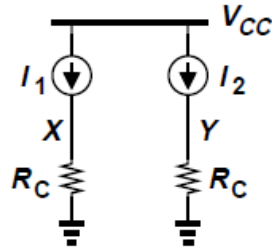


### Problem Set 3: Differential Amplifiers (Chapter 10)

#### Fundamental Concepts:

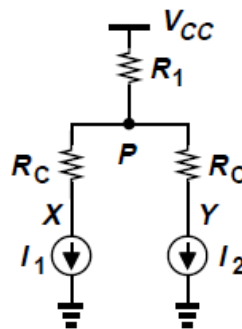
#### Problem 10.4

In the circuit of Fig. 10.60,  $I_1 = I_0 \cos \omega t + I_0$  and  $I_2 = -I_0 \cos \omega t + I_0$ . Plot the waveforms at  $X$  and  $Y$  and determine their peak-to-peak swings and common-mode level.



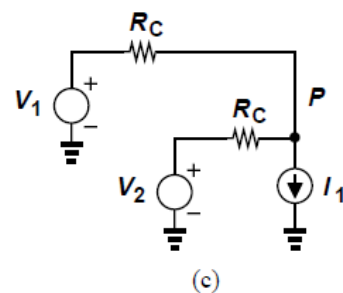
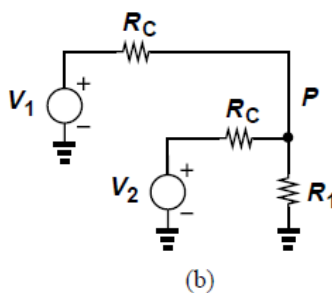
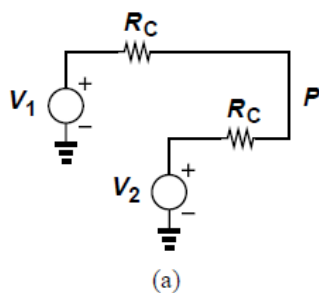
#### Problem 10.5

In the circuit of Fig. 10.60,  $I_1 = I_0 \cos \omega t + I_0$  and  $I_2 = -I_0 \cos \omega t + I_0$ . Plot the waveforms at  $X$  and  $Y$  and determine their peak-to-peak swings and common-mode level. Also, plot the voltage at node  $P$  as a function of time.



#### Problem 10.10

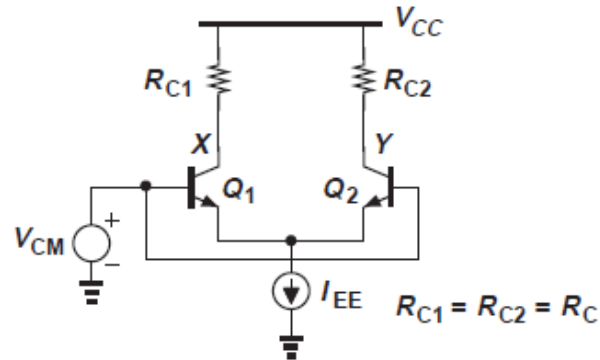
Assuming  $V_1 = V_0 \cos \omega t + V_0$  and  $V_2 = -V_0 \cos \omega t + V_0$ , plot  $V_P$  as a function of time for the circuits shown in Fig. 10.65. Assume  $I_T$  is constant.



## BJT Differential Amplifier:

### Problem 10.12

In Fig. 10.7,  $I_{EE}$  experiences a change of  $\Delta I$ . How do  $V_X$ ,  $V_Y$ , and  $V_X - V_Y$  change?

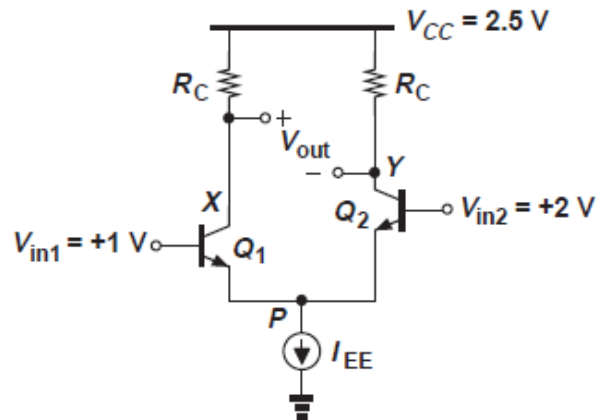


### Problem 10.13

Repeat Problem 12, but assuming that  $R_{C1} = R_{C2} + \Delta R$ . Neglect the Early effect.

### Problem 10.15

In the circuit of Fig. 10.9(b),  $R_C = 500 \Omega$ . What is the maximum allowable value of  $I_{EE}$  if  $Q_2$  must remain in the active region?



### Problem 10.19

Suppose the input differential signal applied to a bipolar differential pair must not change the transconductance (and hence the bias current) of each transistor by more than 10%. From Eq. (10.58), determine the maximum allowable input.

$$I_{C2} = \frac{I_{EE}}{1 + \exp \frac{V_{in1} - V_{in2}}{V_T}}$$

### Problem 10.24

In Example 10.9,  $R_C = 500\ \Omega$ ,  $I_{EE} = 1\text{ mA}$ , and  $V_{CC} = 2.5\text{ V}$ . Assume

$$V_{in1} = V_0 \sin \omega t + V_{CM} \quad (10.214)$$

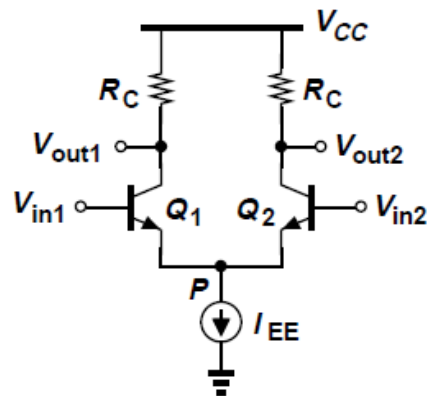
$$V_{in2} = -V_0 \sin \omega t + V_{CM}, \quad (10.215)$$

where  $V_{CM} = 1\text{ V}$  denotes the input common-mode level.

(a) If  $V_0 = 2\text{ mV}$ , plot the output waveforms (as a function of time).

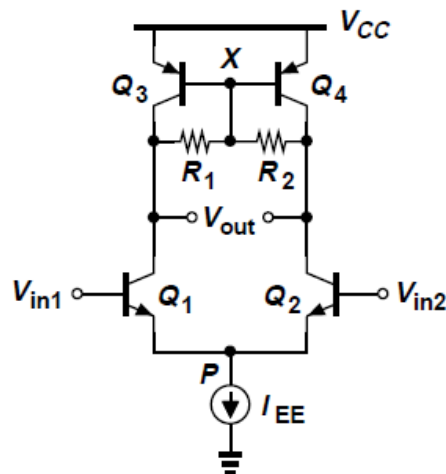
(b) If  $V_0 = 50\text{ mV}$ , determine the time  $t_1$  at which one transistor carries 95% of the tail current.

(c) Find the linear input range.



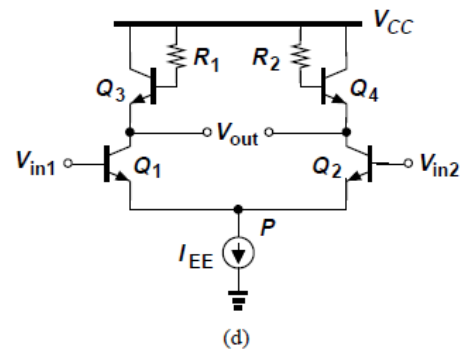
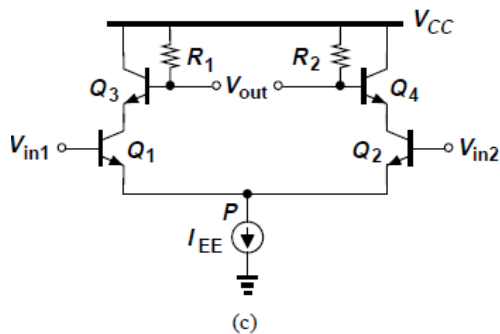
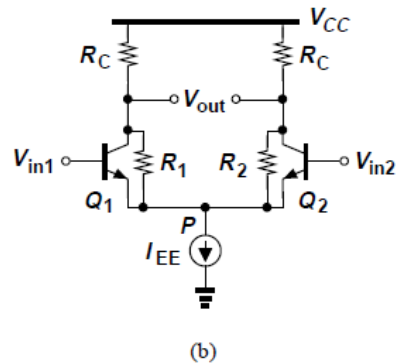
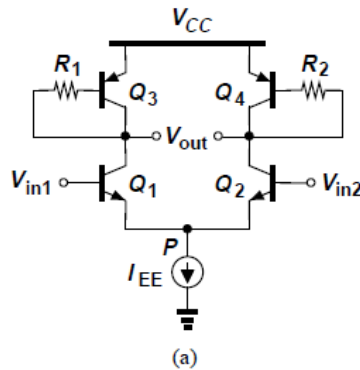
### Problem 10.30

Consider the circuit shown in Fig. 10.68, where  $I_{EE} = 2\text{ mA}$ ,  $V_{A,n} = 5\text{ V}$ ,  $V_{A,p} = 4\text{ V}$ . What value of  $R_1 = R_2$  allows a differential voltage gain of 50?



### Problem 10.32(a)

- . Assuming perfect symmetry and  $V_A < \infty$ , compute the differential voltage gain of each stage depicted in Fig. 10.69.

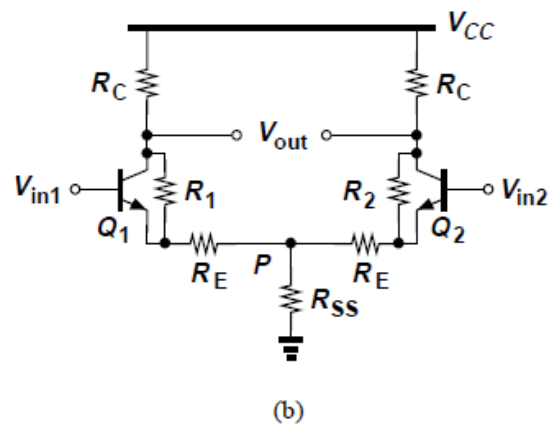
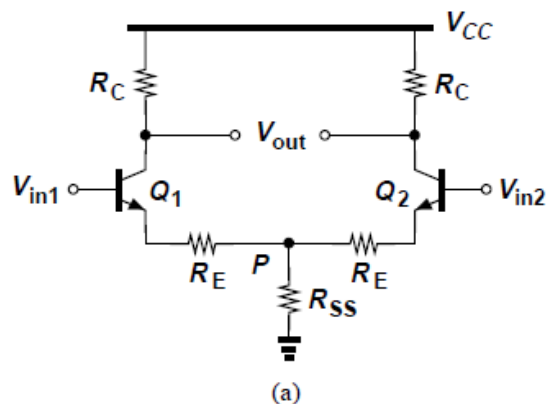


### Problem 10.33(a)

Assuming perfect symmetry and  $V_A < \infty$ , compute the differential voltage gain of each stage depicted in Fig. 10.70. You may need to compute the gain as  $A_v = G_m R_{out}$

(a) Find  $A_{dm}$ ,  $A_{dm-cm}$ ,  $A_{cm-dm}$ ,  $A_{cm}$  and  $CMRR$

(b) Find  $R_{in-dm}$ ,  $R_{in-cm}$ ,  $R_{out-dm}$ ,  $R_{out-cm}$

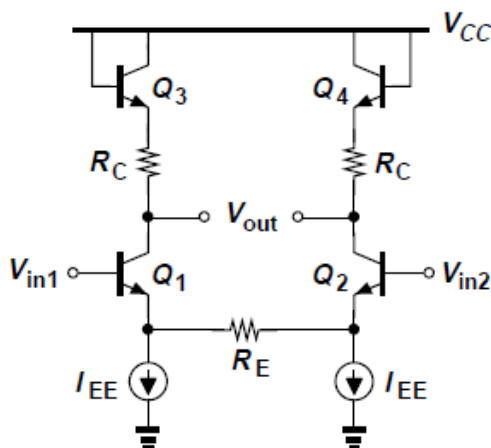


### Problem 10.34

Consider the differential pair illustrated in Fig. 10.71. Assuming perfect symmetry and  $V_A = \infty$ ,

(a) Determine the voltage gain.

(b) Under what condition does the gain become *independent* of the tail currents? This is an example of a very linear circuit because the gain does not vary with the input or output signal levels.

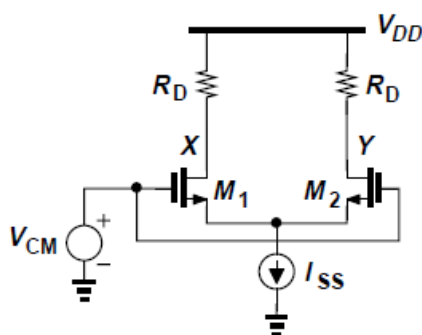


### MOS Differential Amplifier:

#### Problem 10.37

The MOS differential pair of Fig. 10.24 must be designed for an equilibrium overdrive of 200 mV. If  $\mu_n C_{ox} = 100 \mu\text{A}/\text{V}^2$  and  $W/L = 20/0.18$ , what is the required value of  $I_{SS}$ ?

Calculate the common-mode input range assuming  $V_{\text{eff}}$  of  $I_{SS}$  is 150mV,  $V_{TH}=0.5\text{V}$ ,  $V_{DD}=2\text{V}$ , and  $R_D=5\text{k}\Omega$ . Find the linear input range and the output voltage swing.



#### Problem 10.41

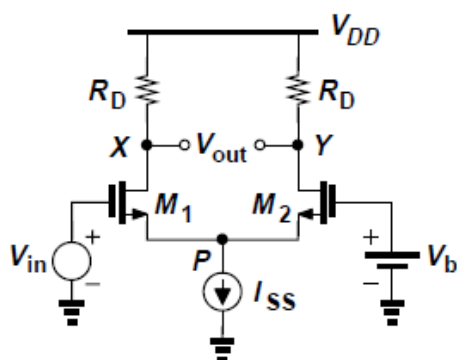
Design an NMOS differential pair for a voltage gain of 5 and a power budget of 2 mW subject to the condition that the stage following the differential pair requires an input CM level of at least 1.6 V. Assume  $\mu_n C_{ox} = 100 \mu\text{A}/\text{V}^2$ ,  $\lambda = 0$ , and  $V_{DD} = 2\text{V}$ . Formulate the trade-off between  $V_{DD}$  and  $W/L$  for a given output common-mode level.

### Problem 10.49

Explain what happens to the characteristics shown in Fig. 10.31 if (a) the gate oxide thickness of the transistor is doubled, (b) the threshold voltage is halved, (c)  $I_{SS}$  and  $W/L$  are halved.

### Problem 10.51

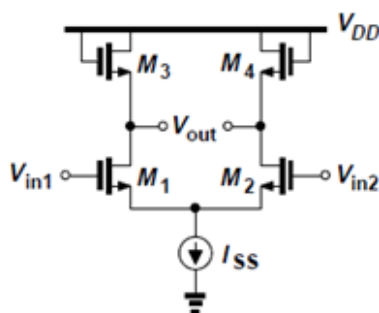
A student who has a single-ended voltage source constructs the circuit shown in Fig. 10.75, hoping to obtain differential outputs. Assume perfect symmetry but  $\lambda = 0$  for simplicity.



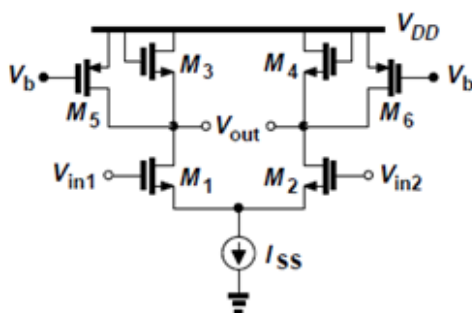
- Viewing  $M_1$  as a common-source stage degenerated by the impedance seen at the source of  $M_2$ , calculate  $v_X$  in terms of  $v_{in}$ .
- Viewing  $M_1$  as a source follower and  $M_2$  as a common-gate stage, calculate  $v_Y$  in terms of  $v_{in}$ .
- Add the results obtained in (a) and (b) with proper polarities. If the voltage gain is defined as  $(v_X - v_Y)/v_{in}$ , how does it compare with the gain of differentially-driven pairs?

### Problem 10.52(c)

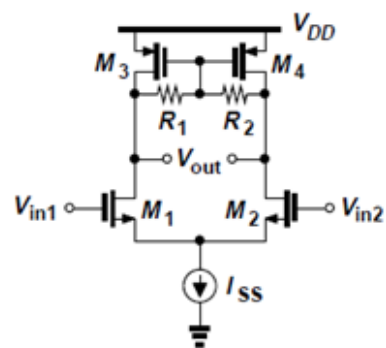
Calculate the differential voltage gain of the circuits depicted in Fig. 10.76. Assume perfect symmetry and  $\lambda > 0$ .



(a)



(b)



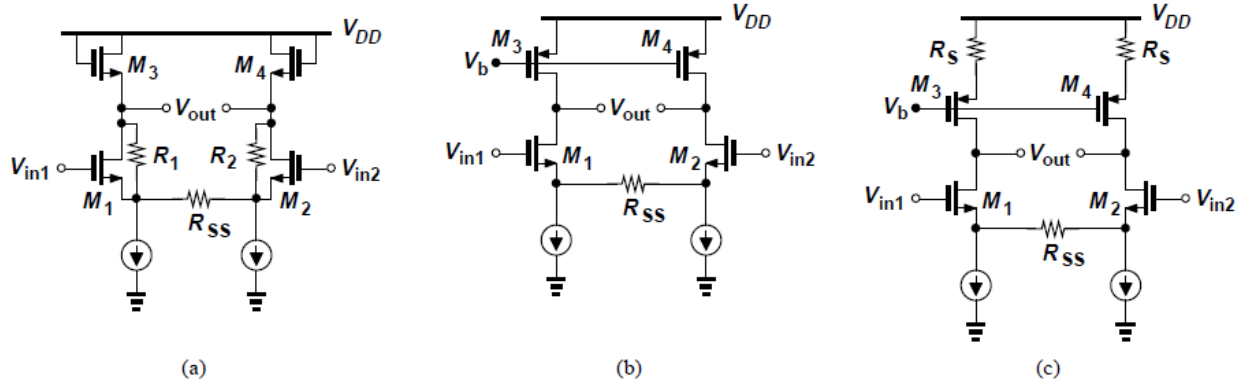
(c)

(a) Find  $A_{dm}$ ,  $A_{dm-cm}$ ,  $A_{cm-dm}$ ,  $A_{cm}$  and CMRR

(b) Find  $R_{in-dm}$ ,  $R_{in-cm}$ ,  $R_{out-dm}$ ,  $R_{out-cm}$

### Problem 10.53(b)

Calculate the differential voltage gain of the circuits depicted in Fig. 10.77. Assume perfect symmetry and  $\lambda > 0$ . You may need to compute the gain as  $A_v = G_m R_{out}$  in some cases.



### Cascode Differential Amplifiers:

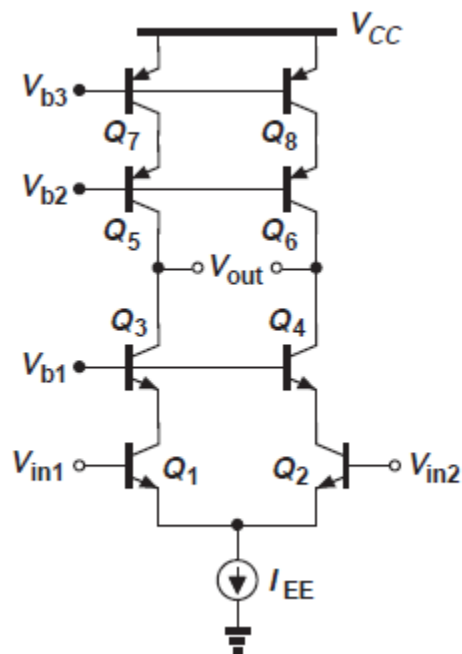
#### Problem 10.60

The cascode amplifier shown below is to operate as with a differential gain of 800. If  $Q_1$ - $Q_4$  are identical and so are  $Q_5$ - $Q_8$ , determine the minimum allowable Early voltage.

Assume  $\beta_n = 2\beta_p = 100$  and  $V_{A,n} = 2V_{A,p}$

**Hint:**

- Show that  $G_m \cong g_{m1}$
- Find  $R_{out}$  (Use the results of problem 7.4 in the Problem Set #2)

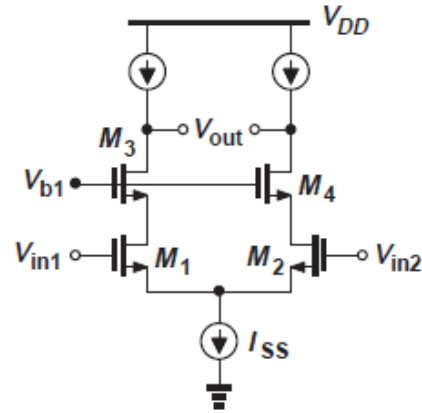


### Problem 10.62

The MOS cascode amplifier shown must provide a differential voltage gain of 300. If  $W/L = 20\mu\text{m}/180\text{nm}$  for  $M_1$ - $M_4$  and  $\mu_n C_{OX} = 100\mu\text{A}/\text{V}^2$ , determine the required tail current. Assume  $\lambda = 0.1\text{V}^{-1}$ .

**Hint:**

- Show that  $G_m \cong g_{m1}$
- Find  $R_{out}$

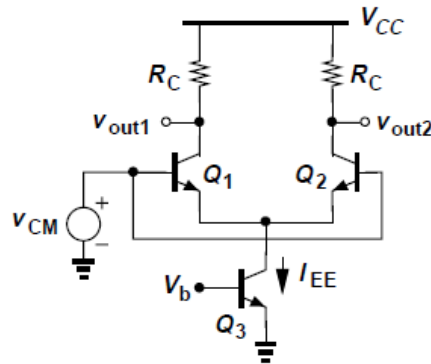


### Common-Mode Rejection:

### Problem 10.66

The bipolar differential pair depicted in Fig. 10.85 must exhibit a common-mode gain of less than 0.01. Assuming  $V_A = \infty$  for  $Q_1$  and  $Q_2$  but  $V_A < \infty$  for  $Q_3$ , prove that

$$R_C I_C < 0.02(V_A + V_T). \quad (10.218)$$



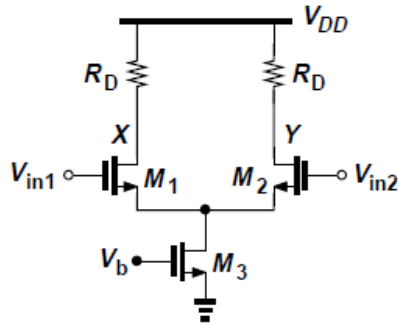
### Problem 10.67

Compute the common-mode gain of the MOS differential pair shown in Fig. 10.86. Assume  $\lambda = 0$  for  $M_1$  and  $M_2$  but  $\lambda \neq 0$  for  $M_3$ . Prove

$$A_{CM} = \frac{R_D I_{SS}}{\frac{2}{\lambda} + (V_{GS} - V_{TH})_{eq.}}, \quad (10.219)$$

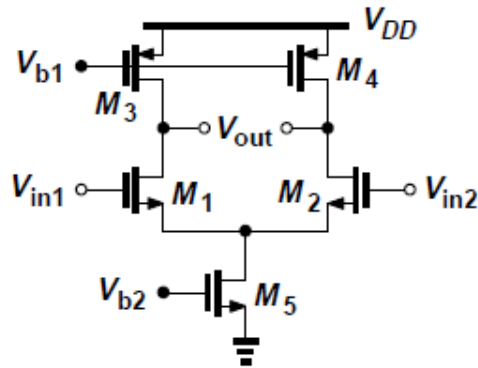
where  $(V_{GS} - V_{TH})_{eq.}$  denotes the equilibrium overdrive of  $M_1$  and  $M_2$ .





### Problem 10.68

Calculate the common-mode gain of the circuit depicted in Fig. 10.87. Assume  $\lambda > 0$ ,  $g_m r_{O} \gg 1$ , and use the relationship  $A_v = -G_m R_{out}$ .



### Problem 10.70

Compute the common-mode-rejection ratio (**CMRR**) of the differential amplifier shown. For simplicity, neglect channel-length modulation in M1 and M2 but not in M3.

