Lecture 10
Unbalanced Fault Analysis

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Single Line-to-Ground (SLG) Faults

- Unbalanced faults unbalance the network, but only at the fault location. This causes a coupling of the sequence networks. How the sequence networks are coupled depends upon the fault type. We’ll derive these relationships for several common faults.
- With a SLG fault only one phase has non-zero fault current -- we’ll assume it is phase A.
Then since
\[
\begin{bmatrix}
I_f^0 \\
I_f^+ \\
I_f^-
\end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} ? \\ 0 \\ 0 \end{bmatrix} \rightarrow I_f^0 = I_f^+ = I_f^- = \frac{1}{3} I_a^f
\]

This means
\[
V_a^f = Z_f I_a^f
\]
\[
\begin{bmatrix}
V_a^f \\
V_b^f \\
V_c^f
\end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_f^0 \\ V_f^+ \\ V_f^- \end{bmatrix}
\]

This means \( V_a^f = V_f^0 + V_f^+ + V_f^- \)

The only way these two constraints can be satisfied is by coupling the sequence networks in series.
With the sequence networks in series we can solve for the fault currents (assume $Z_f=0$)

\[ I^+_f = \frac{1.05 \angle 0^\circ}{j(0.1389 + 0.1456 + 0.25 + 3Z_f)} = -j1.964 = I^-_f = I^0_f \]

\[ I = A \mathbf{I}_s \rightarrow I^f_a = -j5.8 \text{ (of course, } I^f_b = I^f_c = 0) \]

**Line-to-Line (LL) Faults**

- The second most common fault is line-to-line, which occurs when two of the conductors come in contact with each other. With out loss of generality we'll assume phases b and c.

Current Relationships: $I^f_a = 0$, $I^f_b = -I^f_c$, $I^0_f = 0$

Voltage Relationships: $V_{bg} = V_{cg}$
Using the current relationships we get

\[
\begin{bmatrix}
I_f^0 \\
I_f^+ \\
I_f^-
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha & \alpha^2 \\
1 & \alpha^2 & \alpha
\end{bmatrix}
\begin{bmatrix}
I_b^f \\
I_b^f \\
-I_b^f
\end{bmatrix} \rightarrow
\]

\[
I_f^0 = 0
\]

\[
I_f^+ = \frac{1}{3} I_b^f (\alpha - \alpha^2) \quad I_f^- = \frac{1}{3} I_b^f (\alpha^2 - \alpha)
\]

Hence \( I_f^+ = -I_f^- \)

Using the voltage relationships we get

\[
\begin{bmatrix}
V_f^0 \\
V_f^+ \\
V_f^-
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha & \alpha^2 \\
1 & \alpha^2 & \alpha
\end{bmatrix}
\begin{bmatrix}
V_{ag}^f \\
V_{bg}^f \\
V_{cg}^f
\end{bmatrix} \rightarrow
\]

Hence

\[
V_f^+ = \frac{1}{3} \left[ V_{ag}^f + (\alpha + \alpha^2)V_{bg}^f \right]
\]

\[
V_f^- = \frac{1}{3} \left[ V_{ag}^f + (\alpha^2 + \alpha)V_{bg}^f \right] \quad \rightarrow \quad V_f^+ = V_f^-
\]

To satisfy \( I_f^+ = -I_f^- \) & \( V_f^+ = V_f^- \)

the positive and negative sequence networks must be connected in parallel.
Solving the network for the currents we get

\[
I_f^+ = \frac{1.05 \angle 0^\circ}{j0.1389 + j0.1456} = 3.691 \angle -90^\circ
\]

\[
\begin{bmatrix}
I_a^f \\
I_b^f \\
I_c^f
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha^2 & \alpha \\
1 & \alpha & \alpha^2
\end{bmatrix}
\begin{bmatrix}
0 \\
3.691 \angle -90^\circ \\
3.691 \angle 90^\circ
\end{bmatrix}
= \begin{bmatrix}
0 \\
-6.39 \\
6.39
\end{bmatrix}
\]

Solving the network for the voltages we get

\[
V_f^+ = 1.05 \angle 0^\circ - j0.1389 \times 3.691 \angle -90^\circ = 0.537 \angle 0^\circ
\]

\[
V_f^- = -j0.1452 \times 3.691 \angle 90^\circ = 0.537 \angle 0^\circ
\]

\[
\begin{bmatrix}
V_a^f \\
V_b^f \\
V_c^f
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha^2 & \alpha \\
1 & \alpha & \alpha^2
\end{bmatrix}
\begin{bmatrix}
0 \\
0.537 \\
0.537
\end{bmatrix}
= \begin{bmatrix}
1.074 \\
-0.537 \\
-0.537
\end{bmatrix}
\]
Double Line-to-Ground Faults

With a double line-to-ground (DLG) fault two line conductors come in contact both with each other and ground. We'll assume these are phases b and c.

\[ I_a^f = 0 \]
\[ V_{bg}^f = V_{cg}^f = Z_f (I_b^f + I_c^f) \]

DLG Faults, cont'd

From the current relationships we get

\[
\begin{bmatrix}
I_a^f \\
I_b^f \\
I_c^f
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha^2 & \alpha \\
1 & \alpha & \alpha^2
\end{bmatrix}
\begin{bmatrix}
I_f^0 \\
I_f^+ \\
I_f^-
\end{bmatrix}
\]

Since \( I_a^f = 0 \) \( \rightarrow \] \( I_f^0 + I_f^+ + I_f^- = 0 \)

Note, because of the path to ground the zero sequence current is no longer zero.
From the voltage relationships we get

\[
\begin{bmatrix}
V_f^0 \\
V_f^+ \\
V_f^-
\end{bmatrix} = \frac{1}{3}
\begin{bmatrix}
1 & 1 & 1 \\
1 & \alpha & \alpha^2 \\
1 & \alpha^2 & \alpha
\end{bmatrix}
\begin{bmatrix}
V_{ag}^f \\
V_{bg}^f \\
V_{bg}^f
\end{bmatrix} \rightarrow
\]

Since \( V_{bg}^f = V_{cg}^f \) → \( V_f^+ = V_f^- \)

Then \( V_{bg}^f = V_f^0 + (\alpha^2 + \alpha)V_f^+ \)

But since \( 1 + \alpha + \alpha^2 = 0 \) → \( \alpha^2 + \alpha = -1 \)

\( V_{bg}^f = V_f^0 - V_f^+ \)

\[
V_{bg}^f = V_f^0 - V_f^+ \\
= Z_f(I_b^f + I_c^f) \\
\]

Also, since

\( I_b^f = I_f^0 + \alpha^2 I_f^+ + \alpha I_f^+ \)

\( I_c^f = I_f^0 + \alpha I_f^+ + \alpha^2 I_f^+ \)

Adding these together (with \( \alpha + \alpha^2 = -1 \))

\( V_{bg}^f = Z_f(2I_f^0 - I_f^+ - I_f^-) \)  with \( I_f^0 = -I_f^+ - I_f^- \)

\( V_f^0 - V_f^+ = 3Z_fI_f^0 \)
The three sequence networks are joined as follows

Assuming \( Z_f = 0 \), then

\[
I_f^+ = \frac{V^+}{Z^+ + Z^-} = \frac{1.05 \angle 0^\circ}{j0.1389 + j0.092}
\]

\[= 4.547 \angle -90^\circ\]

\[
V_f^+ = 1.05 - 4.547 \angle -90^\circ \times j0.1389 = 0.4184
\]

\[
I_f^0 = -I_f^+ - I_f^- = j4.547 - j2.874 = j1.673
\]

Converting to phase: \( I_b^f = -1.04 + j6.82 \)

\( I_c^f = 1.04 + j6.82 \)
**Example 9.3**

Determine pu and actual value of the current and line voltage at generator bus, when a SLG fault occurs at bus 2.

**Fault Impedance (3-ph Fault)**
Fault Impedance (SLG Fault)

Fault Impedance (LL Fault)

Fault Impedance (DLG Fault)
Fault Variables

- Fault Current ($I_f$)
- Voltage at faulted bus ($V_f = I_f Z_f$)
- Voltage of healthy buses
- Currents flow on transmission lines
- Generators currents

You must calculate the phase values.

Fault Summary

- 3-ph: Positive sequence only
- SLG: Sequence networks are connected in series, parallel to three times the fault impedance
- LL: Positive and negative sequence networks are connected in parallel; zero sequence network is not included since there is no path to ground
- DLG: Positive, negative and zero sequence networks are connected in parallel, with the zero sequence network including three times the fault impedance