

A New On-line Observer-Based Controller for Wheeled Non-holonomic Mobile Robots

M. F. Hassan

Electrical Engineering Dept.,
Kuwait University, Kuwait
m.f.hassan47@gmail.com

E. Aljuwaiser and R. Badr

Electronics and Electrical Communication
Engineering Dept., Cairo University, Egypt
elhamalj@gmail.com

Abstract—In this paper, a new on-line observer-based controller is developed to implement trajectory tracking for wheeled non-holonomic mobile robots. The proposed approach is designed to track a user-specified trajectory which is not required to be pre-specified. A novel estimation technique is presented leading to short transient estimation period with reasonable overshoot which are hard to achieve using available estimators. The tracking and regulation functions are implemented using a two-component control strategy based on the estimated states. Applications of the proposed technique to different case studies are presented to illustrate the effectiveness of the developed approach.

Keywords—Trajectory tracking; Discrete-time nonlinear systems, Constrained system; Nonlinear state estimation.

I. INTRODUCTION

Many control design techniques have been developed in the literature to tackle trajectory tracking control problem such as Lyapunov approach [1], sliding mode control [2] and Backstepping control design [3] among others. Observer-based trajectory tracking techniques have also been investigated. Those techniques include the high-gain observer [4], sliding mode observer [5]. Most of the developed nonlinear observers require excessive computational time and some of them have either restrictive conditions to be satisfied and/or necessitate state transformation which can be either difficult to achieve or impossible. Moreover, they show large overshoot at the start of the estimation process if it is desired to have a short transient period of the estimation error.

In this paper, an on-line observer-based trajectory tracking controller is developed for wheeled non-holonomic mobile robot. The proposed approach allows the system to track the desired trajectory which is not required to be pre-defined and can even be changed on-line from one time interval to another. In order to achieve this objective a novel estimation technique with short transient estimation period and reasonable overshoot is firstly presented as techniques available in the literature cannot achieve that goal.

Using the estimated states, a new on-line observer-

based controller is generated in which the control signal is divided into two components. The tracking of the desired trajectory is guaranteed by the first control component in which the predicted states are used to generate the desired control actions. On the other hand, the second control component is designed to achieve the desired steady state position using the filtered estimate. The effectiveness of this approach is demonstrated through different case studies on the wheeled non-holonomic mobile robot.

The rest of the paper is organized as follows. Section II presents the mathematical model of the wheeled non-holonomic mobile robot. The developed observer-based controller is introduced in section III. The effectiveness and the applicability of the proposed approach are demonstrated in section IV. The paper is finally concluded in section V.

II. THE MATHEMATICAL MODEL OF THE WMR

The discrete-time mathematical model of wheeled non-holonomic mobile robot (WMR), for which the coordinates are shown in Fig. 1, is given by [6]:

$$\begin{aligned} x_{k+1} &= x_k + \Delta T w_k \cos \theta_k - \left(\Delta T^2 / 2 \right) w_k v_k \sin \theta_k \\ y_{k+1} &= y_k + \Delta T w_k \sin \theta_k + \left(\Delta T^2 / 2 \right) w_k v_k \cos \theta_k \quad (1) \\ \theta_{k+1} &= \theta_k + \Delta T v_k \\ \xi_{k+1} &= [x_{k+1} \quad \theta_{k+1}]^T \end{aligned}$$

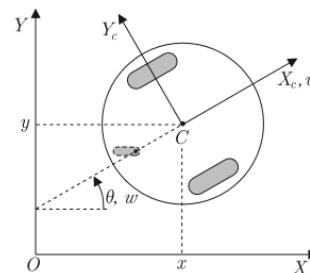


Fig.1 Coordinate system of the WMR.

where x_k, y_k, θ_k are, respectively, the positions and orientation of the center of the axis of the wheels C with respect to a global inertial frame $\{O, X, Y\}$,

$\mathbf{u}_k = [w_k \ v_k]^T$ is the control input, ξ_{k+1} is the output measurement vector, ΔT is the sampling period and $k \in \{0, 1, \dots\}$ is the discrete time.

The objective of this problem is to control the robot so that it follows a specified trajectory and at the end returns back to its parking area.

III. DESIGN OF THE ON-LINE OBSERVER-BASED CONSTRAINED CONTROLLER

Let us consider the following constrained nonlinear discrete-time system:

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k) \quad (2)$$

$$\xi_{k+1} = \mathbf{h}_{k+1}(\mathbf{x}_{k+1})$$

$$\text{Subject to either } \mathbf{G}(\mathbf{x}_k) = \bar{\mathbf{E}}_k \quad (3\text{-a})$$

$$\text{OR } \underline{\omega}_{k+1} \leq \mathbf{G}(\mathbf{x}_{k+1}) \leq \bar{\omega}_{k+1} \quad (3\text{-b})$$

where $\mathbf{x}_k \in \mathbf{R}^n$ is the state vector, $\mathbf{u}_k \in \mathbf{R}^l$ is the control vector, $\xi_k \in \mathbf{R}^r$ is the measured output vector, $\mathbf{f}_k(\mathbf{x}_k): \mathbf{R}^n \rightarrow \mathbf{R}^n$ is a vector of nonlinear functions of the state equation, $\mathbf{h}_{k+1}(\mathbf{x}_{k+1}): \mathbf{R}^n \rightarrow \mathbf{R}^r$ is a vector of nonlinear functions of the output measurements, $\mathbf{G}(\mathbf{x}_k): \mathbf{R}^n \rightarrow \mathbf{R}^m$ is a set of linear or nonlinear functions of the states, m is the number of constraints, $\bar{\mathbf{E}}_k \in \mathbf{R}^m$ is time dependent linear or nonlinear functions to be satisfied by (3-a) and $\underline{\omega}_{k+1}, \bar{\omega}_{k+1} \in \mathbf{R}^m$ are the lower or upper bounds to be satisfied by (3-b). The vector $\mathbf{x}_0 = \mathbf{x}_{k_0} \in \mathbf{R}^n$ represents the initial conditions. The vector functions \mathbf{f}_k and \mathbf{h}_{k+1} are assumed to be smooth of their arguments and differentiable.

Our objective is to design a control signal \mathbf{u}_k to satisfy the set of imposed constraints (3) and reach the desired steady state at the end of the process.

A. The Nonlinear Constrained State Estimator:

In this section, we address the state estimation problem for discrete-time nonlinear dynamical systems. Our objective is to reach zero steady state error between the actual and the estimated states as fast as possible while avoiding large overshoot at the start of the estimation process. For the nonlinear system (2), (3), knowing the estimate $\hat{\mathbf{x}}_{k|k} \in \mathbf{R}^n$, it is desired to estimate the state vector $\hat{\mathbf{x}}_{k+1|k+1}$ as we receive the measurements ξ_{k+1} . To achieve a fast convergence of the estimation error to the zero steady state we impose the following constraint:

$$\underline{\varepsilon} \leq \xi_{k+1} - \hat{\xi}_{k+1|k+1} \leq \bar{\varepsilon} \quad (4)$$

where $\hat{\xi}_{k+1|k+1} \in \mathbf{R}^r$ is the estimated output and

$\underline{\varepsilon} \in \mathbf{R}^r, \bar{\varepsilon} \in \mathbf{R}^r$ are, respectively, the lower and upper bounds of the output estimation errors element by element.

The proposed estimator $\hat{\mathbf{x}}_{k+1|k+1}$ is realized using the following two phase estimator:

Phase I: The Nonlinear State Estimator:

- The predicted estimate of the state vector $\hat{\mathbf{x}}_{k+1|k}$ and the output vector $\hat{\mathbf{y}}_{k+1|k}$ are given by:

$$\hat{\mathbf{x}}_{k+1|k} = \mathbf{f}_k(\hat{\mathbf{x}}_{k|k}, \mathbf{u}_k) \quad (5)$$

$$\hat{\xi}_{k+1|k} = \mathbf{h}_{k+1}(\hat{\mathbf{x}}_{k+1|k})$$

- The filtered estimate of the state vector $\hat{\mathbf{x}}_{k+1|k+1}$ and the corresponding estimated output vector $\hat{\xi}_{k+1|k+1}$ are given by:

$$\hat{\mathbf{x}}_{k+1|k+1} = \hat{\mathbf{x}}_{k+1|k} + \mathbf{L}_{k+1} [\xi_{k+1} - \hat{\xi}_{k+1|k}] \quad (6)$$

$$\hat{\xi}_{k+1|k+1} = \mathbf{h}(\hat{\mathbf{x}}_{k+1|k+1})$$

where, $\mathbf{L}_{k+1} \in \mathbf{R}^{n \times r}$ is the observer gain. To avoid large overshoot, \mathbf{L}_{k+1} is chosen such that the poles of the matrix $\mathbf{A}_{ck} = (\mathbf{I} - \mathbf{L}_{k+1} \hat{\mathbf{C}}_{k+1}) \hat{\mathbf{A}}_k$ are close to those of the matrix $\hat{\mathbf{A}}_k$ at each sampling instant, where:

$$\hat{\mathbf{A}}_k = \left. \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}_k} \right|_{\hat{\mathbf{x}}_{k|k}}, \hat{\mathbf{C}}_{k+1} = \left. \frac{\partial \mathbf{h}_{k+1}}{\partial \mathbf{x}_{k+1}} \right|_{\hat{\mathbf{x}}_{k+1|k}} \quad (7)$$

Phase II: The Updated Estimator

In this phase, if the estimated outputs satisfy (4), we proceed to the next sampling instant of time. On the other hand, if a subset of the imposed constraint (4) is violated, we define a new estimated state vector $\hat{\mathbf{x}}_{k+1|k+1}^i$ which at the start of the update procedure is initialized by:

$$\hat{\mathbf{x}}_{k+1|k+1}^0 = \hat{\mathbf{x}}_{k+1|k+1} \quad (8)$$

The subset of the estimated outputs violating the imposed constraints has to be identified and saturated to the violated lower or the upper bounds. Let:

$$\mathbf{z}_{k+1} = \mathbf{h}'_{k+1}(\mathbf{x}_{k+1}) \quad (9)$$

where $\mathbf{z}_{k+1} \in \mathbf{R}^{\hat{h}}$ is the subset of the violated outputs, \hat{h} is the number of the violated constraints, $\mathbf{h}'_{k+1}(\mathbf{x}_{k+1}) \in \mathbf{R}^{\hat{h}}$ are the corresponding nonlinear measurement functions.

The subset of the violated constraints (9) is treated as a new received subset of the measurement and used to update the estimator iteratively as follows:

$$\hat{\mathbf{x}}_{k+1|k+1}^i = \hat{\mathbf{x}}_{k+1|k+1}^{i-1} + \mathbf{H}_{k+1} [\mathbf{z}_{k+1} - \hat{\mathbf{z}}_{k+1|k+1}^{i-1}] \quad (10)$$

$$\hat{\mathbf{z}}_{k+1|k+1}^{i-1} = \mathbf{h}'_{k+1}(\hat{\mathbf{x}}_{k+1|k+1}^{i-1}) \quad (11)$$

where $\mathbf{H}_{k+1} \in \mathbf{R}^{n \times \hat{h}}$ is a gain matrix to be specified, and i is the iteration number. The corresponding estimation error $\tilde{\mathbf{x}}_{k+1|k+1}^i = \mathbf{x}_{k+1} - \hat{\mathbf{x}}_{k+1|k+1}^i$ is given by:

$$\tilde{\mathbf{x}}_{k+1|k+1}^i = (\mathbf{I} - \mathbf{H}_{k+1} \mathbf{D}_{k+1}) \tilde{\mathbf{x}}_{k+1|k+1}^{i-1} \quad (12)$$

where \mathbf{D}_{k+1} is the Jacobean matrix of \mathbf{h}'_{k+1} with respect to \mathbf{x}_{k+1} .

To guarantee the stability of the error vector $\tilde{\mathbf{x}}_{k+1|k+1}^i$, the gain matrix \mathbf{H}_{k+1} is chosen such that the magnitude of the eigenvalues of the matrix $[(\mathbf{I} - \mathbf{H}_{k+1} \mathbf{D}_{k+1}) \mathbf{A}_{ck}]$ are less than one.

B. Control Design

The feedback control signal which satisfies our design objectives is assumed to take the form:

$$\mathbf{u}_k = \mathbf{u}_{k-1} + \Delta \mathbf{u}_{k-1}^1 + \Delta \mathbf{u}_k^2 \quad (13)$$

where \mathbf{u}_k^1 is the stabilizing control signal to be generated using the gain scheduling approach, whereas \mathbf{u}_k^2 is the required control action to satisfy the imposed constraints (3).

1) Gain Scheduling Control

To design the gain scheduling component \mathbf{u}_k^1 , let:

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \Delta \mathbf{x}_k \quad (14)$$

$$\mathbf{u}_k^1 = \mathbf{u}_{k-1} + \Delta \mathbf{u}_{k-1}^1 \quad (15)$$

Assume that $\mathbf{x}_k, \mathbf{u}_k$ are close to $\mathbf{x}_{k-1}, \mathbf{u}_{k-1}$ so that we can replace $\mathbf{f}_k(\mathbf{x}_{k-1} + \Delta \mathbf{x}_{k-1}, \mathbf{u}_{k-1} + \Delta \mathbf{u}_{k-1}^1)$ by its first order approximation. Then (2) reduces to:

$$\mathbf{x}_k + \Delta \mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \mathbf{u}_{k-1}) + \mathbf{A}_{k-1} \Delta \mathbf{x}_{k-1} + \mathbf{B}_{k-1} \Delta \mathbf{u}_{k-1}^1 \quad (16)$$

where \mathbf{A}_{k-1} and \mathbf{B}_{k-1} are the Jacobian matrices of \mathbf{f}_k with respect to \mathbf{x}_{k-1} and \mathbf{u}_{k-1} , respectively.

Since $\mathbf{x}_k = \mathbf{f}_k(\mathbf{x}_{k-1}, \mathbf{u}_{k-1})$, we pose the following infinite regulator problem to be solved at each sampling point:

$$\min J = \frac{1}{2} \sum_{k=0}^{\infty} (\|\Delta \mathbf{x}_k\|_{\hat{\mathbf{Q}}}^2 + \|\Delta \mathbf{u}_{k-1}^1\|_{\hat{\mathbf{R}}}^2) \quad (17)$$

$$\text{s.t. } \Delta \mathbf{x}_k = \mathbf{A}_{k-1} \Delta \mathbf{x}_{k-1} + \mathbf{B}_{k-1} \Delta \mathbf{u}_{k-1}^1 \quad (18)$$

where $\hat{\mathbf{Q}} \in \mathbf{R}^{n \times n}$ and $\hat{\mathbf{R}} \in \mathbf{R}^{l \times l}$ are respectively a positive semi-definite and a positive definite weighting matrices.

Under the assumptions that \mathbf{A}_{k-1} is bounded for all $\mathbf{x}_{k-1} \in \Omega$ (where Ω is a nonempty domain in the neighborhood of the desired steady state of the system \mathbf{x}_{ss}), the pair $(\mathbf{A}_k, \mathbf{B}_k)$ is controllable, and the pair

$(\hat{\mathbf{Q}}^{1/2}, \mathbf{A}_{k-1})$ is detectable, the minimization of (17) subject to (18) leads to:

$$\Delta \mathbf{u}_{k-1}^1 = -\mathbf{T}_{k-1} \Delta \mathbf{x}_{k-1} \quad (19)$$

where,

$$\mathbf{T}_{k-1} = \hat{\mathbf{R}}^{-1} \mathbf{B}_{k-1}^T \left[\pi_{k-1}^{-1} + \mathbf{B}_{k-1} \hat{\mathbf{R}}^{-1} \mathbf{B}_{k-1}^T \right]^{-1} \mathbf{A}_{k-1} \quad (20)$$

and π_{k-1} is the solution of the algebraic Riccati equation given by:

$$\pi_{k-1} = \hat{\mathbf{Q}} + \mathbf{A}_{k-1}^T \left[\pi_{k-1}^{-1} + \mathbf{B}_{k-1} \hat{\mathbf{R}}^{-1} \mathbf{B}_{k-1}^T \right]^{-1} \mathbf{A}_{k-1} \quad (21)$$

2) Corrective Control Action

Each type of the imposed set of constraints, namely, (the equality and inequality constraints) is treated differently. We start with the equality constraints.

Corrective Control for the Equality Constraints

Starting with \mathbf{x}_k which satisfies the set of equality constraints, let us assume for the moment that we can get an estimate for \mathbf{x}_{k+1} . If the set or a subset of the imposed constraints (3-a) is violated, we pose the following optimization problem to be solved:

$$\min. J = \frac{1}{2} \|\tilde{\mathbf{x}}_{k+1} - \mathbf{x}_{k+1}\|_W^2 + \frac{1}{2} \|\Delta \mathbf{u}_k^2\|_M^2 \quad (22)$$

$$\text{s.t. } \bar{\mathbf{G}}(\tilde{\mathbf{x}}_{k+1}) = \bar{\mathbf{E}}_{k+1} \quad (23)$$

where $W \in \mathbf{R}^{n \times n}$ and $M \in \mathbf{R}^{l \times l}$ are positive definite weighting matrices, and $\tilde{\mathbf{x}}_{k+1} \in \mathbf{R}^n$ is the desired state vector to be adjusted by the control vector $\Delta \mathbf{u}_k^2$ to satisfy (23), $\bar{\mathbf{G}}(\mathbf{x}_k) : \mathbf{R}^n \rightarrow \mathbf{R}^j$ is the subset of the j violated constraints and $\bar{\mathbf{E}}_k \in \mathbf{R}^j$ are the corresponding violated time functions.

Let $\tilde{\mathbf{x}}_{k+1} = \mathbf{x}_{k+1} + \Delta \mathbf{x}_{k+1}$ where $\Delta \mathbf{x}_{k+1}$ is the corrective component to be given by:

$$\Delta \mathbf{x}_{k+1} = \mathbf{B}_{k-1} \Delta \mathbf{u}_k^2 \quad (24)$$

from which:

$$\tilde{\mathbf{x}}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{B}_{k-1} \Delta \mathbf{u}_k^2 \quad (24)$$

The satisfied subset of constraints is then given by:

$$\bar{\mathbf{G}}(\mathbf{x}_{k+1} + \Delta \mathbf{x}_{k+1}) = \bar{\mathbf{E}}_{k+1} \quad (25)$$

By assuming that the vector function $\bar{\mathbf{G}}(\tilde{\mathbf{x}}_{k+1})$ is affine in the neighborhood of $\tilde{\mathbf{x}}_{k+1}, \mathbf{x}_{k+1}$, we get:

$$\bar{\mathbf{G}}(\mathbf{x}_{k+1}) + \bar{\mathbf{D}}_{k+1} \Delta \mathbf{x}_{k+1} = \bar{\mathbf{E}}_{k+1} \quad (26)$$

where $\bar{\mathbf{D}}_{k+1}$ is the Jacobian matrix of $\bar{\mathbf{G}}_{k+1}$ with respect to \mathbf{x}_{k+1} .

Therefore, the optimization problem to be solved is to minimize (22) with respect to $\Delta \mathbf{u}_k^2$ subject to (26) while taking into consideration (24). This leads to:

$$\Delta \mathbf{u}_k^2 = \mathfrak{R}^{-1} \mathbf{B}_{k-1}^T \bar{\mathbf{D}}_{k+1}^T \left(\bar{\mathbf{D}}_{k+1} \mathbf{B}_{k-1} \mathfrak{R}^{-1} \mathbf{B}_{k-1}^T \bar{\mathbf{D}}_{k+1}^T \right)^{-1} * (\bar{\mathbf{E}}_{k+1} - \bar{\mathbf{G}}(\mathbf{x}_{k+1})) \quad (27)$$

where $\mathfrak{R} = \mathbf{B}_{k-1}^T \mathbf{W} \mathbf{B}_{k-1} + \mathbf{M}$

Corrective Control for the Inequality Constraints

In this situation, if the state vector \mathbf{x}_{k+1} satisfies the imposed set of inequality constraints, we let $\Delta \mathbf{u}_k^2 = 0$ and we proceed to the next sampling instant of time. Otherwise, if a subset of q -constraints of the vector $\mathbf{G}(\mathbf{x}_{k+1})$ violates the imposed lower or upper bounds, this subset has to be identified and saturated to the lower or the upper bounds which have been violated. Let us define the vector $\mathbf{m}(\mathbf{x}_{k+1}) : \mathbf{R}^n \rightarrow \mathbf{R}^q$ to be the vector function including the elements of $\mathbf{G}(\mathbf{x}_{k+1})$ corresponding to the violated constraints, $\mathbf{E}_{k+1} \in \mathbf{R}^q$ contains the elements of the lower or the upper bounds which have been violated.

Using the same procedure as above, we get:

$$\Delta \mathbf{u}_k^2 = \mathfrak{R}^{-1} \mathbf{B}_{k-1}^T \bar{\mathbf{D}}_{k+1}^T \left(\bar{\mathbf{D}}_{k+1} \mathbf{B}_{k-1} \mathfrak{R}^{-1} \mathbf{B}_{k-1}^T \bar{\mathbf{D}}_{k+1}^T \right)^{-1} * (\mathbf{E}_{k+1} - \mathbf{m}(\mathbf{x}_{k+1})) \quad (28)$$

where $\bar{\mathbf{D}}_{k+1}$ is the Jacobian matrix of \mathbf{m}_{k+1} with respect to \mathbf{x}_{k+1} .

C. Control Implementation

In this subsection, we discuss the implementation of the proposed controller in real application. The following different cases may take place:

Case 1: The system (2) is required to be regulated while satisfying an imposed set of inequality constraints. In this case, the two control signals $\Delta \mathbf{u}_k^1$, $\Delta \mathbf{u}_k^2$ as given by (19) and (28) have to be applied at the same sampling instant of time at which a subset of the imposed inequality constraints is violated. Therefore, the control signal is given by:

$$\mathbf{u}_k = \mathbf{u}_{k-1} - \hat{\mathbf{T}}_{k-1} \Delta \hat{\mathbf{x}}_{k-1|k-1} + \Delta \mathbf{u}_k^2$$

At sampling instants at which the inequality constraints are satisfied, only the gain scheduling control signal $\Delta \mathbf{u}_k^1$ has to be applied. Hence, the control signal is such that:

$$\mathbf{u}_k = \mathbf{u}_{k-1} - \hat{\mathbf{T}}_{k-1} \Delta \hat{\mathbf{x}}_{k-1|k-1}$$

Case 2: In this case, it is desired for the system (2) to satisfy the set of imposed equality constraints. In this case, the only control signal to be applied is $\Delta \mathbf{u}_k^2$ as given by (27). Therefore, the control signal is given by: $\mathbf{u}_k = \mathbf{u}_{k-1} + \Delta \mathbf{u}_k^2$

D. Design of the On-line Observer-Based Controller

Since the states of the system are not available for direct measurements, the estimated states will be used to calculate the Jacobian matrices resulting from system linearization as well as the incremental change in the state vector $\Delta \mathbf{x}_{k-1}$.

Therefore, $\mathbf{A}_{k-1} = \mathbf{A}_{k-1}(\mathbf{x}_{k-1})$, $\mathbf{B}_{k-1} = \mathbf{B}_{k-1}(\mathbf{x}_{k-1})$, $\boldsymbol{\pi}_{k-1}$, \mathbf{T}_{k-1} , $\bar{\mathbf{G}}(\mathbf{x}_{k+1})$, $\bar{\mathbf{D}}_{k+1}$, $\Delta \mathbf{x}_{k-1}$, $\Delta \mathbf{x}_k$ and \mathbf{x}_{k+1} will be replaced by $\hat{\mathbf{A}}_{k-1} = \hat{\mathbf{A}}_{k-1}(\hat{\mathbf{x}}_{k-1|k-1})$, $\hat{\mathbf{B}}_{k-1} = \hat{\mathbf{B}}_{k-1}(\hat{\mathbf{x}}_{k-1|k-1})$, $\hat{\boldsymbol{\pi}}_{k-1}$, $\hat{\mathbf{T}}_{k-1}$, $\hat{\bar{\mathbf{G}}}(\hat{\mathbf{x}}_{k+1|k})$, $\hat{\bar{\mathbf{D}}}_{k+1}$, $\Delta \hat{\mathbf{x}}_{k-1|k-1}$, $\Delta \hat{\mathbf{x}}_{k|k}$ and $\hat{\mathbf{x}}_{k+1|k}$ respectively in (14)-(28). Also, $\tilde{\mathbf{x}}_{k+1}$ will be replaced by $\hat{\tilde{\mathbf{x}}}_{k+1|k} = \hat{\mathbf{x}}_{k+1|k} + \Delta \hat{\mathbf{x}}_{k+1|k}$.

Accordingly, the control signals $\Delta \mathbf{u}_k^1, \mathbf{u}_k^2$ are such as:

$$\Delta \mathbf{u}_{k-1}^1 = -\hat{\mathbf{T}}_{k-1} \Delta \hat{\mathbf{x}}_{k-1|k-1} \quad (29-a)$$

$$\Delta \mathbf{u}_k^2 = \hat{\mathfrak{R}}^{-1} \hat{\mathbf{B}}_k^T \hat{\boldsymbol{\Lambda}}_{k+1}^T \left(\hat{\boldsymbol{\Lambda}}_{k+1} \hat{\mathbf{B}}_{k-1} \hat{\mathfrak{R}}^{-1} \hat{\mathbf{B}}_{k-1}^T \hat{\boldsymbol{\Lambda}}_{k+1}^T \right)^{-1} \hat{\boldsymbol{\eta}}_{k+1} \quad (29-b)$$

where $\hat{\boldsymbol{\Lambda}}_{k+1} = \begin{cases} \hat{\bar{\mathbf{D}}}_{k+1} & \text{If (3-a) is applicable} \\ \hat{\bar{\mathbf{D}}}_{k+1} & \text{If (3-b) is applicable} \end{cases}$

$\hat{\boldsymbol{\eta}}_{k+1} = \begin{cases} \hat{\bar{\mathbf{E}}}_{k+1} - \hat{\bar{\mathbf{G}}}(\hat{\mathbf{x}}_{k+1|k}) & \text{If (3-a) is applicable.} \\ \mathbf{E}_{k+1} - \mathbf{m}(\hat{\mathbf{x}}_{k+1|k}) & \text{If (3-b) is applicable.} \end{cases}$

IV. SIMULATION RESULTS AND DISCUSSION

The developed on-line observer-based controller is applied to control the behavior of the wheeled non-holonomic mobile robot described in section II. The initial condition of the system is $\mathbf{x}_0 = [0 \ 0 \ 0]^T$, while that of the estimator is $\hat{\mathbf{x}}_{0|0} = [1 \ 1 \ 0]^T$. The weighting matrices of the cost function (17) are $\hat{\mathbf{Q}} = \mathbf{I}_3$, $\hat{\mathbf{R}} = \mathbf{I}_2$ while that of the cost function (22) are $\mathbf{W} = \text{diag}[100 \ 100 \ 100]$ and $\mathbf{M} = \text{diag}[100 \ 100]$.

Case study 1: In this case, it is desired for the robot to carry out the following tasks:

a) The robot will move from the starting position at \mathbf{x}_0 , tracking the circular spiral path described by:

$$\begin{aligned} x_k &= 2e^{0.5k} \cos(0.4\pi k) \\ y_k &= 2e^{0.5k} \sin(0.4\pi k) \end{aligned} \quad (30)$$

b) At a certain instant of time, the robot will arrive to a point on the peripheral of the ellipse given by:

$$\begin{aligned} x_k &= 5.5 \cos(0.4\pi k) \\ y_k &= 3 \sin(0.4\pi k) \end{aligned} \quad (31)$$

Once that happens, the robot will track the elliptical trajectory instead of the spiral.

c) After 60 sec., the robot returns back to the point $(x_0, y_0) = (0, 0)$ while tracking the following elliptical spiral path:

$$\begin{aligned} x_k &= 5.5e^{-0.2k} \cos(0.4\pi k) \\ y_k &= 3e^{-0.2k} \sin(0.4\pi k) \end{aligned} \quad (32)$$

d) Finally, the robot is required to be of a certain alignment in its parking area which is $\theta_{k_f} = 0$. Therefore, it rotates in its parking position until it achieves this alignment.

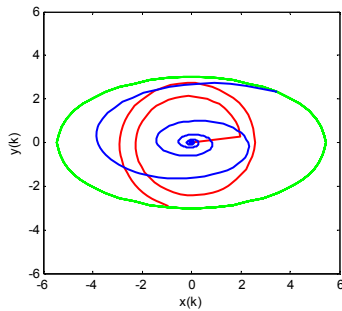


Fig. 2 The trajectory in the X-Y plane (Case study 1).

In this case study, the corrective control action is the only required control component to accomplish tasks (a), (b) and (c) while the gain scheduling control component is used to align the robot in its parking position. Figure 2 shows the path of the robot in the X-Y plane. In this figure, the robot moves on the circular path (red trajectory). Then the robot moves on the elliptical path (green trajectory) for 20 sec. Finally, the robot tracks the elliptical spiral path (blue line) till it reaches the point (0,0) in the X-Y plane and then, it rotates to achieve a certain alignment in its parking position. The average computational time per sample was found to be 1.2×10^{-4} sec. which is much less than the sampling period (0.01 sec.).

Case study 2: The robot is required to fulfill the tasks (a) and (b) which are the same as for case study 1. After 60 sec. it is desired for the robot to return back directly to its starting position x_0 without following any pre-specified trajectory. However, it was observed that the robot trajectory exceeded the elliptical area during the final parts of its trip to reach the steady state position. Assuming that for security reasons, it is desired to restrict the robot to move only inside the elliptical area. In this case, the following inequality constraint is imposed after 60 sec.:

$$(x / 5.5)^2 + (y / 3)^2 \leq 1 \quad (33)$$

To handle the added inequality constraint (33), the gain scheduling component and the corrective control signals are applied simultaneously. Again, using the same legend as in case study 1, Fig. 3 show the trajectory in the X-Y plane. The average

computational time required to execute the proposed controller is 1.75×10^{-4} sec per sample.

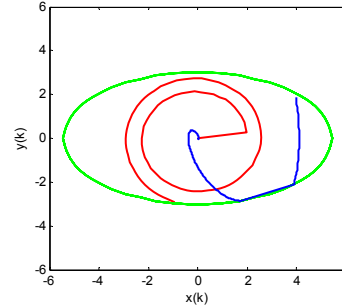


Fig. 3 The trajectory in the X-Y plane (Case study 2).

V. CONCLUSION

In this paper, a new on-line observer-based controller is developed and successfully applied to implement trajectory tracking for wheeled non-holonomic mobile robots. This technique does not require pre-specified trajectories. It applies a two-component control strategy to fulfill the assigned tracking and regulation tasks while using the estimated states resulting from a new proposed estimator. Simulation results of different case studies are presented to show the fulfillment of the design objectives with very reasonable computational times.

REFERENCES

- [1] R. Fareh, M. Saad, S. Khadraoui and T. Rabie, "Lyapunov-Based Tracking Control for Nonholonomic Wheeled Mobile Robot", International Journal of Electrical, Computer, Energetic, Electronic and Communication Engineering, vol.10, pp. 973-978, 2016.
- [2] Z. Zhao, J. Yang, S. Li, Z. Zhang and L. Guo, "Finite-time super-twisting sliding mode control for Mars entry trajectory tracking" Journal of the Franklin Institute, vol.352, pp. 5226-5248, 2015.
- [3] K. Djamel, M. Abdellah, and A. Benallegue, "Attitude Optimal Backstepping Controller Based Quaternion for a UAV", Mathematical Problems in Engineering, (2016)
- [4] C. Sun, G. Hu and L. Xie, "Robust consensus tracking for a class of high-order multi-agent systems", International Journal of Robust and Nonlinear Control, vol.26, pp. 578-598, 2016.
- [5] M. Cui, W. Liu, H. Liu, H. Jiang, and Z. Wang, "Extended state observer-based adaptive sliding mode control of differential-driving mobile robot with uncertainties", Nonlinear Dynamics, vol.83, pp. 667-683, 2016.
- [6] A. Loria, J. Dasdemir and N.A. Jarquin, "Leader-Follower Formation and Tracking Control of Mobile Robots Along Straight Paths", IEEE Transactions on Control Systems Technology, vol.24, pp. 727-732, 2016.