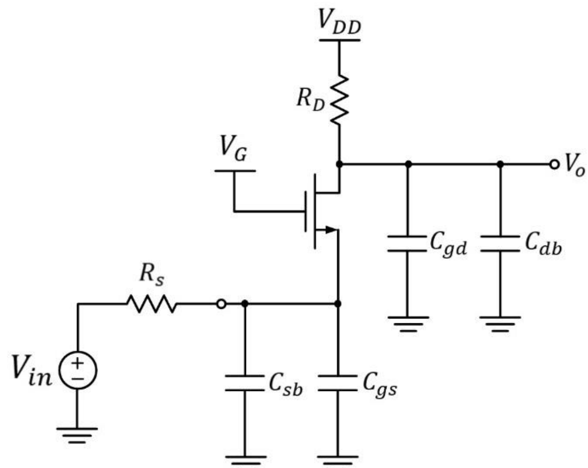
$$V_x\, s_z C_{gd} = g_m V_x \quad \rightarrow \quad s_z = g_m / C_{gd}$$

Frequency Response

Common Gate Configuration

All the parasitic capacitances of Q_1 are connected either from the input node to ground and or from output node to ground.

Because the input and output nodes are isolated. Miller's approximation is not needed



Because this amplifier has no Miller multiplication of capacitances, it achieves a wide bandwidth.

Frequency Response

Common Gate Configuration

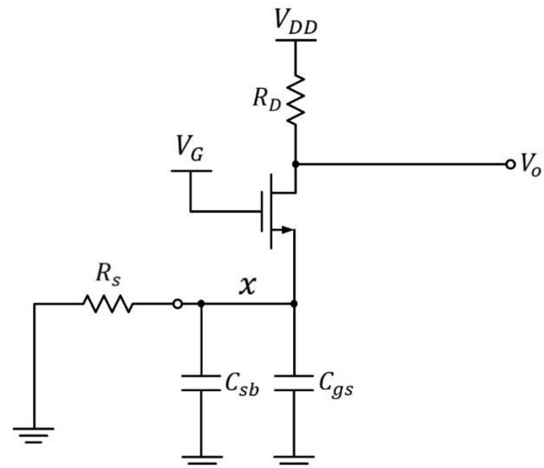
For the input node x

$$C_x = C_{gs} + C_{sb}$$

$$R_x = R_s // 1/g_m$$

$$\omega_{px} = \frac{1}{R_x C_x}$$

$$= \frac{1}{[R_s // 1/g_m][C_{gs} + C_{sb}]}$$



Frequency Response

Common Gate Configuration

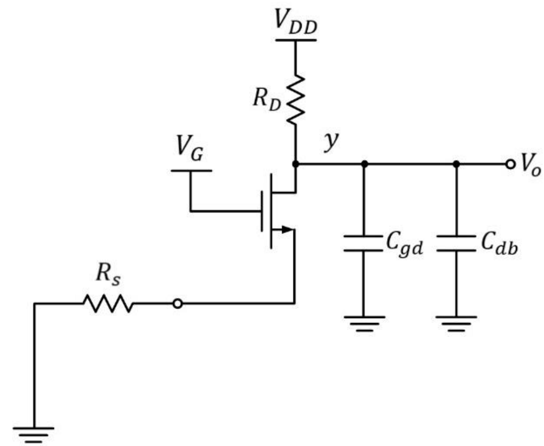
For the input node y

$$C_y = C_{gd} + C_{db}$$

$$R_y = R_D$$

$$\omega_{py} = \frac{1}{R_y C_y}$$

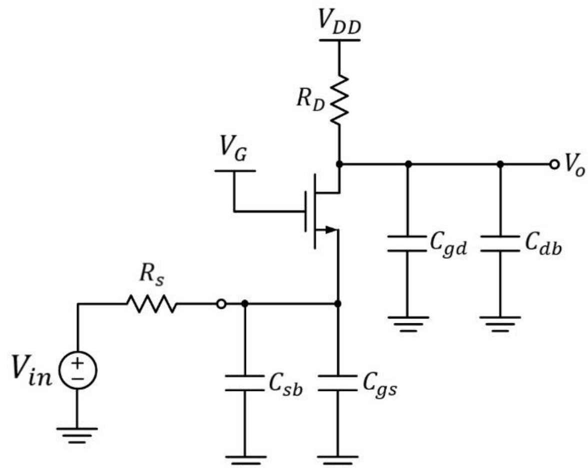
$$= \frac{1}{R_D\left[C_{gd} + C_{db}\right]}$$



Frequency Response

Common Gate Configuration

By knowing the two poles of the amplifier, the transfer function can be written as



$$A(s) = \frac{A_M}{(1 + s/\omega_{px})(1 + s/\omega_{py})}$$

$$= \frac{R_D/(1/g_m + R_s)}{(1 + [R_s// 1/g_m][C_{gs} + C_{sb}]s)(1 + R_D[C_{gd} + C_{db}]s)}$$

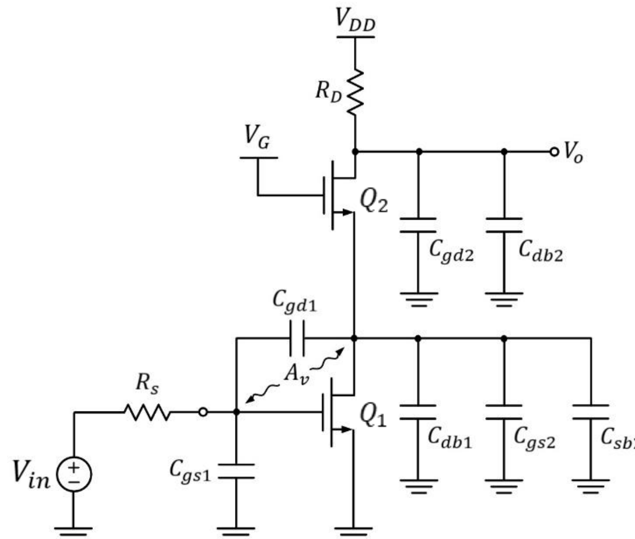
Frequency Response

Cascode Amplifier

The Miller approximation of C_{gd1} is determined by the mid-band frequency gain

$$A_v = -\frac{g_{m1}}{g_{m2}}$$

The Miller multiplication effect is less significant in cascode amplifier than in common source configuration.



Frequency Response

Cascode Amplifier

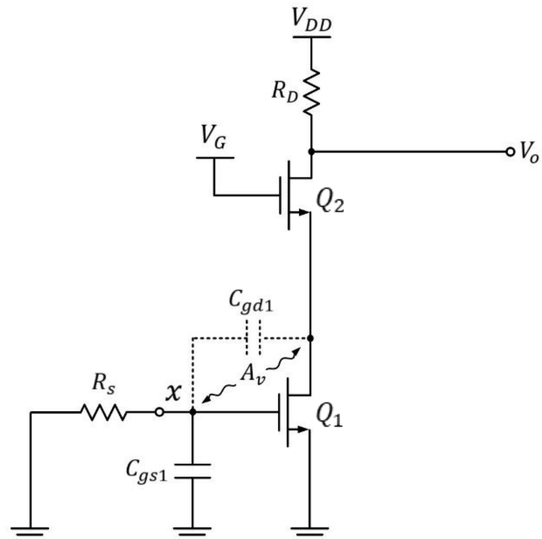
For the output node x

$$C_x = C_{gs1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right) C_{gd1}$$

$$R_x = R_s$$

$$\omega_{px} = \frac{1}{R_x C_x}$$

$$= \frac{1}{R_s \left[C_{gs1} + \left(1 + \frac{g_{m1}}{g_{m2}}\right) C_{gd1} \right]}$$



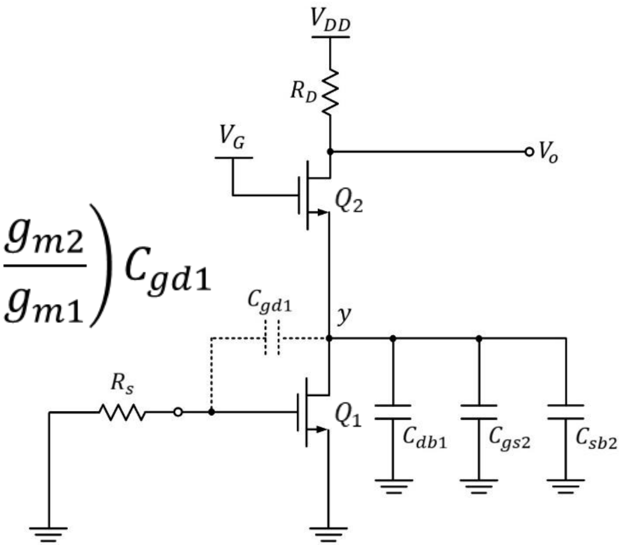
Frequency Response

Cascode Amplifier

For the output node y

$$R_y = 1/g_{m2}$$

$$C_y = C_{gs2} + C_{db1} + C_{sb2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right) C_{gd1}$$



$$\omega_{py} = \frac{1}{R_y C_y}$$

$$= \frac{g_{m2}}{C_{gs2} + C_{db1} + C_{sb2} + \left(1 + \frac{g_{m2}}{g_{m1}}\right) C_{gd1}}$$

Frequency Response

Cascode Amplifier

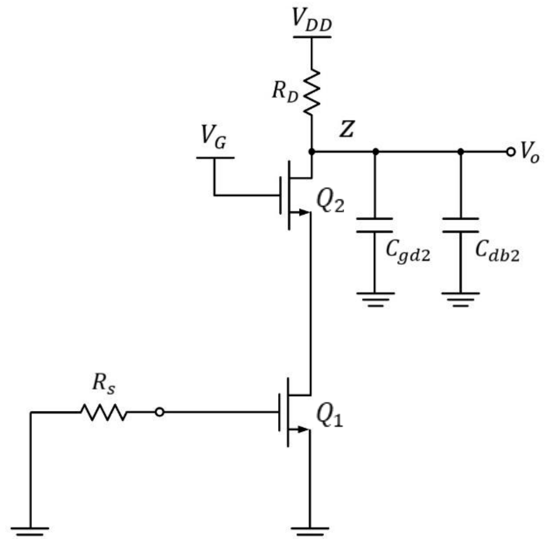
For the output node z

$$C_z = C_{gd2} + C_{db2}$$

$$R_z = R_D$$

$$\omega_{pz} = \frac{1}{R_Z C_z}$$

$$= \frac{1}{R_D\left[C_{gd2} + C_{db2} + C_L\right]}$$



Frequency Response

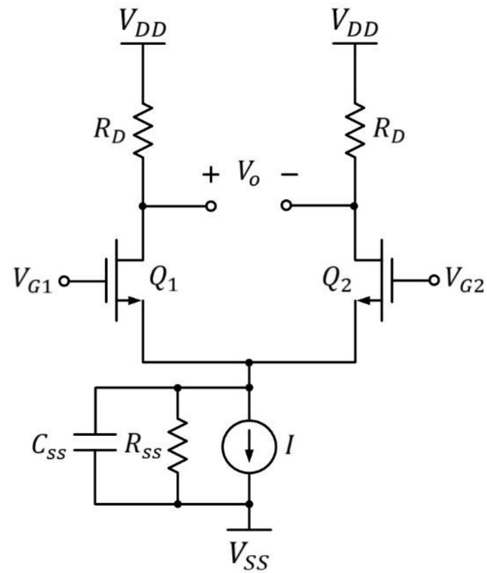
Differential Pair

Passive Loaded Differential Pair

C_{SS} is the total capacitance between the common source node and ground

C_{SS} includes C_{db} and C_{gd} of Q_S, as well as C_{sb} of Q_1 and C_{sb} of Q_2.

This capacitance can be significant, especially if **wide** transistors are used for Q_S, Q_1, and Q_2.



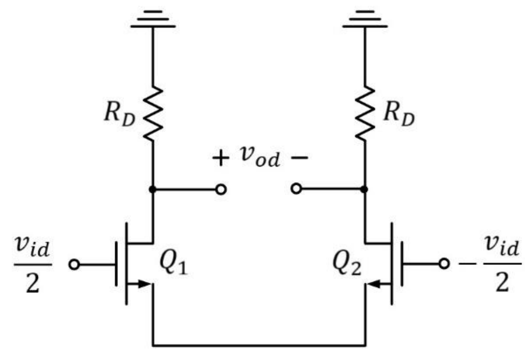
Frequency Response

Differential Pair

Passive Loaded Differential Pair

When a differential signal v_{id} is applied, a virtual ground appears at the common sources

The differential half circuit can be used to determine the frequency dependence of the differential gain



The frequency characteristics of the differential half circuit is **identical** to that of the common source amplifier

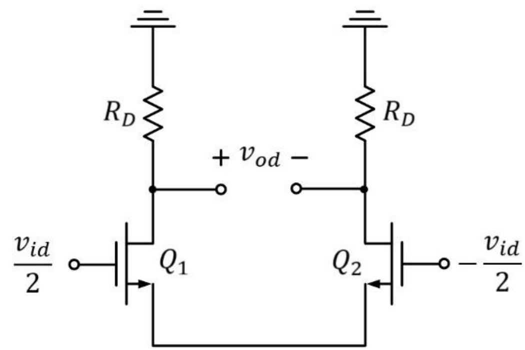
Frequency Response

Differential Pair

Passive Loaded Differential Pair

$v_{id}/2$ and $-v_{id}/2$ are multiplied by the **same** transfer function to generate the output

$$\left|\frac{v_{od}/2}{v_{id}/2}\right| = \frac{g_m R_D}{(1 + s/\omega_{p1})(1 + s/\omega_{p2})}$$



The number of poles in $A_d(s)$ is equal to that of each signal path (not the sum of the number of poles in the two paths).

Frequency Response

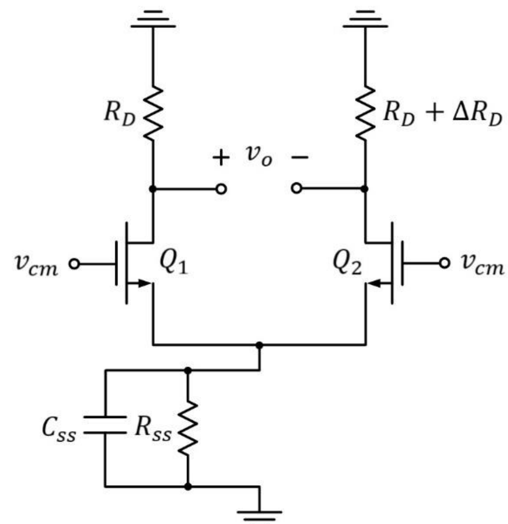
Differential Pair

Passive Loaded Differential Pair

When a common mode signal v_{cm} is applied, C_{ss} and R_{ss} contribute in the common mode gain

For the case where there is a mismatch ΔR_D between the two drain resistances, the common mode gain was

$$A_{cm} = \frac{v_o}{v_{cm}} = \frac{-g_m \Delta R_D}{1 + 2g_m R_{SS}} \simeq -\frac{\Delta R_D}{2R_{SS}}$$



Frequency Response

Differential Pair

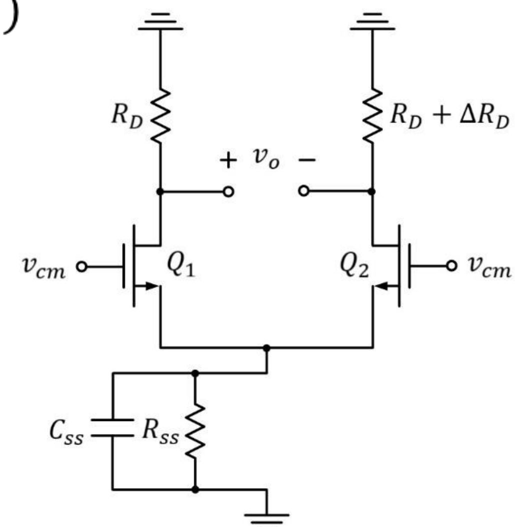
Passive Loaded Differential Pair

Replacing R_{ss} by $Z_{ss} = R_{ss}//(1/sC_{ss})$

$$A_{cm} \simeq -\frac{\Delta R_D}{2Z_{SS}}$$

$$= -\frac{\Delta R_D}{2}\left(\frac{1}{R_{SS}} + sC_{ss}\right)$$

$$= -\frac{\Delta R_D}{2R_{SS}}(1 + \underbrace{sR_{ss}C_{ss}}_{\omega_Z})$$



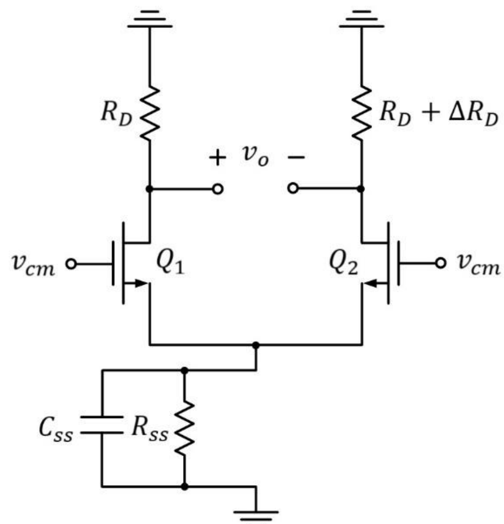
Frequency Response

Differential Pair

Passive Loaded Differential Pair

A_{CM} has zero at

$$\omega_z = \frac{1}{R_{ss}C_{ss}}$$



This zero is at a frequency much **lower** than those of the other poles and zeros of the circuit.

This zero **dominates** the frequency dependence of A_{cm} and $CMRR$.

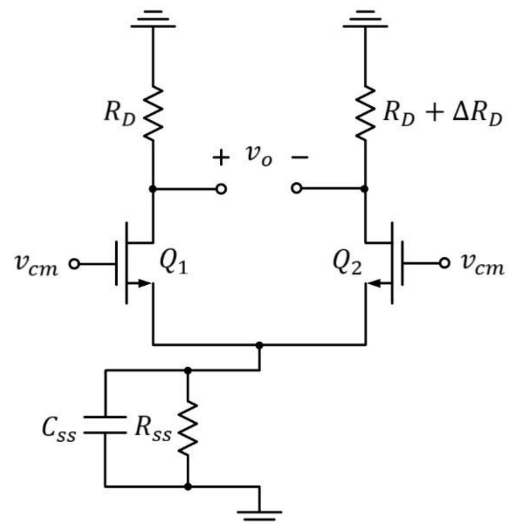
Frequency Response

Differential Pair

Passive Loaded Differential Pair

The common mode gain increases at the rate of 20 dB/decade starting at a relatively low frequency

The increase in the common mode gain causes the $CMRR$ of the differential amplifier to **decrease**



Frequency Response

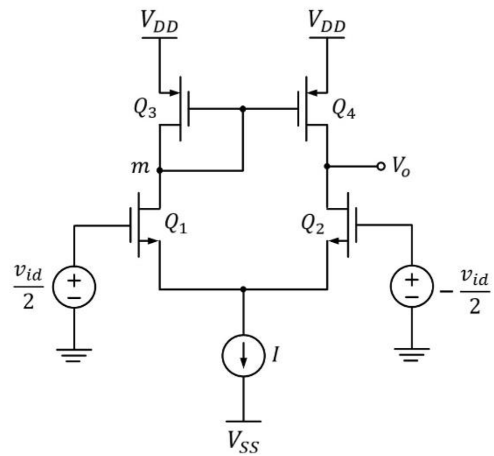
Differential Pair

Active Loaded Differential Pair

This differential pair contains two signal paths with **different** transfer functions.

The path consisting of Q_1, Q_3, and Q_4 includes a pole at node m given by

$$\omega_{pm} = g_{m3}/C_m$$



The capacitance C_m arises from C_{gs3}, C_{gs4}, C_{db3}, C_{db1}, and the Miller effect of C_{gd1} and C_{gd4}

Frequency Response

Differential Pair

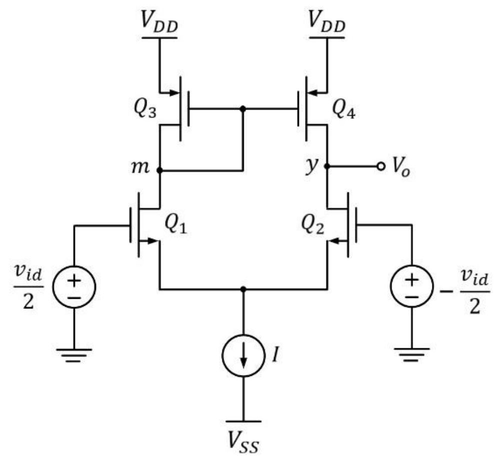
Active Loaded Differential Pair

The pole associated with node m is called the **mirror** pole

The two paths contain the output pole at node y which is given by

$$\omega_{py} = 1/\left([r_{o2}//r_{o4}]C_y\right)$$

The capacitance C_y arises from C_{db2}, C_{db4}, and the Miller effect of C_{gd2} and C_{gd4}



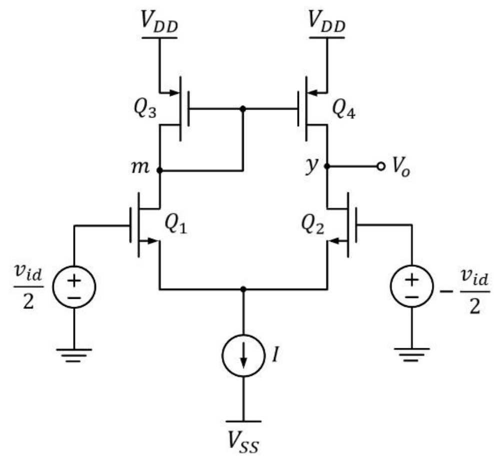
Frequency Response

Differential Pair

Active Loaded Differential Pair

The transfer function of the path consisting of Q_2 is given by

$$\frac{v_{o2}}{-v_{id}/2} = \frac{A_M}{(1 + s/\omega_{py})}$$



While the transfer function of the path consisting of Q_1, Q_3, and Q_4 is given by

$$\frac{v_{o1}}{v_{id}/2} = \frac{A_M}{(1 + s/\omega_{pm})(1 + s/\omega_{py})}$$

Frequency Response

Differential Pair

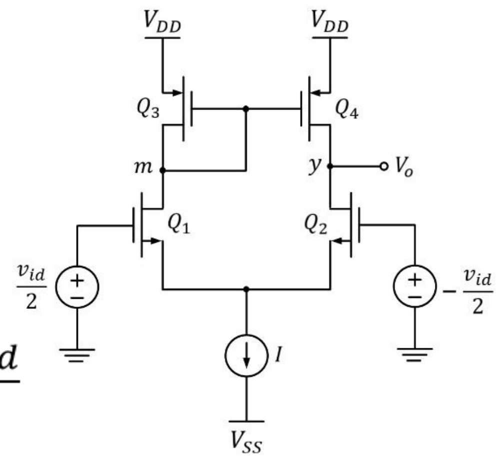
Active Loaded Differential Pair

The total output is given by

$$v_o = v_{o1} - v_{o2}$$

$$= \frac{A_M}{(1 + s/\omega_{pm})(1 + s/\omega_{py})} \frac{v_{id}}{2} + \frac{A_M}{(1 + s/\omega_{py})} \frac{v_{id}}{2}$$

$$= \frac{A_M(1 + s/[2\omega_{pm}])}{(1 + s/\omega_{pm})(1 + s/\omega_{py})} v_{id}$$



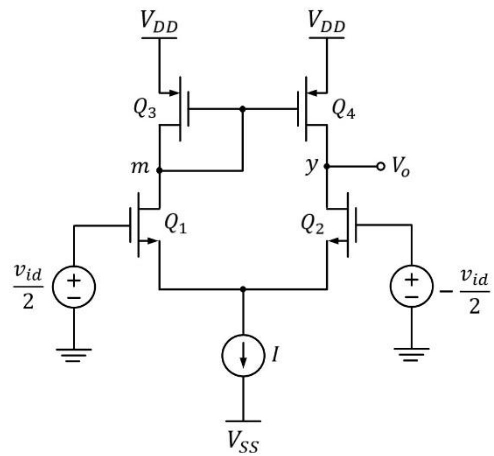
Frequency Response

Differential Pair

Active Loaded Differential Pair

The active loaded differential pair amplifier has a zero at

$$\omega_z = 2\omega_{pm}$$



The total transfer function of any amplifier contain **all** the poles that appears in any signal path

The common pole appears as a **single** pole in the total transfer function