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Citation: Journal of Applied Physics 116, 024308 (2014); doi: 10.1063/1.4886596
View online: http://dx.doi.org/10.1063/1.4886596
View Table of Contents: http://scitation.aip.org/content/aip/journal/jap/116/2?ver=pdfcov
Published by the AIP Publishing
Analytical modeling of the radial p-n junction nanowire solar cells

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(Received 16 May 2014; accepted 18 June 2014; published online 11 July 2014)

In photovoltaic solar cells, radial p-n junctions have been considered a very promising structure to improve the carrier collection efficiency and accordingly the conversion efficiency. In the present study, the semiconductor equations, namely Poisson’s and continuity equations for a cylindrical p-n junction solar cell, have been solved analytically. The analytical model is based on Green’s function theory to calculate the current density, open circuit voltage, fill factor, and conversion efficiency. The model has been used to simulate p-n and p-i-n silicon radial solar cells. The validity and accuracy of the present simulator were confirmed through a comparison with previously published experimental and numerical reports. © 2014 AIP Publishing LLC.

[http://dx.doi.org/10.1063/1.4886596]

I. INTRODUCTION

Recently, radial structures have attracted great interest to enhance the solar energy conversion. Planar junction solar cells must have a minority carrier diffusion length long enough to allow for effective collection of photo-generated carriers over the entire depth of light absorption. For the materials that have low diffusion lengths relative to the optical thickness, radial p-n junction nanorod geometry with radius in nano-size has the potential to improve the cell efficiency due to the independence between carrier transport and light absorption directions.

In silicon solar cells, improvement in External Quantum Efficiency (EQE) and absorption efficiency was observed experimentally when using Si Nanowire Solar Cells (NW SCs).¹ Additionally, when the performance of this NW SCs has been calculated using numerical simulations and compared to conventional planar p-i-n thin-film Si solar cells, NW SCs achieve nearly 80% efficiency enhancement.²

Besides, Cd (Se, Te) nanorod arrays were observed to have improved fill factors relative to their planar counterparts. Furthermore, the spectral response data indicate that the nanorod array electrodes behave as if they had a much longer minority carrier collection length than planar electrodes that is made using identical materials fabrication processes.³ It was shown by numerical simulation that the device design using nanowire CdS can yield a substantial (∼25%) enhancement in the power conversion efficiency of this cell.³

It is worth mentioning that during the year 2013 several studies were done concerning the vertically aligned NW radial p-n and p-i-n junctions. For instance, Si NWs were investigated using the commercially available well known packages, namely COMSOL multiphysics and 3-D TCAD numerical simulator. COMSOL has been used to optimize the length and the doping level of the Si-NWs array.⁵ The 3-D TCAD numerical simulator has been utilized to study the performance of the 10 µm, in long, crystalline amorphous silicon core-shell (c-Si/a-Si/AZO/Glass) NWs solar cells that reached photogenerated current up to 22.94 mA/cm² and conversion efficiency of 13.95%.⁶

Furthermore, silicon radial p-n and p-i-n junction NWs arrays were experimentally fabricated by low temperature epitaxial growth process using silane-based chemical vapor deposition and they achieved solar energy conversion efficiency of 10% under AM 1.5 G illumination.⁷ Additionally, Si-NWs arrays with excellent light trapping property were fabricated by metal-assisted chemical etching technique and achieved a short circuit current density of 37.13.⁸ Concerning the NWs contacts, wrap-around top Ag contacts for radial junction Si-NWs solar cells were fabricated and obtained a fill factor of 65.4%, which is higher than that of conventional top contacts.⁹

Although many numerical and experimental comparative studies were performed to prove the enhancement achieved when applying nanorod geometry in organic cells and inorganic cells, analytical studies were rarely discussed.¹⁰⁻¹² In 2005, Kayes and Atwater showed analytically that extremely large efficiency gains from 1.5% to 11% are possible by applying the radial p-n junction nanorod geometry.¹³

Even though the analytical model for cell parameters is not enough to know the detailed parameters at each point inside the cell such as carrier concentrations and current density, it is a sufficient and a quick tool to calculate the important parameters of the photovoltaics (PV) solar cells such as Isc, Voc, FF, and efficiency.

Using Green’s function in the present model reduces the need for uniform generation assumption used in conventional analyses.¹⁴ It depends on calculating the Green’s function

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that satisfies the required equations and boundaries but with point source, then the actual generation source can be applied to get a general solution.

II. THEORY/CALCULATION

In our analysis, we considered a radial structure with p-type core and n-type outer shell, as shown in Figure 1. The following assumptions are considered in the analysis: (1) the ratio between the rod length and diameter is very high, (2) recombination of carriers is due to bulk traps and surface traps, (3) the doping is low and uniform, (4) carrier transport is only in the radial direction, and (5) incident light is normal to the top surface with zero reflectance. The energy band diagram at thermal equilibrium is shown in Figure 2. The solar cell total current, I, takes the form

$$I = I_{\text{dark}}(V) - I_{\text{sc}},$$  \hspace{1cm} (1)

where $I_{\text{sc}}$ is the short circuit current and $I_{\text{dark}}(V)$ is the voltage dependent dark current.

The continuity equation for electrons in the p-type region takes the form

$$\frac{\partial n}{\partial t} = G_L - U_n + \frac{1}{q} (\nabla \cdot J_e),$$  \hspace{1cm} (2)

where $G_L = \int \alpha F_o \exp(-xz) \, d\lambda$ represents the generation rate of carriers due to incident light assuming zero reflectance of light at the upper surface, where $x$ is the absorption coefficient, $F_o$ is the incident flux density. $U_n = \frac{q}{\tau_n}$ represents the net recombination rate of the minority carriers in the quasi neutral region assuming low-level of injection. $\Delta n$ is the excess carriers’ concentration due to illumination and $\tau_n$ is the electron recombination time. $J_e = qD_e \nabla n$, represents the electron current density assuming diffusion only and $D_e$ is the electron diffusion constant.

Using cylindrical coordinates and assuming no dependence on $\phi$ and $z$, Eq. (2) for excess electrons in the p-type region at steady state can be written in the form

$$\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{L_e^2} \right) \Delta n = -g_{01}. $$  \hspace{1cm} (3)

Here, $L_e = \sqrt{\frac{\tau_n D_e}{q}}$ is the electron diffusion length and $g_{01} = \frac{G_L}{D_e}$. It is worthy to mention that Eq. (3) represents an inhomogeneous Helmholtz equation.

Similarly, the continuity equation for excess holes in the n-type region can be written as

$$\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{L_h^2} \right) \Delta p = -g_{02}, $$  \hspace{1cm} (4)

where $g_{02} = \frac{G_L}{D_h}$ and $D_h$ is the hole diffusion constant.

On the other hand, the voltage dependant parameters $\Delta r_1$, $\Delta r_2$, $\Delta r_3$, and $r_4$ are calculated by solving Poisson’s equation in all regions, as stated in Appendix A.

A. Dark current calculations

$$I_{\text{dark}} = I_{\text{de}} + I_{\text{sh}} + I_r,$$  \hspace{1cm} (5)

$$I_{\text{dark}} = \left( I_{o}^{e} + I_{o}^{h} \right) \left( \exp \left( \frac{V}{V_i} \right) - 1 \right) + I_{\text{dep}}^{e} \left( \exp \left( \frac{V}{2V_i} \right) - 1 \right),$$  \hspace{1cm} (6)

where $V_i$ is the thermal potential, $I_{o}^{e}$, $I_{o}^{h}$ are the electrons and holes reverse saturation current, respectively, and $I_{\text{dep}}^{e}$ is the depletion region recombination current. It is approximately defined, from the planar structure, \footnote{The depletion region recombination current is} as

$$I_{\text{dep}}^{e} = \frac{q n_i 2 \pi (\Delta r_3 + \Delta r_2)^2 L}{(2 \sqrt{\tau_n \tau_p})}.$$  \hspace{1cm} (7)

The boundary conditions for $\Delta n$ and $\Delta p$ are

FIG. 1. The structure of radial p-n junction, where $L$ is the nanorod length, $R_0$ is the radius of the metallurgical junction, $R$ is the total radius of the rod, and $R_n = R - R_p$.
\[ \Delta n \mid_{r=0} \text{is finite, and } \Delta n \mid_{r=R} = \Delta n \mid_{r=r_a} = n_{po} \left( \exp \left( \frac{V}{V_t} \right) - 1 \right), \quad (8) \]

\[ \Delta \rho \mid_{r=R} = \Delta n \mid_{r=R} = n_{po} \left( \exp \left( \frac{V}{V_t} \right) - 1 \right) \]

\[ D_h \frac{\partial}{\partial r} (\Delta \rho) \bigg|_{r=r} = -S_p \Delta \rho \bigg|_{r=R}. \quad (9) \]

Here, \( S_p \) is the holes surface recombination velocity, \( n_{po} \) and \( p_{po} \) are the minority carrier concentrations at thermal equilibrium, and \( \Delta n \) and \( \Delta \rho \) represent the excess carrier concentrations due to the bias only.

Solving the homogeneous equations (3) and (4) with \( g_{01} = 0 \) and \( g_{02} = 0 \), we get expressions for the concentration and the current density as follows:

\[ \Delta n = n_{po} \left( \exp \left( \frac{V}{V_t} \right) - 1 \right) \left( \frac{I_0(r)}{I_0(L_n)} \right), \quad (10) \]

where \( I_0 \) is the modified Bessel function of order zero of the first kind.

\[ J^d_d = qD_e \frac{\partial \Delta n}{\partial r} \bigg|_{r=r_a}, \quad (11) \]

\[ A = \begin{cases} 
K_{-1}(\beta_3) - \frac{S_L P K_0(\beta_3)}{D_h} \\
L_{-1}(\beta_3)K_0(\beta_2) + K_{-1}(\beta_3)Io(\beta_2) + \frac{S_L P K_0(\beta_2)}{D_h}(Io(\beta_3)K_0(\beta_2) - Io(\beta_2)K_0(\beta_3)) 
\end{cases} \quad (17) \]

\[ B = \frac{1}{K_0(\beta_3)} - \frac{Al(\beta_2)}{K_0(\beta_2)}. \quad (18) \]

**B. Short circuit current calculations**

\[ I_{sc} = \int (I^e_{sc}(\lambda) + I^h_{sc}(\lambda) + I^{lep}_{sc}(\lambda)) d\lambda, \quad (19) \]

where \( I^{lep}_{sc}(\lambda) \) is the contribution of the depletion region to the light-generated current. It was calculated by assuming that all absorbed photons in the depletion region generate carriers that are all collected.\(^{13}\)

\[ I^{lep}_{sc}(\lambda) = qF_0(\lambda)(1 - \exp(-\lambda L_2))((R_p + \Delta r_2)^2 - r^2_1). \quad (20) \]

To get \( I^e_{sc} \) and \( I^h_{sc} \), we have to solve the continuity equations (3) and (4), at \( V = 0 \).

Using the Green’s function theory, we defined a Green’s function which satisfies the same equation, but from a point source located at \( r' \), so the equation becomes homogeneous when \( r \neq r' \).

So, the equations in the p-type region would be as follows\(^{17}\) (see Appendix B):

\[ F^e = \int J^e_d r d\varphi dz = 2\pi r_d \frac{L}{L_n} \left( qD_e n_{po} \left( \exp \left( \frac{V}{V_t} \right) - 1 \right) I_1(\beta_1) \right). \quad (12) \]

where \( I_{-1} \) is the modified Bessel function of order \(-1\) of the first kind.

\[ \Delta \rho = p_{po} \left[ \exp \left( \frac{V}{V_t} \right) - 1 \right] [Al(\beta_2) - Bk(\beta_2)]. \quad (13) \]

With \( \beta_1 = \frac{r^4}{L_n}, \quad \beta_2 = \frac{R_p + \Delta r_2}{L_p}, \quad \beta_3 = \frac{R}{L_p}, \quad (16) \]

\[ \begin{aligned}
\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{L_n^2} \right) G_1(r,r') &= \frac{1}{r} \delta (r-r') \\
\Delta n &= \int r'(-g_{01})G_1(r,r') dr' \\
&+ r' \Delta n(r') \left|_{surface} \right. - r' G_1 \left|_{surface} \right. \frac{\partial \Delta n(r')}{\partial r'}.
\end{aligned} \quad (21) \]

According to the Dirichlet boundary conditions (B.C) \( \Delta n \mid_{r=r_a} = 0 \), where \( \Delta n \) represents the excess carrier concentrations due to illumination only with zero bias, the Green’s function can derived as shown in Appendix C

\[ G_1(r,r') = \frac{K_0(\beta_1)}{Io(\beta_1)} \times \left[ \frac{Io(r < L_n)}{L_n} - \frac{Io(\beta_1)}{K_0(\beta_1)} \right]. \quad (23) \]

Here \( r < \) and \( r > \) are the smaller and larger of \( r \) and \( r' \).
\[
\Delta n(r, \lambda) = -g_0 L_n \left[ r I_{1}(r) \left( \frac{r}{L_n} \right) - \frac{I_0(\beta_1)}{K_0(\beta_1)} K_1(\frac{r}{L_n}) \right] + \left\{ r_4 I_0 \left( \frac{r}{L_n} \right) \left( I_1(\beta_1) + \frac{I_0(\beta_1)}{K_0(\beta_1)} K_1(\beta_1) \right) \right\} \nonumber
\]

\[
- \left\{ r I_0 \left( \frac{r}{L_n} \right) \left( I_1(\beta_1) + \frac{I_0(\beta_1)}{K_0(\beta_1)} K_1(\frac{r}{L_n}) \right) \right\}
\]

\[
\left( r_4 I_1(\beta_1) \frac{K_0(\beta_1)}{K_0(\beta_1)} I_{-1}(\beta_1) \right) + \left( r_4 I_1(\beta_1) K_{-1}(\beta_1) \right)
\]

(24)

On the other hand, the boundary conditions in the n-type region would be:

\[
G_2(r, r') = \text{const} \times \xi_1(r < \xi_2(r >)
\]

\[
= \text{const} \left( I_0 \left( \frac{r < \xi_2}{L_p} \right) + c_1 K_0 \left( \frac{r > \xi_2}{L_p} \right) \right)
\]

\[
\times \left( I_0 \left( \frac{r > \xi_2}{L_p} \right) c_2 K_0 \left( \frac{r > \xi_2}{L_p} \right) \right),
\]

(26)

\[
\Delta p(r, \lambda) = \left( -g_0 \right) \int_{R_p+\Delta r_2}^{R_p} r' G_2(r, r') dr' + \Delta p(R) \left( 1 + \frac{R}{R_p} G_2(r, r') \right)|_{r=R},
\]

(27)

\[
I_{sc}^P = -q D\pi 2 \pi (R_p + \Delta r_2)(F_1 + F_2).
\]

(28)

Here,

\[
F_1 = -F_0 (1 - e^{-2L}) \text{const} (l_{-1}(\beta_2)
\]

\[
- c_1 K_{-1}(\beta_2) (R I_1(\beta_2) - c_2 K_1(\beta_2))
\]

\[
- (R_p + \Delta r_2)(I_1(\beta_2) - c_2 K_1(\beta_2))),
\]

(29)

\[
F_2 = \Delta p(R) \frac{S_p}{L_p} D_n \text{const} (l_{-1}(\beta_2) - c_1 K_{-1}(\beta_2))
\]

\[
\times (I_0(\beta_3) + c_2 K_0(\beta_3)),
\]

(30)

\[
F_{3R} = \int_{R_p}^{R_p} r'(-F_0 (1 - e^{-2L})) G_2(r, r') dr' \quad |_{r=R}
\]

\[
= \text{const} \times L_p (-F_0 (1 - e^{-2L})) (I_0(\beta_3)
\]

\[
+ c_2 K_0(\beta_3)) (R I_1(\beta_2) - c_2 K_1(\beta_2))
\]

\[
- (R_p + \Delta r_2)(I_1(\beta_2) - c_2 K_1(\beta_2))).
\]

(35)

From the above analysis, we calculate the short circuit current, the I–V relation, the open-circuit voltage \(V_{oc}\), the fill factor \(FF = \frac{P_{max}}{V_{oc} I_{sc}}\), and the conversion efficiency \(\eta = \frac{P_{max}}{P_{in}}\), where \(P_{in}\) is the overall incident power density.

### III. RESULTS AND DISCUSSION

To check the validity of our analysis, a comparison was done between our results and previously published works. The parameters of Si are listed in Table I, when the cell is illuminated by AM1.5G standard spectrum of ASTM G-173 global tilt solar spectrum. In all comparisons, we defined the error as [our analysis—published \(\times 100\%\)

<table>
<thead>
<tr>
<th>T (K)</th>
<th>E_g (eV)</th>
<th>(1.17 - (4.73 \times 10^{-4}) \times T^2/(T + 636)) (Ref. 20)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_i (cm(^{-3}))</td>
<td>(10^{17})</td>
<td>(6.2 \times (10^{15}) \times T^2)</td>
</tr>
<tr>
<td>N_i (cm(^{-3}))</td>
<td>(10^{17})</td>
<td>(3.5 \times (10^{15}) \times T^4)</td>
</tr>
<tr>
<td>(\nu_r)</td>
<td>11.7</td>
<td>(\mu_r (cm^2/Vs) (D_r = V_r \mu_r))</td>
</tr>
<tr>
<td>(\nu_i (cm^{-3}))</td>
<td>(10^{10})</td>
<td>(\mu_i (cm^2/Vs) (D_i = V_i \mu_i))</td>
</tr>
</tbody>
</table>

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The present analysis was compared with previously reported experimental study of Si-NWs. In that study, the nanowire radius equals 100 nm with n-type shell thickness of 10 nm and length of 1 μm. The used doping concentrations $N_d$ and $N_a$ were equal to $2.68 \times 10^{20} \text{cm}^{-3}$ and $1.5 \times 10^{16} \text{cm}^{-3}$, respectively, under AM1.5 G illumination. As shown in Figure 3, the analysis achieves good agreement, with an error of $J_{sc}$ and $V_{oc}$ equal 3.7% and 1.7%, respectively. Basically, the calculations assume ideal solar cell with zero series resistance, infinite shunt resistance, and zero reflection that surely have other non-ideal values in such experimental work. On the other hand, the non-ideal parameters were assumed; series resistance equals $R_s = 20 \Omega$, shunt resistance equals $R_{sh} = 20 \Omega$, surface recombination velocity equals $S_p = 3 \times 10^3 \text{cm/s}$, and reflectivity of normal incident light between air/SiO$_2$ $R_{\text{ref}} = \left(\frac{n_2-n_1}{n_2+n_1}\right)^2$. In this case, error of $J_{sc}$ decreased to 0.03% and that of $V_{oc}$ decreased from to 1.3%.

Another comparison was accomplished with previously published numerical study of radial Si p-i-n junction that used a simulator built on COMSOL Multiphysics. We used total radius, p-type radius, intrinsic thickness, and n-type thickness equal 120 nm, 80 nm, 20 nm, and 20 nm, respectively. The length of the NW equals the diffusion length, and the doping concentration equals $10^{18} \text{cm}^{-3}$. The J-V characteristic under dark conditions for our analytical work and the previously published numerical work is shown in Figure 4. A good match is achieved between both results. Figure 5 shows the J-V characteristic under AM1.5 G illumination with a good match between our results and the published one. The difference in $J_{sc}$ and $V_{oc}$ is calculated to be 4.1% and 0.5%, respectively.

IV. CONCLUSION

In the present study, an analytical model for the radial p-n junction nanowires solar cells was successfully established. We solved analytically the semiconductor equations, namely Poisson’s equation and the continuity equations for nanorods p-n junction PV solar cells. This analytical solution is implemented using Green’s function theory that helps in decreasing the dependence on many assumptions used in previous conventional studies.

Afterwards, a detailed study for Si-NWs solar cells was presented in a good agreement with previously published experimental and numerical results.

A detailed study of different nanorods materials under different conditions of operation could take place that reflects the advantage of our analytical models to characterize any radial p-n junction solar cell. In further work, this study can be extended using different ways of contacts/semiconductor connections on the advances in fabrication of such nanorods PV cells.

APPENDIX A: SOLVING POISSON’S EQUATION

Assuming homojunction case, the Poisson’s equation will be as follows:

$$\nabla^2 \psi = -\frac{\rho}{\varepsilon}, \quad (A1)$$
where \( \psi \) is the potential, \( \rho \) is the charge density in each region, and \( \varepsilon \) is the permittivity of the material of the rod.

At \( r < r < R_p \) (in the p-type depletion region),

\[
\nabla^2 \psi = -\frac{q(-N_0)}{\varepsilon}.
\]

(A2)

So, we obtain

\[
\psi_p = q \frac{N_0}{2\varepsilon} \left( \frac{R_p^2 - r^2}{2} - r^2 \ln \frac{R_p}{r_4} \right).
\]

Similarly in n-type region, we obtain

\[
\psi_n = q \frac{N_0}{2\varepsilon} \left( \frac{R_p^2 - (R_p + \Delta r)^2}{2} - (R_p + \Delta r)^2 \ln \frac{R_p + \Delta r}{r_4} \right).
\]

(A4)

Then, we can get the radii from the relation

\[
\psi_n = \psi_p + \psi_n = V_b - V,
\]

where \( V_b \) is the built-in potential which, for non-degenerate semiconductors, equals \( V_b \ln \frac{N_0N_v}{n_0^2} \).

APPENDIX B: DERIVING THE CONCENTRATION EXPRESSION FROM GREEN’S FUNCTION THEORY

\[
\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{L_n^2} \right) G_1(r, r') = \frac{1}{r} \delta(r - r'),
\]

(B1)

\[
\left( \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{L_n^2} \right) \Delta n = -g_{o1},
\]

(B2)

\[
\frac{\partial}{\partial r} r \frac{\partial}{\partial r} \Delta n + \frac{r'}{L_n} \Delta n = -r' g_{o1}.
\]

(B3)

Multiply Eq. (B3) by \( G_1(r, r') \), then integrate w.r.t. \( dr' \)

\[
\int dr' G_1(r, r') \frac{\partial}{\partial r'} \Delta n(r') - \int dr' r' \frac{\partial}{\partial r'} G_1(r, r') \frac{\Delta n(r')}{L_n^2} = \int dr' r' (-g_{o1}) G_1(r, r').
\]

(B4)

Integrate the first term by parts, we get

\[
G_1(r, r') \frac{r \partial}{\partial r} \Delta n(r') \big|_{\text{surface}} - \int dr' \left( -r' \frac{\partial}{\partial r'} \Delta n(r') \right) \frac{G_1(r, r')}{L_n^2},
\]

(B5)

\[
G_1(r, r') \frac{r \partial}{\partial r} \Delta n(r') \big|_{\text{surface}} - \left( \frac{r' \partial G_1(r, r')}{\partial r'} \right) \Delta n(r') \big|_{\text{surface}}
+ \int \Delta n(r') \frac{\partial}{\partial r'} G_1(r, r') \, dr'.
\]

(B6)

Substitute the first term from Eq. (B6) into Eq. (B4), we get

\[
\int \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{L_n^2} \Delta n(r') \, dr' = \frac{\Delta n(r)}{L_n}
\]

\[
= \int dr' r' \left( -g_{o1} \right) G_1(r, r') - G_1(r, r') r' \frac{\partial \Delta n(r')}{\partial r'} \big|_{\text{surface}}
+ r' \frac{\partial G_1(r, r')}{\partial r'} \Delta n(r') \big|_{\text{surface}}.
\]

(B7)

APPENDIX C: DERIVATION OF GREEN’S FUNCTION IN P-TYPE REGION

Assuming that \( G_1(r, r') \) is the green’s function that satisfies the same equations and boundaries of \( \Delta n(r) \), but with point source located at \( r' \).

\[
\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{L_n^2} \right) G_1(r, r') = \frac{1}{r} \delta(r - r').
\]

(C1)

So, when \( r \neq r' \), the equation will be homogeneous, and it will be at the same form of modified Bessel function equation of order zero.

Suppose that \( \zeta_1 (x) \) is some linear combination of \( Io \) and \( K0 \), which satisfy the boundary conditions for \( r < r' \), and suppose that \( \zeta_2 (x) \) is another linearly independent combination for \( r > r' \).

The symmetry of Green’s function in \( r \) and \( r' \) requires that

\[
G_1(r, r') = \zeta_1 \left( r < \frac{L_n}{\ln L_n} \right) \zeta_2 \left( r > \frac{L_n}{\ln L_n} \right).
\]

(C2)

Although \( G_1 \) is continuous at \( r = r' \), but the derivative is not. So, to get the discontinuity of the slope, integrate Eq. (C1) with respect to \( r \) from \( (r = r' - \epsilon) \) to \( (r = r' + \epsilon) \) with \( \epsilon \) represents a very small positive value.

\[
\frac{dG_1}{dr} \bigg|_{r' + \epsilon} - \frac{dG_1}{dr} \bigg|_{r' - \epsilon} = \frac{1}{r'},
\]

(C3)

Also, we can get the slope discontinuity from Eq. (C2) as follows:

\[
\frac{dG_1}{dr} \bigg|_{r' + \epsilon} - \frac{dG_1}{dr} \bigg|_{r' - \epsilon} = \frac{1}{\ln L_n} \left( \zeta_1 \zeta_2' - \zeta_1' \zeta_2 \right) \bigg|_{r = r'} = \frac{1}{\ln L_n} W[\zeta_1, \zeta_2],
\]

(C4)

where \( \zeta_i (x) = \frac{d\zeta_i}{dx} \) and \( W[\zeta_1, \zeta_2] \) is the Wronskian of \( \zeta_1 (x) \) and \( \zeta_2 (x) \).

From the boundary conditions at the p-type region of \( 0 < r < R_p \), \( G_1 \) should be as follows:

\[
G_1(r, r') = \text{const} \left\{ \text{lo}(r < L_n) \int \frac{1}{L_n} \left\{ \text{lo}(r > L_n) \frac{L_n}{K_0(b_1)} K_0 \left( \frac{r'}{L_n} \right) \right\} \right\}.
\]

(C5)

Then, we get the constant value from the slope discontinuity, and the fact that Wronskian term is proportional to \( (1/x) \) for all values of \( x \), so, we can use the limiting forms of modified Bessel function for small and large \( x \).
\[ W[I_m(x), K_m(x)] = -\frac{1}{x}, \quad (C6) \]
\[ \text{const} = \frac{K_0(\beta_1)}{I_0(\beta_1)}. \quad (C7) \]