

What is the "x" which occurs in "sin x"? Being an essay towards a conceptual Foundations of Mathematics

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(Submitted on 16 Oct 2016)

As Lebesgue implies, measure is the most fundamental concept of science. It is not accidental that the principal branch of mathematics is that which deals with measure: recently Analysis, before it Geometry. The transition of the study of measure from geometry to analysis is a long process which is yet to be investigated from both the logical and the conceptual points of view. Trigonometry, thanks to Eudoxus' theory of ratio and proportion, was developed within geometry. Now it involves analysis too. To analytically deal with the trigonometric functions, several mathematicians answered Hardy's key question "The whole difficulty lies in the question, What is the x which occurs in $\cos x$ and $\sin x$?".

Building on the works of Eudoxus and Lebesgue, it is shown that (to the best of my knowledge) all of these answers are logically wrong or conceptually unsound. In contrast, based on Eudoxus' theory, measure in general and measuring angles in particular are investigated, and an answer to this question, which is both logically correct and conceptually sound, is presented. To comprehend the philosophical and historical background of the subject, briefly several problems are considered which need further investigation and several questions are raised which are yet to be answered. In an appendix a correction of the proof of proposition XII.2 of Euclid's Elements, which practically defines " π ", is given.