

AERO-ELASTIC ANALYSIS AND OPTIMIZATION OF COMPOSITE PLATE WINGS

Mohamed A. Mahran¹, Hani M. Negm², Karam Y. Maalawi³ and Adel M. Elsabbagh⁴

¹Aerospace Engineering Department, Cairo University, Giza 12613, Egypt

Email: abdu_aerospace@eng.cu.edu.eg, Web Page <http://scholar.cu.edu.eg/?q=mohamedabdou>

²Aerospace Engineering Department, Cairo University, Giza 12613, Egypt

Email: hnegm_cu@hotmail.com

³Mechanical Engineering Department, National Research Center, Dokki 12311, Cairo, Egypt

Email: nrc.aero@gmail.com

⁴ASU Sound and Vibration Lab, Design and Production Engineering Department, Ain Shams University, Abbaseya, Cairo 11517, Egypt.

Email: aelsabbagh@eng.asu.edu.eg

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Abstract

A mathematical model for aero-elastic analysis and optimization of composite plate-like wings is developed. The required steady and unsteady aerodynamic analyses are performed using the vortex and doublet lattice methods, respectively. The finite element model for structural analysis is formulated using a shell-linear triangular element with five degrees of freedom per node. A MATLAB code is developed to implement the theoretical aero-elastic model and validated by comparing its results with other publications in the literature. The model has been implemented to study the effects of forward-sweep angle and span-wise thickness taper on the wing divergence and flutter speeds. An optimization model is developed to tailor the composite wing to improve its divergence speed. The objective function of the proposed optimization model is measured by maximizing the wing divergence speed while maintaining the total mass at a constant known value. Design variables are chosen to be the composite layers thicknesses and orientations. The results show that the effect of the former is negligible with respect to that of the latter. The optimization has been performed by involving MATLAB optimization toolbox function which uses the sequential quadratic programming algorithm.

1. Introduction

“Aero-elastic tailoring is the embodiment of directional stiffness into an aircraft structural design to control aero-elastic deformation, static or dynamic, in such a fashion as to affect the aerodynamic and structural performance of that aircraft in a beneficial way”. The first aero-elastic application was presented by Munk in 1949 for a propeller made from wood [1]. Since then, there has been a lot of interest in aero-elasticity to overcome the difficulty arising from the flexibility of the structure due to the light weight of the aircraft.

There has been a lot of research work in the field of optimizing aircraft structures from the aero-elastic point of view. Kennedy and Martins [2]. presented a coupled aero-structural optimization model for aircraft wing design optimization. They used a 3-D panel code to obtain the aerodynamic loads and a finite element code to perform the structural analysis. The connection between the structural and aerodynamic models is done using a general model. They developed aero-elastic optimization models for real wings. Mahran, et al. studied the aero-elastic characteristics of tapered [3] and composite [4] plate wings. They obtained the steady aerodynamic loads using the vortex lattice method and the

unsteady aerodynamic loads using the doublet lattice method. The finite element method was applied to obtain the structural response. The aerodynamic model was coupled with the finite element model using the finite element shape functions [5].

The aim of the present work is to use a coupled aerodynamic-structural model for the analysis and optimization of composite plate-like wings. The effect of three wing design parameters on the divergence and flutter speeds are studied. These parameters are the wing forward-sweep angle, the wing thickness, and the orientation angles of the composite layers. An optimization model for the tapered wing has been formulated. The objective function is measured by maximizing the wing divergence speed subject to mass constraints. Side constraints are imposed on the design variables in order to avoid having odd optimal solutions. A final flutter check on the optimal wing design was performed.

2. Aero-elastic analysis

The aero-elastic analysis is divided into two parts; static aero-elasticity (divergence) which studies the interaction between the steady aerodynamic forces and the elastic loads, and the dynamic aero-elasticity (flutter) which studies the interaction between the unsteady aerodynamic forces, elastic forces and the inertial loads. The aeroelastic model of the present work is documented in details in References [4]–[6].

Figure 1 illustrates the wing geometry in which c_r is the wing root chord, c_t is the wing tip chord, s is the wing half span, Λ_w is the wing sweep angle and ρ_∞ and U_∞ are the air flow density and speed, respectively. It also represents the triangular-shell finite element in the global coordinates (X, Y, Z) , the local coordinates (x, y, z) and the area coordinates (L_1, L_2, L_3) . It gives the convention of the composite fiber orientation angle θ .

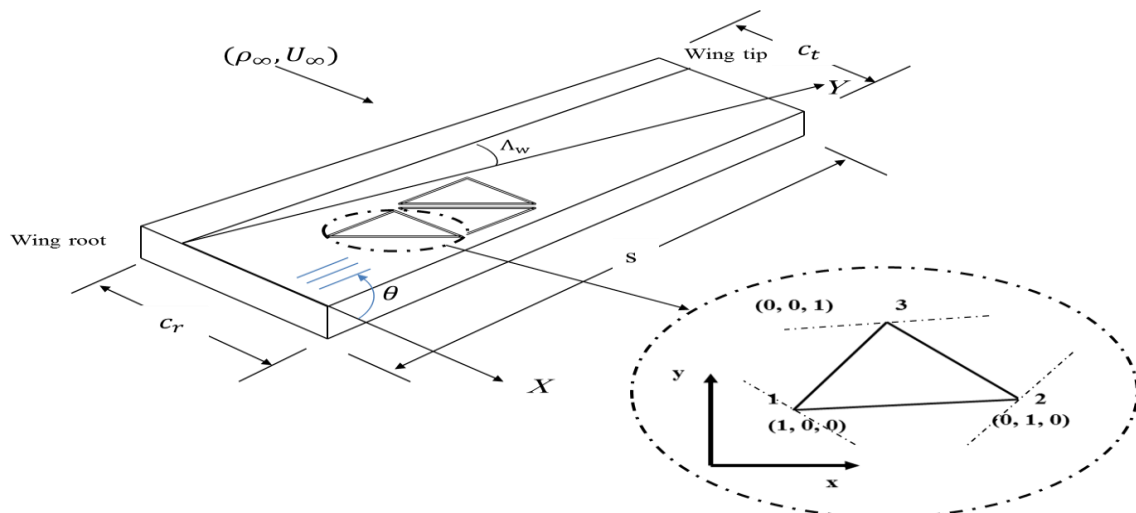


Figure 1. Wing geometry with the finite element coordinates

3. Optimization model formulation

3.1 The objective function

Two important, but rather conflicting, objectives are usually addressed in early design stages of an airplane wing: high stiffness level and light structural weight. In this study, we consider the optimization problem of maximizing the overall stiffness level without weight penalty. Maximization

of the wing stiffness will be measured by maximization of the critical flight speed at which wing divergence occurs.

3.2 Pre-assigned parameters and design variables

Figure 2 defines the selected pre-assigned parameters and design variables of the plate wing model. The pre-assigned parameters that are not changed during the optimization process are selected to be: wing chord taper ratio Δ_c , wing thickness taper ratio Δ_t , wing chord and semispan, c , s , and the number of layers in the p^{th} panel Ni_p where $i = 1, 2, \dots, n_i$ and $p = 1, 2, \dots, N_p$. The wing design variables that can be changed during the optimization process are chosen to be: fiber angle in the i^{th} layer within the p^{th} panel and thickness h_i of each lamina inside the p^{th} panel.

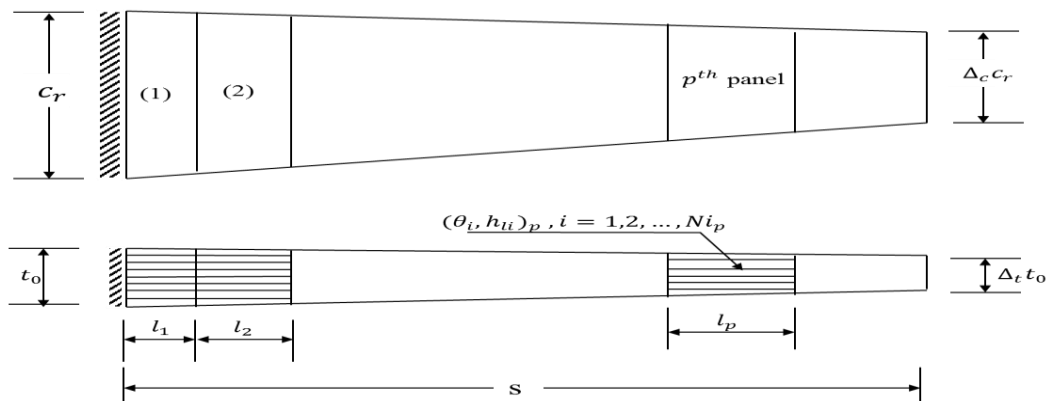


Figure 2. Definition of pre-assigned parameters and design variables

4. Case studies

The wing model to be considered in this study is a composite plate having rectangular plan-form and divided into three panels; inboard, middle and outboard. Each panel has pre-selected length and number of laminas as shown in **Table 1** and **Figure 3**. Two types of wings are considered; the first is an experimental wing which has available experimental data, while the second is a full scale wing which has flight conditions available. Both wings have the same material where the problem considers a composite cantilevered plate wing made of graphite/epoxy with material properties;

$$E_L = 98.0 \text{ Gpa}, E_T = 7.90 \text{ Gpa}, G_{LT} = 5.60 \text{ Gpa}, \nu_L = 0.28, \rho = 1520 \text{ kg/m}^3, t = 0.134 \times 10^{-3} \text{ mm}.$$

Table 1. Plate wing model.

Parameter [m]	Experimental wing	Full scale wing
l_1	0.0762	1.2
l_2	0.1144	1.8
l_3	0.1144	1.8
c	0.0762	1.229

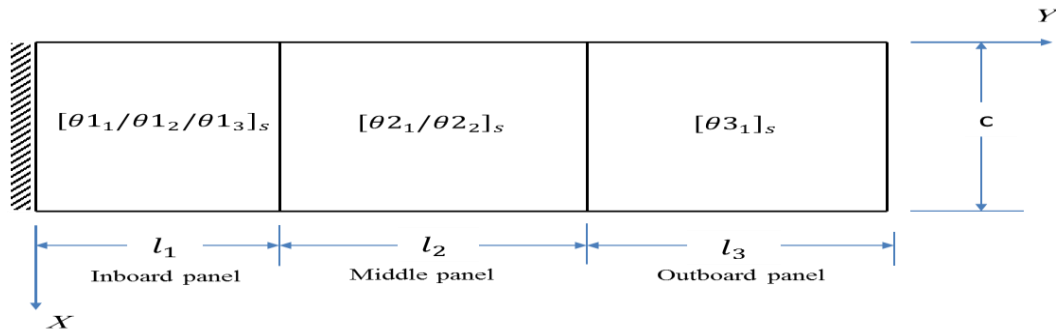


Figure 3. The plate wing model geometry

Design constraints

A design that meets all the requirements placed on it is called a feasible design. We stipulated an equality mass constraint is $M = M_0$. The mass of the wing is maintained constant through the optimization process. The wing mass is calculated using the following equation,

$$M = \rho_m V = \rho_m c (t_1 l_1 + t_2 l_2 + t_3 l_3) \quad (1)$$

where c is the wing chord, t_p is the thickness of the p^{th} panel and l_p is the length of p^{th} panel.

4.1 Parametric study on the effect of laminate layup on critical flight speeds

4.1.1 Analysis of the experimental wing

In this section, the effect of forward-sweep angle on divergence and flutter of composite plate-like wings is studied. A forward sweep angle of 30 degrees is taken as an example. **Table 2** gives the critical speeds for the same wing with and without forward sweep for different ply angles. The wing model used in Ref. [7] is used with the plan-form shown in **Figure 4**. This wing is selected because its experimental data are available in many publications which enables comparison of results.

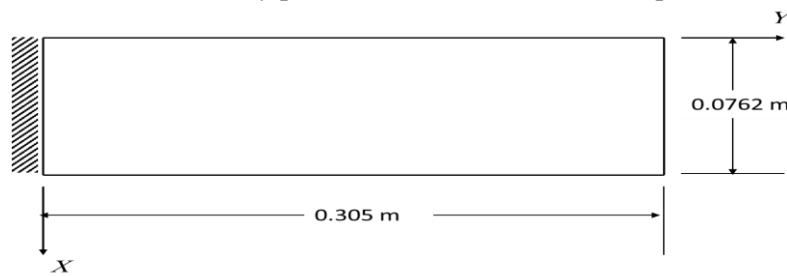


Figure 4. Experimental wing plan-form geometry.

Table 2. Effect of laminate layup on critical flight speeds for unswept and swept with sweep angel [-30°]

Laminate configuration	Divergence Speed [m/s]		Flutter Speed [m/s]	
	Straight Wing	Swept-forward wing [-30°]	Straight Wing	Swept-forward wing [-30°]
$[0_2 / 90]_s$	25.4	9.3	26.4	55.7
$[45 / -45 / 0]_s$	No	35.6	47.5	34.9

$[45_2 / 0]_s$	No	No	27.8	32.6
$[-45_2 / 0]_s$	12.7	7.5	29.1	28.1
$[30_2 / 0]_s$	No	18.3	27.4	30.2
$[-30_2 / 0]_s$	12.8	7.7	48.1	34.3

4.1.2 The analysis of full scale wing of a trainer aircraft

The wing studied in the previous section is a small wing that is suitable for experimental studies. It is not a real wing that has known maximum flight speed and other flight conditions stated in the literature. In this section a full scale plate wing will be considered for a light trainer that has the geometry given in **Table 1**. The main mission of the airplane is patrolling and search on the ground, as well as basic training flight. In the present analysis and optimization, the wing is divided into three span-wise panels with different thicknesses and panel lengths [1.2, 1.8, and 1.8] m as shown in **Figure 3**. The purpose of this section is to study the effect of different laminas fiber orientations on the aero-elastic characteristics of the wing. The wing has a thickness taper ratio of 1/3, and is divided into three panels with the thicknesses and properties given in **Table 3**. The model represents a composite cantilevered plate wing made of graphite/epoxy.

Table 3 The model wing properties

The geometric properties	
Area (S)	11.8 [m ²]
Aspect Ratio (AR)	7.8
Taper Ratio (TR)	1
Span (b)	9.6 [m]
Sweep back angle at c/4 Λ_w	0 [deg]
Washout angle (α_w)	0
Lamina thickness in each panel	[0.0064, 0.0074, 0.0096] [m]
Structural mass	500 [kg]
Flow Properties	
Density	0.9093 [kg/m ³]
Max speed	108 [m/s]
The model boundary condition	
Constraint	The wing root is fixed

The divergence and flutter analyses of the wing are performed for six cases of different angle orientations. The analyses results are listed in **Table 1**.

Table 4. Flutter and divergence speeds of straight wing of training aircraft

Laminate configuration [inboard], [middle], [outboard]	Divergence Speed [m/s]	Flutter Speed [m/s]
$[0_2 / 90]_s$ $[0 / 90]_s$ $[0]_s$	125.7	166.2587
$[\pm 45 / 0]_s$ $[\pm 45]_s$ $[0]_s$	179.5	No flutter
$[45_2 / 0]_s$ $[45_2]_s$ $[0]_s$	156.6	No flutter
$[-45_2 / 0]_s$ $[-45_2]_s$ $[0]_s$	112.5	No flutter
$[30_2 / 0]_s$ $[30_2]_s$ $[0]_s$	151.8	167.2252

$[-30_2 / 0]_s$ $[-30_2]_s$ $[0]_s$	112	No flutter
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4.2 Optimization using MATLAB tool box

The optimization is done using the MATLAB toolbox. The MATLAB function is known as “fmincon”. It uses the sequential quadratic programming algorithm (SQP) [8].

4.2.1 Aero-elastic optimization of experimental wing

In the first stage a constrained optimization model has been formulated. A number of cases were studied using the developed model. The wing geometry is given in **Figure 3** and **Table 1**. Three different cases are presented in this section. In the first case, the thickness of the lamina h_l is assumed constant along span and is included as one design variable in the optimization model. In the second case, each lamina has a different thickness, and hence three thickness values are considered in the optimization problem. In the third case the wing is defined with thickness taper. Therefore, the design variables are reduced into eight. The results of these cases are documented in **Table 5**.

The associated optimization problem is cast in the following form:

Find the design variables vector $\mathbf{X} = (\theta_i, h_i)_p, i = 1, 2, \dots, n_{ip}, p = 1, 2, \dots, N_p$, with total number of design variables N, which minimizes the objective function:

$$F(\mathbf{X}) = - U_D \quad (2)$$

Subject to the constraints:

Mass:

$$M = M_0$$

Side :

$$\mathbf{X}_l \leq \mathbf{X} \leq \mathbf{X}_u$$

where \mathbf{X}_l and \mathbf{X}_u are the lower and upper bounds imposed on the various design variables to avoid having odd-shaped or unrealistic designs in the attained optimal solution. M_0 is the mass of a known baseline design and U_D is the wing divergence speed. n_{ip} is the number of layers of the p^{th} panel.

$$\min_{\mathbf{X}} -U_D(\mathbf{X}) \text{ such that } \begin{cases} M = M_0 \\ \mathbf{X}_l \leq \mathbf{X} \leq \mathbf{X}_u \end{cases} \quad (3)$$

Table 5. Three case studies

	1 st Case	2 nd Case	3 rd Case
Design variables	$\mathbf{X} = \left\{ \begin{matrix} \theta_{1_1}, \theta_{2_1}, \theta_{3_1} \\ \theta_{1_2}, \theta_{2_2}, \theta_3, h_i \end{matrix} \right\}$	$\mathbf{X} = \left\{ \begin{matrix} \theta_{1_1}, \theta_{2_1}, \theta_{3_1}, \theta_{1_2}, \\ \theta_{2_2}, \theta_3, h_{i1}, h_{i2}, h_{i3} \end{matrix} \right\}$	$\mathbf{X} = \left\{ \begin{matrix} \theta_{1_1}, \theta_{2_1}, \theta_{3_1}, \\ \theta_{1_2}, \theta_{2_2}, \theta_3, h_{i0}, \Delta_t \end{matrix} \right\}$
Base line design	W = 0.0178 kg U _D = 3.2415 m/s	W = 0.0178 kg U _D = 3.2415 m/s	W = 0.0178 kg U _D = 3.2415 m/s
Optimum design	$\left\{ \begin{matrix} 40.7, 65.3, -4.6, 53.6, \\ 61.2, 60.2, 0.134e-3 \end{matrix} \right\}$ U _D = 235.5 m/s	$\left\{ \begin{matrix} 25.6, 76.4, -2.5, 40.5, 73.8, 65.3, \\ 0.1961e-3, 0.0519e-3, \\ 0.1757e-3 \end{matrix} \right\}$ U _D = 317.4 m/s	$\left\{ \begin{matrix} 47.5, 24.4, -9.2, 49.4, \\ 21.3, 71, 0.1358e-3, 0.3548 \end{matrix} \right\}$ U _D = 267.2 m/s

In the first case, the lamina thickness remains constant due to the mass constraints, while the divergence speed increased to 235.5 m/s. In the second case, the divergence speed increased to 317.4 m/s which is better than the first one. However, the thickness of the tip panel is greater than that of the middle one which is impractical. Therefore, the model is improved in the last case based on a thickness taper.

4.3 Aero-elastic optimization of full scale wing of trainer aircraft (unconstrained optimization)

It was shown in the previous section that the changes in the thicknesses of the fiber layers has small effect on the divergence speed of the wing compared with the effect of angles orientation. In this section a full scale wing is considered. The goal of the optimization is to choose the best orientation angles which maximize the wing divergence speed. The wing used in the optimization is the same wing of the trainer aircraft used in the previous section.

The wing is divided into three panels, as mentioned before, in which the root-panel has three symmetric laminas, the middle panel has two symmetric laminas, and the tip panel has one symmetric lamina. The design variables are $\mathbf{X} = \{\theta_{1,1}, \theta_{2,1}, \theta_{3,1}, \theta_{1,2}, \theta_{2,2}, \theta_3\}$. The initial design is defined as $\mathbf{X}_0 = \{0, 50, 0, 0, 50, 50\}$ in which all the angles are in [degrees]. The wing is defined with thickness taper ratio $\Delta_t = \frac{1}{3}$. In the analysis it is divided into three panels with lamina thicknesses $\{0.0064, 0.0074, 0.0096\}$ m. The material properties of this wing were defined in the previous section. The wing has a swept-forward angle equal to 30 degrees. The optimization model is defined in Eq. (4). The upper and lower bounds of the design variables in the present optimization model are changed to $-\frac{\pi}{3}$ to $\frac{\pi}{3}$. This is to prevent significant decrease in the wing flutter speed.

$$\min_{\mathbf{x}} -U_D(\mathbf{X}) \text{ such that } \begin{cases} \mathbf{X}_l = \left[-\frac{\pi}{3} & -\frac{\pi}{3} & -\frac{\pi}{3} & -\frac{\pi}{3} & -\frac{\pi}{3} & -\frac{\pi}{3} \right] \\ \mathbf{X}_u = \left[\frac{\pi}{3} & \frac{\pi}{3} & \frac{\pi}{3} & \frac{\pi}{3} & \frac{\pi}{3} & \frac{\pi}{3} \right] \\ \mathbf{X}_l \leq \mathbf{X} \leq \mathbf{X}_u \end{cases} \quad (4)$$

The divergence speed of the initial design is 67.1 m/s, which is lower than the maximum speed of the wing. The optimization problem was executed using the MATLAB toolbox. The optimum design was found to be $\mathbf{X} = \{47.5, 44.6, 49.9, 60, 60, 49.9\}$ and the divergence speed is 1.4579E3 m/s. On the other hand it is important to study the effect of the new fiber orientations on the flutter speed to insure that the wing is safe. The flutter analysis of the initial and final design shows that the wing is free from flutter.

5. Conclusions and discussions

An optimization model for the orientation angles is formulated where the objective function is measured by maximizing the wing divergence speed without mass penalty. The design variables are the fiber orientation angles and the thicknesses of the composite layers. A small scale experimental wing and a full scale wing of a trainer aircraft are taken as case studies. Each of which is divided into three panels with 6, 4, 2 layers in each panel, respectively. The wing layers are assumed to be symmetric with respect to the mid-plane of the wing, and hence the total number of design variables are reduced. The sequential quadratic programming algorithm implemented in the *fmincon* function in

MATLAB optimization toolbox is used to find the optimum solution; the best fiber configuration that makes the wing free from any aero-elastic instability.

Results show that the fiber orientation angles of the composite wings have a great effect on the aero-elastic stability. This is clear in the analyses of the small and full scale wings. In case of the small wing, the difference between the divergence of the straight wing and the forward swept wing for $[0_2 / 90]_s$ orientation angles is smaller than the same difference for $[45 / -45 / 0]_s$ and $[30_2 / 0]_s$ angle orientations. In case of flutter, in some cases the flutter speed of the straight wings is greater than that of the swept back wings like with $[-30_2 / 0]_s$ orientation angles, and in other cases the flutter speed of the swept back wings is greater that of the straight wings like with $[0_2 / 90]_s$ orientation angles.

The wing stresses are divided into chord-wise stresses, transverse stresses and torsional stresses. The best fiber angles are 0° angle for the longitudinal stresses, 90° angle for the transverse stresses and $\pm 45^\circ$ angles for the torsional stresses. The analysis of the composite wing indicates that the latter configuration can be the best because it gives the maximum divergence and flutter speeds for the experimental and full scale wings. Additionally, the wing thickness has a small effect on the divergence and flutter speeds comparing with the composite fiber orientation.

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