Abstract—Diffusion stills have been effective in water desalination. The present work represents a model of the distillation process by using vertical single-effect diffusion stills. A semi-analytical model has been developed to model the process. A software computer code using Engineering Equation Solver EES software has been developed to solve the equations of the developed model. An experimental setup has been constructed, and used for the validation of the model. The model is also validated against former literature results. The results obtained from the present experimental test rig, and the data from the literature, have been compared with the results of the code to find its best range of validity. In addition, a parametric analysis of the system has been developed using the model to determine the effect of operating conditions on the system's performance. The dominant parameters that affect the productivity of the still are the hot plate temperature that ranges from (55- 90 °C) and feed flow rate in range of (0.00694-0.0211 kg/m$^2$-s).

Keywords—Analytical Model, Solar Distillation, Sustainable Water Systems, Vertical Diffusion Still.

I. INTRODUCTION

FRESH water is the nature’s gift and plays a key role in the development of an economy of a society. It is now a global crisis, but the solar still represents a solution for the domestic scale [1]. The thermal analysis of the solar still is important to have a better understanding of how to enhance the productivity of the still. There have been many studies about the calculations of the vertical diffusion solar still. Dunkle [2] is one of the earliest who studied Vertical Multiple Effect Diffusion Still. He developed experimentally the performance of a five-effect multiple-effect diffusion still. His study is limited to certain conditions, and also faced some problems with the experimental setup. Cooper and Appleyard [3] simplified the Multiple Effect Diffusion (MED) system by combining the solar collector and the diffusion still into one unit. Selcuk [4] performed indoor tests for a Multiple-effect, tilted solar still to study the performance of the still. Elsayed et al. [5] studied theoretically three-effect diffusion and developed a mathematical model as well as an experimental study on a three-effect multiple–effect diffusion–still. He verified the calculated results with the experimental results, and then [5] presented a study of parametric conditions on the performance of an ideal diffusion still and also developed numerical correlations to predict the performance of the still. Grater et al. [7] investigated experimentally a multi-effect still for hybrid solar/fossil desalination. Tanaka et al. [6] constructed eleven-effect vertical multiple-effect diffusion still. Tanaka et al. [9] performed a parametric study and investigated the performance of the vertical multiple-effect diffusion still. He listed different heat and mass transfer correlation to be employed in the analytical model, and found that [7] is the closest correlation to the experimental work. There are some researchers who investigated the single-effect diffusion still. Bouchekima et al. [8] presented a theoretical analysis of heat and mass transfer inside a solar distiller and compared it with the experimental measurements of the distiller performance.

II. SCOPE OF WORK

The literature shows a significant amount of research has been conducted on vertical diffusion solar stills. Some of literature was about to develop numerical and experimental models and to verify them against each other. Some others were theoretically focused on the heat and mass transfer inside the diffusion gap. The former studies reveal that there is no simple semi-analytical model that can predict the performance of vertical diffusion stills.

The scope of study is to investigate vertical single-effect-diffusion still as shown below in Fig. 1, by developing a semi-analytical model of the diffusion still utilizing flat sheets with wick. The study also includes construction of an experimental setup to verify the semi-analytical model. The model is used to investigate the parameters that influence the still productivity and increase the distilled yield such as: (i) the hot plate temperature, (ii) the feed water temperature, (iii) the cold plate temperature, (iv) the mass feed flow rate.

![Fig. 1 A schematic diagram of vertical single-effect diffusion still](image-url)
III. METHODOLOGY

The thermal analysis of proposed model is developed for vertical single-effect diffusion still. For accurate analysis of the performance of the single-effect-diffusion still, it is important to study theoretically the two-dimensional heat and mass transfer in the still, in both the direction perpendicular to the plates and the direction of saline flowing over the plates.

In the presented analytical model, as shown in Fig. 1, the model consists of two isothermal plates facing each other vertically. One plate represents the heat source of the system and the other represent the heat sink. One of the plates is the hot plate, and the other one is the cold plate. A feed saline film flows down over the hot plate, where its temperature is raised up.

The saline film evaporates and diffuses within the gap between the two plates and then condenses over the cold plate through the gap between the two plates to evaporate a portion of the saline and condense over the cold plate.

There is a closed hot water cycle that is circulated over the hot plate to heat up the hot plate. Similarly, there is a cold water cycle that circulates over the cold plate at the condensation side. There are three utilized sprayers that spray out the water over both plates inside each cycle to keep the plates at constant temperature.

The test rig is built up to monitor the still's yield and to investigate the design parameters that could be changed throughout the experiment. It is constructed to examine the performance of the vertical diffusion still. The experimental results will be used to validate the analytical model.

Fig. 2 shows a schematic diagram of the test rig. There are two loops that are responsible for evaporation and condensation processes. The first loop is a closed loop which is responsible for providing the still with the heat necessary to evaporate the feeding film. The second loop is an open loop to provide the system with the heat sink required to achieve an efficient condensation process. The feeding system is considered simple, as it is preferred to have an overhead tank which contains saline/brackish water to be fed to the feeding boxes in controlled rates; the flow rates should be controlled by the use of valves to guarantee equal feed rates to the boxes in case of multiple-effect mode.

The brine and condensate can be easily collected from the apparatus sides using graduated cylinders over a known period of time. An inclined groove is machined from both sides in opposite directions to enable easy collection.

IV. THE ANALYTICAL MODEL

The presented analytical model depends mainly on the heat transfer correlations that are expected to describe the energy flow in the investigated system. The model is considered a two-dimensional model seeking simplicity with adequate accuracy. The structure of the model consists of two isothermal plates facing each other vertically; one represents the heat source of the system and the other the heat sink. Over the hot plate, a saline film flows down to extract its energy and the mechanism of mass transfer through the gap between the two plates to evaporate a portion of the saline and condense it over the cold plate, to get salt-free water on that side.

![Fig. 2 A schematic diagram of the test rig of the diffusion still](image)

![Fig. 3 A schematic diagram for the thermal resistance of a single-diffusion still](image)

The thermal circuit shown in Fig. 3 explains the thermal energy flow through the system from the heat source to the heat sink. As noticed from the circuit the dominant mode of heat transfer on the saline film is the conductive one and the convective is neglected while a combination of convective heat and mass transfer describes energy flow through the gap.

The heat source which is the hot water sprays, inside the hot water container, that is directed to the evaporating plate provides the plate with continuous source of heat as:

$$Q_{in} = \dot{m}_{HS}C_{p,HS}(T_{HSI} - T_{HSO})$$

where $\dot{m}_{HS}$ is the mass flow rate of the hot water, the specific heat of the hot water at an average temperature of the inlet hot
water temperature, \(T_{HSO}\) and hot water temperature at the exit, \(T_{HSE}\).

The heat transferred through the apparatus from the heat source (hot water sprays) to the heat sink (cold water sprays), exhibits 5 stages of transfer:

a) Through the evaporator (first vertical plate)
b) Through the saline film
c) Through the air gap
d) Through the condensate film
e) Through the condenser (second vertical plate)

Consequently, there are formulas that describe each transfer stage; especially the middle stage involves heat and mass transfer as shown in Fig. 3.

\[Q_{in} = \frac{T_{HSE} - T_{Sth}}{R_p}\]  
\[R_p = \frac{\delta_e}{A \times k_p}\]  

The thermal conductivity of the saline film \(k_{Sth}\) is evaluated at average temperature \(\frac{T_{HSE} + T_{Sth}}{2}\).

\[Q_{1} = \frac{T_{Sth} - T_{Sfilm}}{R_{Sth}}\]  

The thermal resistance of the evaporating film \(R_{Sth}\) can be calculated as:

\[R_{Sth} = \frac{\delta_{Sth}}{A \times k_{Sth}}\]  

The thermal conductivity of the saline film \(k_{Sth}\) is evaluated at an average temperature \(\frac{T_{HSE} + T_{Sth}}{2}\);

The saline film thickness can be estimated according to the Nusselt's theory as

\[\delta_{Sth} = \frac{L \times \rho_{Sth}(\mu_{Sth}^{1/2} - \rho_v^{1/2}) \mu_{Sth}^{1/2} \rho_v^{1/2}}{2 \times \mu_{Sth}^{1/2} \mu_{Sth}^{1/2} (T_{Sth} - T_{Sgap})} \left( \delta_{e,in}^5 - \delta_{e, out}^5 \right)\]  

where \(L\) is the length of saline film, \(\rho_{Sth}\), and \(\mu_{Sth}\) are the density and viscosity evaluated at average temperature \(\frac{T_{Sth} + T_{Sgap}}{2}\), respectively. \(\delta_{e,in}\) and \(\delta_{e, out}\) are the thickness of the saline film at the partition inlet and outlet, respectively.

The thickness of the saline feed at the upper edge of the hot plate, \(\delta_{e,in}\), can be evaluated using an empirical correlation deduced from the rectangular weir equation:

\[\delta_{in} = \left(\frac{m_{in}}{g_3021}\right)^{1/2}\]  

The thickness at outlet \(\delta_{e, out}\), can be evaluated as

\[\delta_{e, out} = \left(\delta_{e, in}^4 - L \times \rho_{Sth}(\mu_{Sth}^{1/2} - \rho_v^{1/2})^2 \mu_{Sth}^{1/2} \mu_{Sth}^{1/2} (T_{Sth} - T_{Sgap})\right)^{1/2}\]  

While \(\rho_v\) is the density of water vapor at saturation temperature, \(h_{fg}\) is the modified latent heat that can be calculated by:

\[h_{fg} = h_{fg}^{mod} + 0.68 C_p(T_{Sth} - T_{Sgap})\]  

where \(C_p\) is the specific heat of evaporating saline film estimated at the average temperature \(\frac{T_{Sth} + T_{Sgap}}{2}\).

C. Third Stage: Heat Transfer through the Air Gap

This stage includes the transfer of sensible and latent heat phases, the sensible heat is transferred in three modes: conduction, convection, and radiation. The latent heat is transferred to the condensing side through mass diffusion.

\[Q_{in} = Q_k + Q_c + Q_r + Q_l\]  

The terms \(Q_k, Q_c, Q_r,\) and \(Q_l\) represent the portion of heat transferred by conduction, convection, radiation, and latent heat, respectively.

The conduction heat transfer across the air gap is simply obtained by applying Fourier's law:

\[Q_k = \frac{T_{Sgap} - T_{ gap}}{R_k}\]  

The thermal resistance of air gap due to conduction \(R_k\) is estimated by:

\[R_k = \frac{\delta_{ gap}}{A \times k_{ gap}}\]  

The thermal conductivity of air \(k_{ gap}\) is obtained at average temperature \(\frac{T_{Sgap} + T_{ gap}}{2}\).

The convection heat transfer \(Q_c\) and the radiation heat transfer \(Q_r\), across the gap are neglected due to the small gap. The latent component is estimated using

\[Q_l = m_{evap} \times h_{fg, gap}\]  

\[m_{evap} = \frac{(M_w + 10^5 \times \rho_v^{1/2} \phi_{gap}^{P_{gl}} \phi_{gl}^{P_{ti}})}{8 \times (R_{t} + 273) \phi_{gap}^{P_{gl}} \phi_{gl}^{P_{ti}}}\]  

where \(P_{gl}\) and \(P_{ti}\) are partial pressures, and the diffusion coefficient \(\zeta\) can be calculated using:

\[\zeta = 0.911 \times \frac{10^{-5} \times (R_{t} + 273)^{1/2}}{P_v \times \frac{T_{c}-P_{ti}}{2}}\]  

where \(M_w\) is the molecular weight of water-vapor, and \(P_v\) is the total pressure (kPa), and \(T_c\) is the cold plate temperature.
Fig. 4 describes the structure of the analytical code.

\[ T_p, T_{p2}, T_{sw,in}, \delta_p, R_w, \delta_w, P_{in}, L, m_{sw,in}, \delta_{GP} \]

State initial guesses for \( T_{GP}, T_{ef}, T_{GP2}, T_{DF} \)

Substitute in (3), (5), (7) to get \( R_p, R_w, R_{eff}, \delta_{in} \)
Use (6), (8), (9) as a function to get the value of \( \delta_{ef}, \delta_{DF} \)
Substitute to get \( \delta_{GP}, R_{GP} \)
Use (13)-(16) as a function to get the value of \( m_{evap} \)
Calculate \( h_{in}, R_m, R_{GP} \)
Estimate \( Q_{in}, Q_o, Q_b, Q_D \) using (2), (4), (10), (11)

Calculate new values for \( T_{ef}, T_{GP}, T_{GP2}, T_{DF} \)
Calculate the relative percentage error for each temperature
\[ \epsilon_{ef}, \epsilon_{GP}, \epsilon_{GP2}, \epsilon_{DF} \]

Y

If \( \epsilon_{ef}, \epsilon_{GP}, \epsilon_{GP2}, \epsilon_{DF} \geq 10^{-4} \)

Evaluate Condensate to feed ratio

Fig. 4 Flow chart of the analytical developed code

V. RESULTS AND DISCUSSION

The analytical model is developed to predict the yield of vertical diffusion solar still.

The results of the experimental model are presented in Table I. Fig. 5 shows a comparison between the experimental results obtained by the test bench and the theoretical results obtained by the analytical model at different feed flow rate for the change of hot plate temperature with respect to the condensate to feed ratio (CF). The experimental and analytical results are in a quite agreement.

The validation of the analytical code against literature and the experimental results was in a good agreement (almost all the points are located in the region of maximum deviation of 15%). It is now important to investigate the influence of the design parameters of the diffusion still. There are different parameters such as the hot plate temperature, cold plate temperature, the feed water temperature, mass feed rate and diffusion gap.

Table II shows the experimental results of condensate to feed ratio against the experimental condensate to feed ratio results (CF).

Figs. 6-8 show the relations of influential parameters to the productivity of the still. It is obvious, that the most dominant parameter is the hot plate temperature and the diffusion gap thickness.

There are many parameters that influence the productivity of the vertical diffusion stills. Not all of them should be changed in the experimental setup to verify the mathematical model but at least two could be employed to give a validation to the model and study effect of other parameters change.
In the present experimental model, the change of the saline feed flow rate and the hot plate temperature have been considered, while the distance between the two isothermal plates, the cold plate temperature, and the inlet feed temperature are kept constant.

The code has been used to show the effect former parameters. Fig. 6 shows the effect of decreasing the cold plate temperature which lead to a significant increase in the condensate to feed ratio. But the decrease in air gap thickness is related strongly to the saline feed flow rate and as it is limited by the distance between the two plates. Thus, air gap thickness of 4 mm is applicable for a very limited range of feed flow rates.

The change in feed inlet temperature is observed on Fig. 8. It is clear that increasing the saline feed temperature leads to an increase in the condensate to feed ratio. But the increase is noticed to be very slight compared to the other former factors.

### V. CONCLUSION

The main aim of presented analytical model is to contribute for a better understanding of the vertical diffusion stills. A semi-analytical model has been developed and has also been validated against former authors [5]. In addition an experimental setup is constructed for validation.

The experimental setup has been installed to investigate the performance of the flat sheets with wick. The results of the analytical and experimental models for the flat sheet have shown a good agreement. Moreover, a parametric study has

<table>
<thead>
<tr>
<th>Feed flow rate [ml/min] (Kg/m²s)</th>
<th>Hot temperature [°C]</th>
<th>Condensate [ml/min]</th>
<th>Feed [ml/min]</th>
<th>CF</th>
<th>Experimental CF</th>
<th>Theoretical CF</th>
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### Fig. 7 The relation between the feed water temperature and the CF at (T_p= 30 °C, T_sw_in= 20 °C, m_sw_in= 0.0151 kg/m²-s)

The change in feed inlet temperature is observed on Fig. 8. It is clear that increasing the saline feed temperature leads to an increase in the condensate to feed ratio. But the increase is noticed to be very slight compared to the other former factors.

### Fig. 8 The relation between the feed water temperature and the CF at (T_p= 30 °C, T_sw_in= 20 °C, Delta_gap = 10 mm)

The main aim of presented analytical model is to contribute for a better understanding of the vertical diffusion stills. A semi-analytical model has been developed and has also been validated against former authors [5]. In addition an experimental setup is constructed for validation.

The experimental setup has been installed to investigate the performance of the flat sheets with wick. The results of the analytical and experimental models for the flat sheet have shown a good agreement. Moreover, a parametric study has
been conducted and showed the dominant and influential parameters that affect the performance of the still.

There are four parameters that influence the productivity of the still. All of the four parameters could be varied by the user. The productivity increases when the hot plate temperature increases and air gap thickness decreases.

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