AER 633
Analysis of Composite Materials and Structures

Lecture 1: Introduction

24/9/2019

Course Objectives:

- Learning the fundamental characteristics and properties of continuous-fibers composite materials.
- Learning the basic analysis of plate structures made of composite materials by investigating three types of problems (Bending, vibration, and Buckling).
- The ability to develop basic computer codes to analyze composite plate structures.
- The ability to read and understand published scientific articles in the field of composite structures.

Text books: (*Downloadable from pdfdrive.com*)

1- R. M. Jones ,“ Mechanics of Composite Materials”, 2nd edition, CRC Press, 1998.

2- J. N. Reddy ,“ Mechanics of Laminated Composite Plate and Shells: Theory and Analysis”, 2nd edition, CRC Press, 2003.

Topics Covered by the Course:

1. Micromechaincs of continuous-fiber composites.
2. Analysis of composite plate structures using the Classical Laminated Theory (CLT).
3. Analysis of composite plate structures using the First-Order Shear Deformation Theory (FSDT).
4. Analysis of woven composites.
5. Analysis of sandwich plates.
6. Failure of composite materials.
7. Nanocomposites.

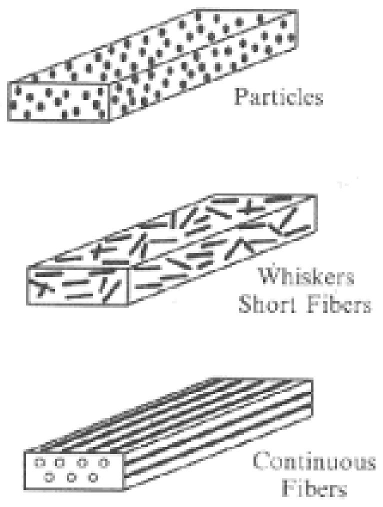
Grading and Assessment Policy:

- Assignments (Computer Codes) 40%
- Midterm-Exam 20%
- Final Exam 40%

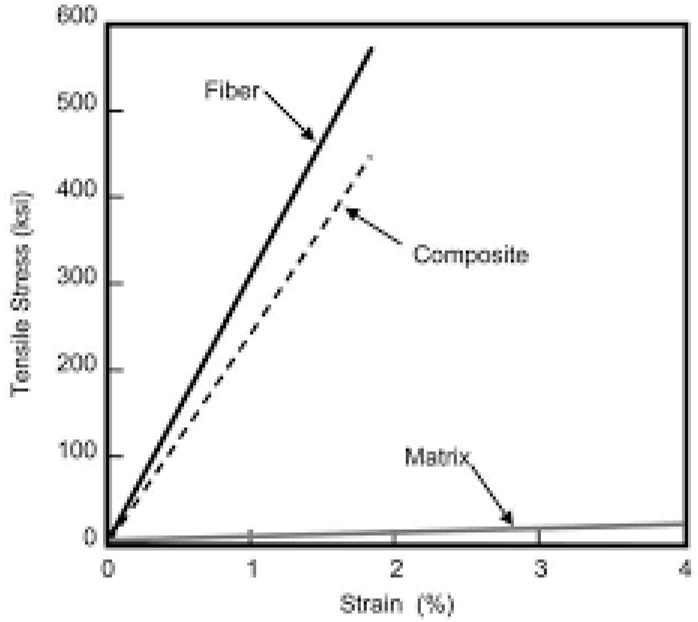
Important Note: Final exam requires the possession of a laptop.

Definition of a Composite Material:

- A composite material is typically composed of two materials (Constituents) ; One material is called "***Matrix***" which can be metal or polymer, while the other material is called "***Reinforcement***" which is usually a type of ceramics.
- The Matrix material function is to act as a bonding element to bond the reinforcements and to transfer the applied load to them, and in the case of metal matrix, it also carries a part of the applied loads.
- The Reinforcement's function is to improve the overall properties of the composite materials (e.g. stiffness, strength, toughness, fatigue, etc) which can be continuous or discontinuous (Particles and whiskers).



Constitutive Equations of Unidirectional Continuous-Fiber Composites:



For a linear elastic material : $\sigma_{ij} = C_{ijkl}\,\varepsilon_{kl}$ $\quad i,j = 1,2,3$

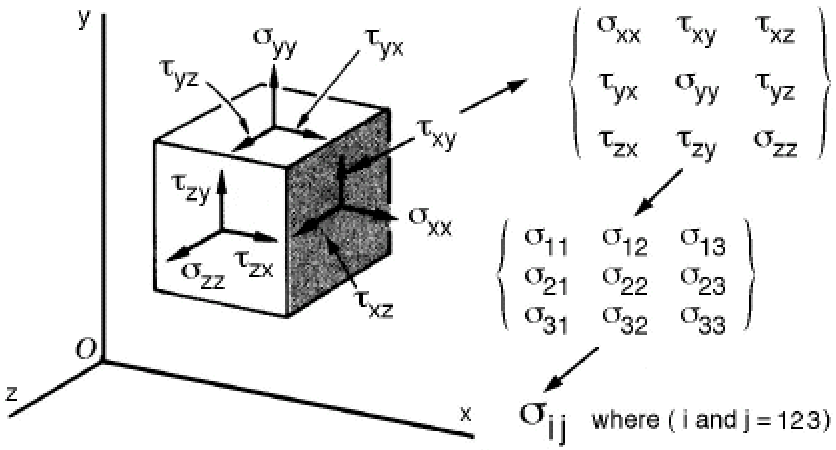
σ_{ij} : Stress Components.
ε_{kl} : Strain Components.
C_{ijkl} : Stiffness Coefficients. (81 Coefficients)

Energy density function (*W*):

$$W = \int \sigma_{ij}\, d\varepsilon_{ij}$$

$$\therefore\ \sigma_{ij} = \frac{\partial W}{\partial \varepsilon_{ij}} \quad \rightarrow \therefore\ C_{ijkl} = \frac{\partial^2 W}{\partial \varepsilon_{ij}\partial \varepsilon_{kl}} \rightarrow \therefore\ C_{ijkl} = C_{klij}$$

(Reduces the independent Coefficients to 45)



Taking advantage of the symmetry of the stress and strain components:

$$C_{ijkl} = C_{jikl} \quad \text{and}\ C_{ijkl} = C_{ijlk}$$

Reduces the coefficients to 21 independent coefficients, so the stress-strain relation can be written in a matrix form as follows:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} C_{1111} & & & & & \\ C_{2211} & C_{2222} & & & & \\ C_{3311} & C_{3322} & C_{3333} & & & \\ C_{2311} & C_{2322} & C_{2333} & C_{2323} & & \\ C_{1311} & C_{1322} & C_{1333} & C_{1323} & C_{1313} & \\ C_{1211} & C_{1222} & C_{1233} & C_{1223} & C_{1213} & C_{1212} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{Bmatrix}$$

Engineering Notation (Voigt Notation):

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{Bmatrix} \rightarrow \begin{Bmatrix} \sigma_{1} \\ \sigma_{2} \\ \sigma_{3} \\ \sigma_{4} \\ \sigma_{5} \\ \sigma_{6} \end{Bmatrix} \quad \text{and} \quad \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ 2\varepsilon_{23} \\ 2\varepsilon_{13} \\ 2\varepsilon_{12} \end{Bmatrix} \rightarrow \begin{Bmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \varepsilon_{3} \\ \varepsilon_{4} \\ \varepsilon_{5} \\ \varepsilon_{6} \end{Bmatrix}$$

The Constitutive relations become (Triclinic):

$$\begin{Bmatrix} \sigma_{1} \\ \sigma_{2} \\ \sigma_{3} \\ \sigma_{4} \\ \sigma_{5} \\ \sigma_{6} \end{Bmatrix} = \begin{bmatrix} C_{11} & & & & & \\ C_{21} & C_{22} & & & & \\ C_{31} & C_{32} & C_{33} & & & \\ C_{41} & C_{42} & C_{43} & C_{44} & & \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{1} \\ \varepsilon_{2} \\ \varepsilon_{3} \\ \varepsilon_{4} \\ \varepsilon_{5} \\ \varepsilon_{6} \end{Bmatrix}$$

Monoclinic Material (one plane of symmetry):

For 1-2 plane of symmetry:

$$\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_3\\\sigma_4\\\sigma_5\\\sigma_6\end{Bmatrix}=\begin{bmatrix}C_{11} & & & & & \\ C_{21} & C_{22} & & & & \\ C_{31} & C_{32} & C_{33} & & & \\ 0 & 0 & 0 & C_{44} & & \\ 0 & 0 & 0 & C_{54} & C_{55} & \\ C_{61} & C_{62} & C_{63} & 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_3\\\varepsilon_4\\\varepsilon_5\\\varepsilon_6\end{Bmatrix}$$

(Three planes of symmetry) (Orthotropic):

$$\begin{Bmatrix}\sigma_1\\\sigma_2\\\sigma_3\\\sigma_4\\\sigma_5\\\sigma_6\end{Bmatrix}=\begin{bmatrix}C_{11} & & & & & \\ C_{21} & C_{22} & & & & \\ C_{31} & C_{32} & C_{33} & & & \\ 0 & 0 & 0 & C_{44} & & \\ 0 & 0 & 0 & 0 & C_{55} & \\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\\varepsilon_2\\\varepsilon_3\\\varepsilon_4\\\varepsilon_5\\\varepsilon_6\end{Bmatrix}$$

Transversely isotropic material about axis-1:

$$\begin{Bmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\\ \sigma_4\\ \sigma_5\\ \sigma_6\end{Bmatrix}=\begin{bmatrix}C_{11} & & & & & \\ C_{21} & C_{22} & & & & \\ C_{21} & C_{32} & C_{22} & & & \\ 0 & 0 & 0 & (C_{22}-C_{23})/2 & & \\ 0 & 0 & 0 & 0 & C_{66} & \\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\\ \varepsilon_4\\ \varepsilon_5\\ \varepsilon_6\end{Bmatrix}$$

Which is the constitutive equations form for a unidirectional composite material with the 1-axis being aligned along the fibers directions.

Coordinate Transformation of the Stiffness Coefficients

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{zy} \\ \sigma_{zx} \\ \sigma_{xy} \end{Bmatrix} = \underbrace{\begin{bmatrix} c\theta^2 & s\theta^2 & 0 & 0 & 0 & -s2\theta \\ s\theta^2 & c\theta^2 & 0 & 0 & 0 & s2\theta \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c\theta & s\theta & 0 \\ 0 & 0 & 0 & -s\theta & c\theta & 0 \\ s\theta c\theta & -s\theta c\theta & 0 & 0 & 0 & c\theta^2 - s\theta^2 \end{bmatrix}}_{[T]} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix}$$

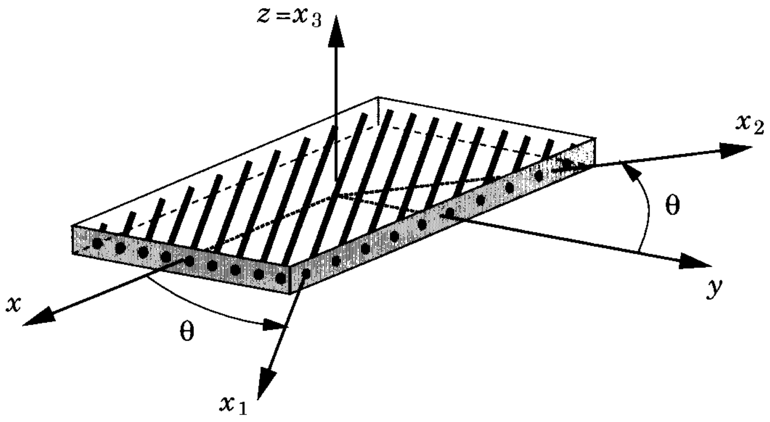
Note:

$c\theta = cos\theta$

$s\theta = sin\theta$

Strain components are transformed in a similar way, so the stiffness coefficients in the transformed axes will be as follows:

$$[\bar{C}] = [T][C][T]^T$$



Detailed expression for $[\bar{C}]$ can be found on pages 96-97 in Ref[2]

End of Lecture 1

Home Work

-Please Read pages 89-99 in Ref [2], and prove that $C_{44} = \frac{C_{22}-C_{23}}{2}$ for transversely isotropic material about axis-1.

-Define the following terms: Homogenous Material, non-homogenous material, anisotropic, orthotropic, transversely isotropic, and isotropic material.