Airframe Design and Construction

Fuselage stresses – example

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Fuselage ultimate bending moment - Example

Consider the fuselage shown in figure. It is a portion of a *2024–T3 aluminum alloy* tapered fuselage of length *90 inch*. The fuselage is made up of a *0.05 inch thickness skin, 0.064 inch thickness floor plate and six stringers of the types shown.*

The frame spacing is *18 inch*. *A cutout at one side of the fuselage is made*. This cutout is bounded by the *Frames at Stations (18)* and *(36) and Stringers numbered (1)* and *(5) (elements 2, 3 and 4 are removed).*

Skin riveting are single lines *countersunk of diameter 5/32" spaced at 1.0" and edge distance 0.4",* while floor riveting are brazier of the same diameter and spacing.

Calculate the stresses of the fuselage cross-section at *Station (0), taking into consideration the shear lag and inter-rivet buckling effects, for the loading (-5000,-6000, 20000) at (90, 5, 20).*

Fuselage ultimate bending moment - Example

Calculate the shear flow at *Station (0)* assuming that the stringer forces at *Station (0)* and *Station (18)* are as follows:

Stringer no.	*1*	*3*	*5*	*7*	*9*	*11*
Force (lb):Station (0)	*–18,400*	*–9,440*	*3,940*	*30,500*	*6,600*	*-13,200*
Force (lb):Station (18)	*-25,200*	*0.0*	*5,700*	*28,000*	*7300*	*-15,800*

Fuselage ultimate bending moment - Example

- *Material: aluminum alloy 2024-T3*
- *Fuselage length* (L_F)*: 90 inch*
- *Skin thickness* (t_s)*: 0.05 inch*
- *Floor thickness* (t_f) *: 0.06 inch*
- Frame spacing (FS): 18 inch
- Cutout *(elements 2, 3 and 4 are removed).*
- Skin riveting: single lines *countersunk* d_r *=5/32" spaced at 1.0" and edge distance 0.4",*
- floor riveting: brazier of the same diameter and spacing.

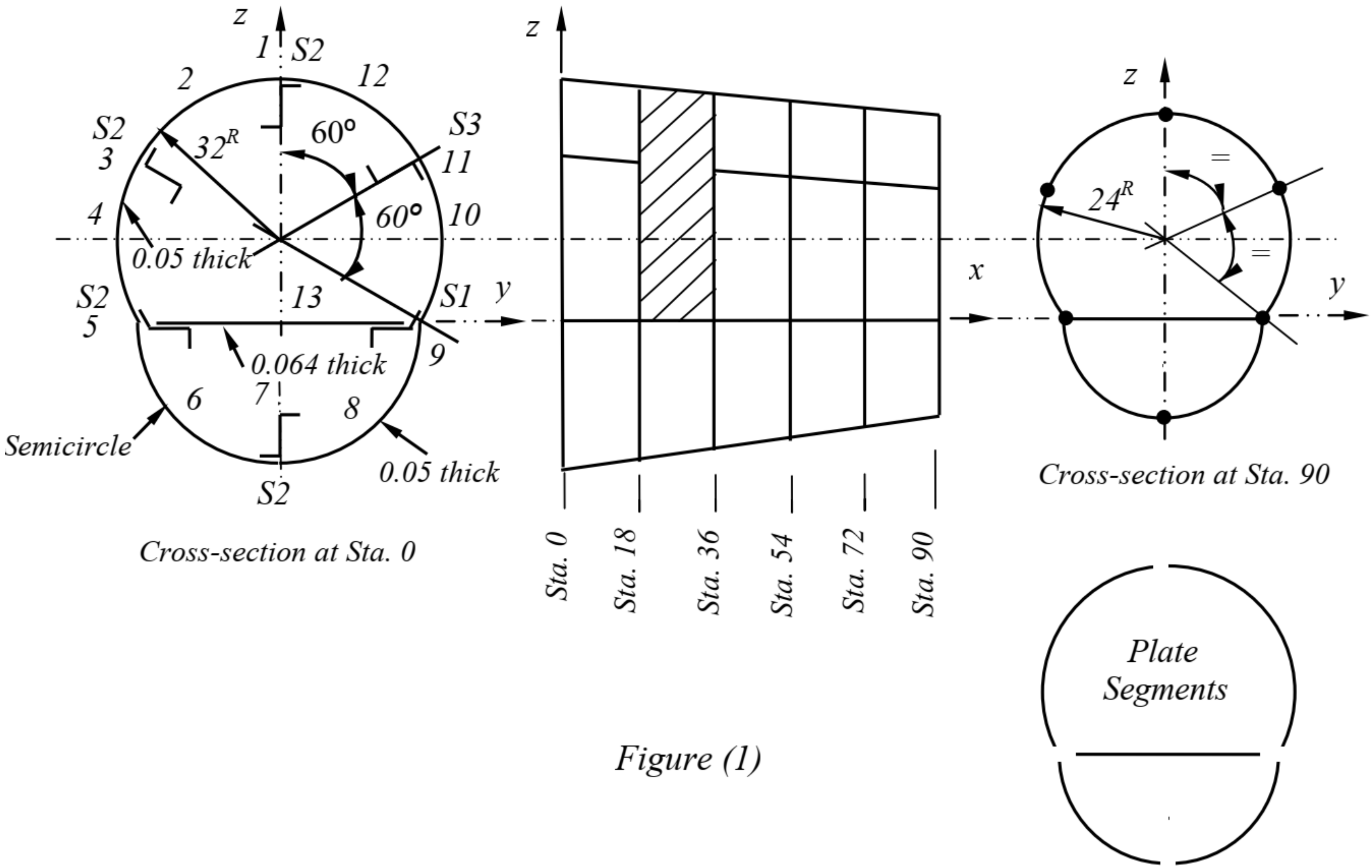


Figure (1)

Fuselage ultimate bending moment - Example

$$\sigma_b = \frac{P}{A} - \left(K_3 M_z - K_1 M_y\right) Y - \left(K_2 M_y - K_1 M_z\right) Z$$

$$K_1 = \frac{I_{xz}}{I_x I_z - I_{xz}^2} \quad K_2 = \frac{I_z}{I_x I_z - I_{xz}^2} \quad K_3 = \frac{I_x}{I_x I_z - I_{xz}^2}$$

$$w = 1.90t\sqrt{E/F_{ST}} \qquad w_1 = .62t\sqrt{E/F_{ST}}$$

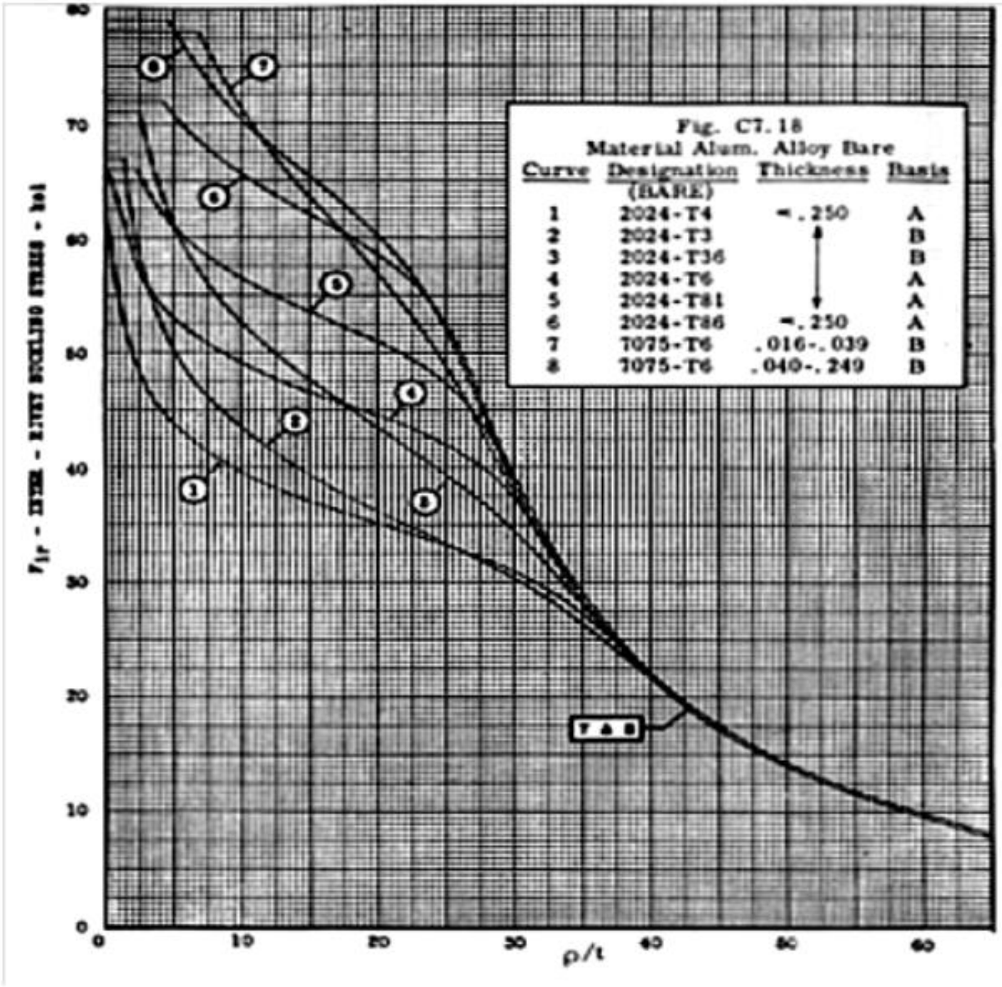
psi	F_{tu}	F_{su}	F_{cy}	E
2024–T3	65,000	49,000	40,000	$10.7x10^6$
2014–T6	62,000	46,000	52,000	$10.7x10^6$
17–4 PH	180,000	108,000	165,000	$27.5\ x10^6$

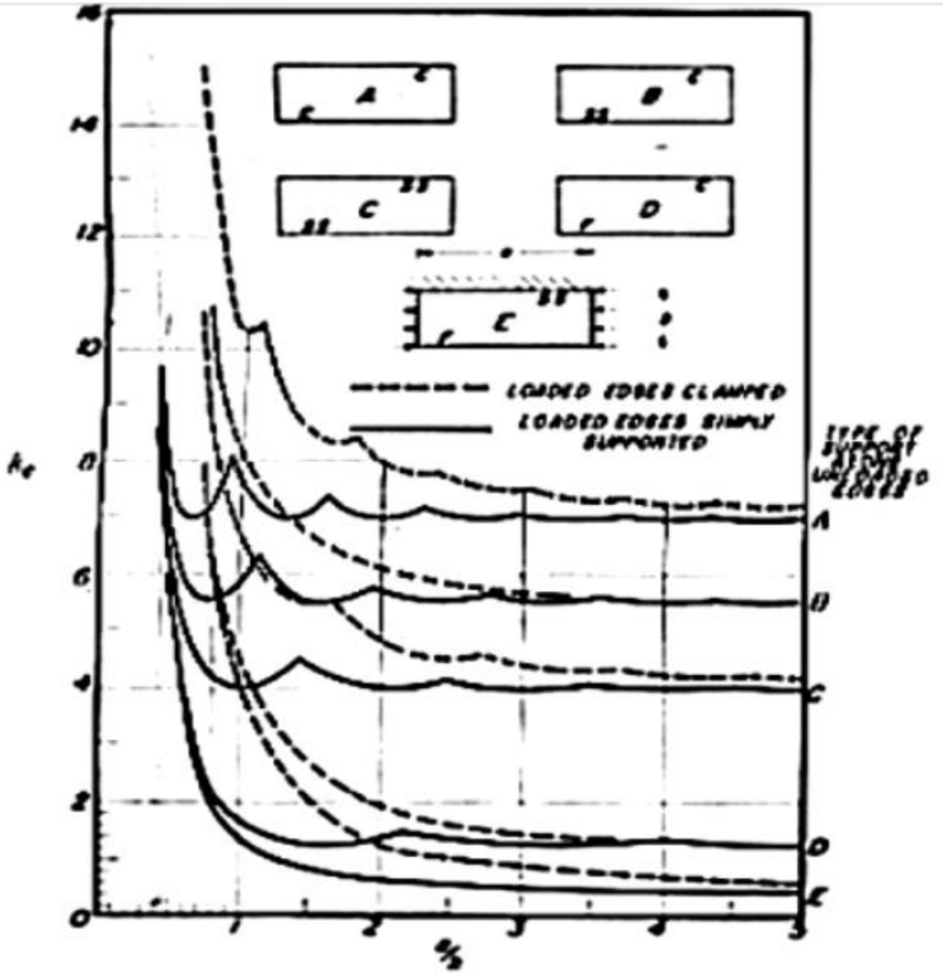
Stringer Type	Failing Stress (psi)	Failing Strain	Area (in^2)	Height (in)	Thick. (in)
S1	− 37,685	− 0.006	0.135	1.0	0.05
S2	− 40,616	− 0.0059	0.18	1.125	0.064
S3	− 34,455	− 0.00475	0.08	0.875	0.04

E = 10.7E6 psi

	ε^3	ε^2	ε	1
	− 0.006 <= ε < − 2.8155E-3			
S1	0.4236	6.8366	36.8524	28.8098
S2	− 5.9 E-3< ε < − 2.8155E-3			
	0.2363	4.5535	29.0559	20.8374
S3	− 4.75 E-3< ε < − 2.8155E-3			
	0.3599	5.5560	28.4280	13.7918
	Linear : − 2.8155E-3 <= ε <= 4.8544E-3			
S1 , S2 , S3	0	0	10.7	0
S1 , S2 , S3	4.8544E-3 < ε < 9.75E-3			
	0.08423	− 2.5758	26.2167	− 24.0833
	ε > = 9.75E-3			
	σ = 64,737			

Fuselage ultimate bending moment - Example





Fuselage ultimate bending moment - Example

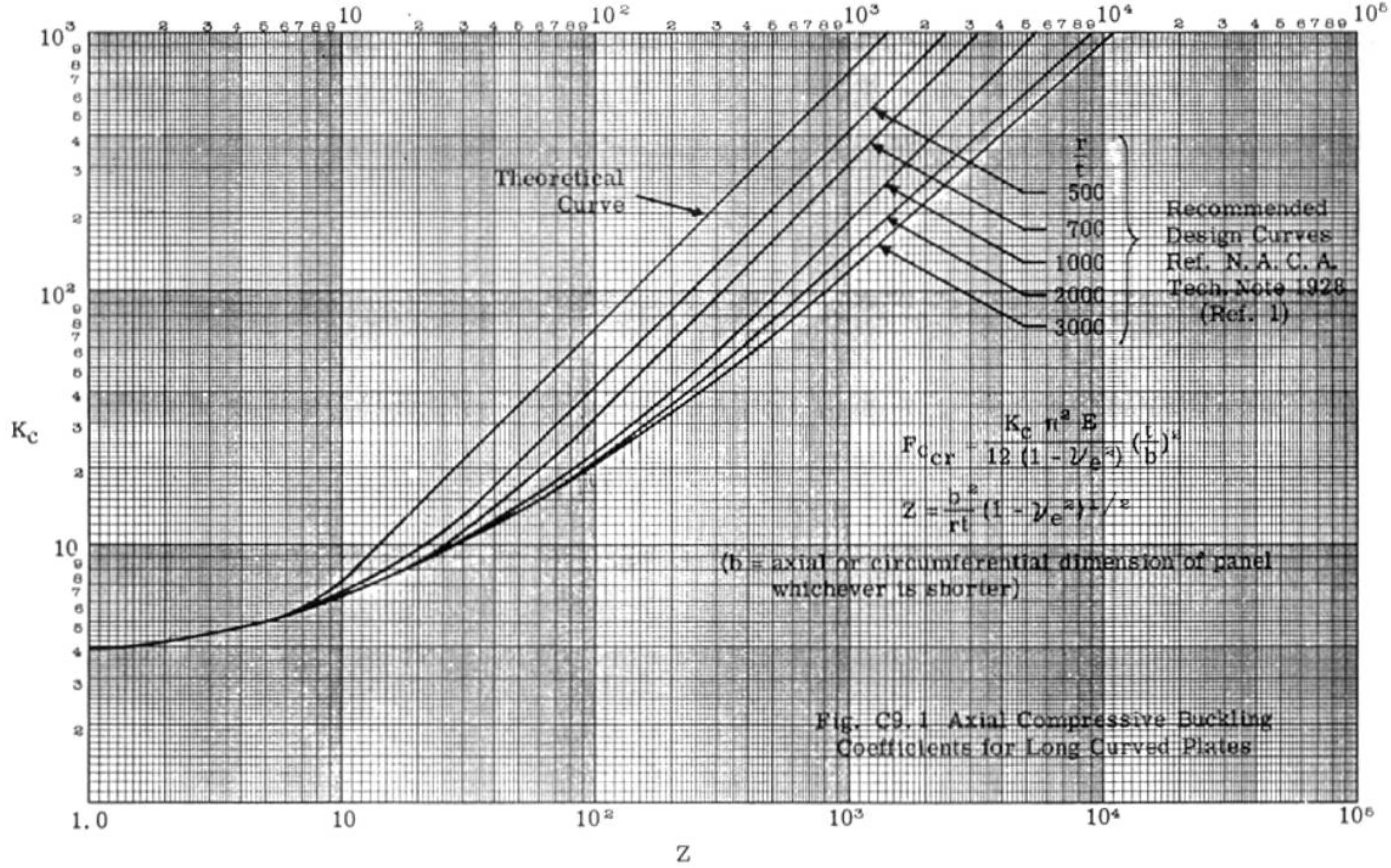
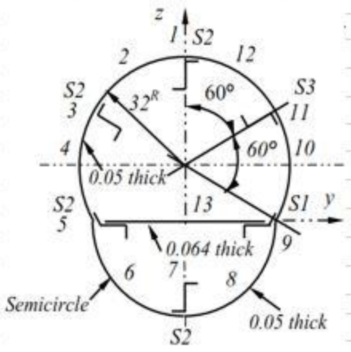


Fig. C9.1 Axial Compressive Buckling Coefficients for Long Curved Plates

Given



Given:												
	FS	18	fuselage L	90	t_skin	0.05	t_floor	0.064				
	E=	10700000	ν=	0.3			p	1				
Solution:		R(90)	24									
	R1=	32	R2=	27.712813	dΘ1	1.047198	dΘ2	1.570796				
					b1=	33.48414	b2=	43.53118				
Stringers Properties			H=	16	bf=	55.11313						
St. type	**Failing stress [psi]**		**Failing strain**		**Area [in^**	**Hieght [in**	**t [in]**					
S1	-37685		-0.006		0.135	1	0.05					
S2	-40616		-0.0059		0.18	1.125	0.064					
S3	-34455		-0.00475		0.08	0.875	0.04					
	ε^3	ε^2	ε	1								
	− 6 <= ε < -2.8155					Rcg_skin 1	30.557749					
S1	0.4236	6.8366	36.8524	28.8098		Rcg_skin 2	24.950298					
							Shear-lag effect		31.975			
	− 5.9 < ε < -2.8155					Ksl	0.22319					
S2	0.2363	4.5535	29.0559	20.8374								
	− 4.75 < ε < -2.8155					**Inter-Rivite Buckling**						
S3	0.3599	5.556	28.428	13.7918	**upper skin**		p=	1	c=	1		
							p/t=	20	p/teq=	40	Fir=	22000
	− 2.8155 < ε <= 4.8544											
S1, S2, S3	0	0	10.7	0	**Floor**		p=	1	c=	3		
							p/t=	15.625	p/teq=	18.0422	Fir=	36000
S1, S2, S3	**-4.8544 < ε <= 9.75**					**the critical stress of buckled skin**						
	0.08423	-2.5758	26.2167	-24.0833		**upper skin**		r/t=	639.5			
	ε >= 9.75							**σcr**	5223.4			
	σ = 64737											
Px=	**-5000**	**Pz=**	**20000**	**XL=**	**90**	**Floor**		a	18	bavg	53.362	a/b
Py=	**-6000**	**yL=**	**5**	**ZL=**	**20**			Kc	11	**σcr**	153.0204	0.33732

First trial

				Initial Trial								
N0.	type	A	Rcg	Θ	y'	Z'	**K**sl	**A**eff	**A**eff Y'	**A**eff Z'	Y	Z
1	S2, R1	0.18	31.4125	1.5707963	0	47.4125	1	0.18	0	8.53425	0.013383	54.1623
2	skin 1-3	0.075	30.5577	2.0943951	-15.27887454	42.46379	1	0.075	-1.14592	3.184784	-15.26549	49.2135
3	S2, R1	0.18	31.4125	2.6179939	-27.204023	31.70625	0.22319	0.040174	-1.0929	1.273773	-27.19064	38.456
4	skin 3-5	0.075	30.5577	3.1415927	-30.55774907	16	1	0.075	-2.29183	1.2	-30.54437	22.7498
5	S2, R2	0.18	31.4125		-27.12531292	-0.064	1	0.18	-4.88256	-0.01152	-27.11193	6.68576
6	Skin 5_7, R2	2.176559	24.9503	3.9269908	-17.64252465	-17.6425	1	2.176559	-38.4	-38.4	-17.62914	-10.893
7	S2, R2	0.18	27.1253	4.712389	0	-27.1253	1	0.18	0	-4.88256	0.013383	-20.376
8	Skin 7_9, R2	2.176559	24.9503	-0.785398	17.64252465	-17.6425	1	2.176559	38.4	-38.4	17.655908	-10.893
9	S1, R2	0.135	31.4125		27.18781292	-0.057	1	0.135	3.67035	-0.0077	27.201196	6.69276
10	skin 9-11	0.075	30.5577	0	30.55774907	16	1	0.075	2.29183	1.2	30.571132	22.7498
11	S3, R2	0.08	31.5375	0.5235988	27.31227617	31.76875	1	0.08	2.18498	2.5415	27.325659	38.5185
12	skin 1-11	0.075	30.5577	1.0471976	15.27887454	42.46379	1	0.075	1.14592	3.184784	15.292258	49.2135
13	Skin 5_9, R2	3.52724			0	0	1	3.52724	0	0	0.013383	6.74976
∑								8.975533	-0.12012	-60.5827		

		Initial Trial					
N0.	AY^2	AZ^2	AYZ	**σ**linear	**F**x	**M**y	**M**z
1	3.2E-05	528.04	0.1305	-55586	-10005	541920	133.9031
2	17.4776	181.65	-56.35	-55092	-4131.9	203346	-63075.7
3	29.702	59.412	-42.01	-47701	-1916.3	73695	-52106.6
4	69.9719	38.816	-52.12	-32737	-2455.3	55857.7	-74995.9
5	132.31	8.0459	-32.63	-15397	-2771.4	18528.8	-75137.5
6	676.446	258.25	417.96	5278.8	11489.7	125155	202553.6
7	3.2E-05	74.729	-0.049	20150	3627	73902.1	-48.54
8	678.501	258.25	-418.6	15751	34282.3	373429	-605286
9	99.8872	6.0471	24.577	715.33	96.5695	-646.32	-2626.81
10	70.0946	38.816	52.161	-14600	-1095	24910.4	33474.61
11	59.7353	118.69	84.203	-31585	-2526.8	97328.8	69046.65
12	17.539	181.65	56.444	-46023	-3451.7	169873	52785.01
13	0.00063	160.7	0.3186	-7411	-26142	176450	349.853
∑	1851.67	1913.1	34.055		-5000	1933749	-514933

Y_{cg} =	-0.013382977	I_{yy} =	1913.1024	K_1 =	9.62E-06
Z_{cg} =	-6.749758692	I_{zz} =	1851.6652	K_2 =	0.000523
		I_{yz} =	34.05489	K_3 =	0.00054
Xf=	90	**I** =	3541265.4		
Yf=	5.013382977	f_1=	296.77904	σ_{axial}=	-557.07
Zf=	26.74975869	f_2=	-1016.075		
		M_y =	1933748.8		
		M_z =	-514933.1		
		M_{ex}=**Mx** =	0		

second trial

					First Trial			
N0.	**Type**	**A**	**y'**	**Z'**	σ_{linear}	W_{eff}	A_{tot}	K_{sl}
1	S2, R1	0.18	0.01338	54.162259	-55586.01589	0.521663	0.206083	1
3	S2, R1	0.18	-27.1906	38.456009	-47700.87144	0.656219	0.212811	0.22319
5	S2, R2	0.18	-27.1119	6.6857587	-15396.55478	1.2522	0.24261	1
6	Skin 5_7, R2	2.176559	-17.6291	-10.89277	5278.837642	0	2.176559	1
7	S2, R3	0.18	0.01338	-20.37555	20149.99331	0	0.18	1
8	Skin 7_9, R2	2.176559	17.6559	-10.89277	15750.70056	0	2.176559	1
9	S1, R2	0.135	27.2012	6.6927587	715.3298237	0	0.135	1
10	skin 9-11	0.075	30.5711	22.749759	-14599.66025	0	0.075	1
11	S3, R2	0.08	27.3257	38.518509	-31585.08142	0.608955	0.110448	1
13	Skin 5_9, R2	3.52724	0.01338	6.7497587	-7411.359359	0	3.52724	1
∑								

No need for true stress correction, because the applied stress are lower than the proportional limit i.e. Keff = 1

					Buckled Skin	
N0.	**b'**	**A**	**y'**	**Z'**	σ_{linear}	K_{eff}
2	32.89520088	1.64476	-15.26549	49.2135457	-55092.2019	0.094812
4	31.9038322	1.5951916	-30.54437	22.7497587	-32737.45887	0.1595542
12	32.61435534	1.6307178	15.29226	49.2135457	-46023.30259	0.1134947
∑						
∑tot						

						First Trial							
N0.	K_{eff}	A_{eff}	A_{eff} Y'	A_{eff} Z'	**Y**	**Z**	**AY^2**	**AZ^2**	**AYZ**	σ_{linear}	F_x	M_y	M_z
1	1	0.2061	0.0028	11.162	0.65733	52.3918	0.089046	565.678	7.0973	-39352	-8109.69	424881	5330.773
3	1	0.0475	-1.291	1.8266	-26.547	36.6856	33.4726	63.9233	-46.26	-33767	-1603.85	58838	-42576.8
5	1	0.2426	-6.578	1.622	-26.468	4.91532	169.9614	5.86154	-31.56	-10118	-2454.62	12065	-64968.8
6	1	2.1766	-38.37	-23.71	-16.985	-12.663	627.9302	349.026	468.15	5083.9	11065.3	140122	187946.5
7	1	0.18	0.0024	-3.668	0.65733	-22.146	0.077776	88.2801	-2.62	16093	2896.66	64149	-1904.07
8	1	2.1766	38.429	-23.71	18.2999	-12.663	728.8967	349.026	-504.4	12994	28282.1	358142	-517558
9	1	0.135	3.6722	0.9035	27.8451	4.92232	104.6725	3.27094	18.503	2053	277.152	-1364.2	-7717.34
10	1	0.075	2.2928	1.7062	31.2151	20.9793	73.07861	33.0099	49.115	-9135	-685.153	14374	21387.11
11	1	0.1104	3.0181	4.2543	27.9696	36.7481	86.40317	149.151	113.52	-21592	-2384.82	87638	66702.62
13	1	3.5272	0.0472	23.808	0.65733	4.97932	1.524078	87.453	11.545	-4084	-14406.3	71733	9469.734
∑		8.877	1.2246	-5.803			1826.106	1694.68	83.107		12876.8	1E+06	-343888

						Buckled Skin							
N0.		A_{eff}	A_{eff} Y'	A_{eff} Z'	**Y**	**Z**	**AY^2**	**AZ^2**	**AYZ**	σ_{linear}	F_x	M_y	M_z
2		0.15594	-2.3805	7.6745	-14.6215	47.4431	33.338954	351.0039	-108.18	-39096	-6096.691	289246	-89143.01
4		0.25452	-7.7741	5.7903	-29.9004	20.97932	227.54935	112.0221	-159.66	-22836	-5812.218	121936	-173787.7
12		0.18508	2.8303	9.1083	15.93621	47.4431	47.002864	416.582	139.931	-32245	-5967.893	283135	95105.584
∑		0.59554	-7.3244	22.573			307.89117	879.608	-127.9		-17876.8	694318	-167825.1
∑tot		9.47254	-6.0999	16.771			2133.9972	2574.288	-44.796		-5000	#######	-5.12E+05

Y_{cg} =	-0.644	I_{yy} =	2574.29	K_1 =	-8.2E-06	f_1=	224.18	**Xf=**	90	M_y =	1.92E+06
Z_{cg} =	1.7704	I_{zz} =	2134	K_2 =	0.000389	f_2=	-743.8	**Yf=**	5.65733	M_z =	-5.12E+05
		I_{yz} =	-44.796	K_3 =	0.000469			**Zf=**	24.9793		
		I =	5491517	σ_{axial}=	-527.8417						