Airframe Design and Construction

Straight and curved sheet buckling

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Sheet Buckling

➢ The fuselage skin is usually consists of curved sheet panels and some times there are some straight sheets.

➢ If these curved sheets has no stiffeners, failure will occur due to buckling.

➢ When stiffener elements are presented with the fuselage skin, the buckling loads have to be calculated and compared with the ultimate loads of the aircraft structures.

In the present lecture, we are going to study how to calculate the buckling stresses for straight and curved sheets.
Buckling of straight sheets
Buckling under compression

The general buckling equation under compression load takes the form,

$$
\sigma_c = \frac{K_c \pi^2 E}{12(1 - \nu^2)} \left(\frac{t}{b}\right)^2
$$

- $K_c$: Buckling coefficient under compression
- $E$: Young’s Modulus
- $\nu$: Poisson ratio
- $t$: plate thickness
- $b$: panel width
Buckling Compression strength of straight sheets

For straight sheets the buckling coefficient $K_c$ is calculated in terms of the plate aspect ratio

$$\alpha = \frac{a}{b}$$

Or $K_c$ can be calculated from the equation

$$K_c = \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2$$

$$b = \min(b_{avg}, frame\ spacing)$$
Shear buckling of straight sheets

\[ \sigma_s = \frac{K_s \pi^2 E}{12(1 - v^2)} \left( \frac{t}{b} \right)^2 \]
Bending buckling of straight sheets

\[ \sigma_b = \frac{K_b \pi^2 E}{12(1 - v^2)} \left( \frac{t}{b} \right)^2 \]
Buckling of Monocoque Circular Cylinders
Buckling of Monocoque circular cylinders

The term monocoque cylinders means a thin walled cylinders without longitudinal stringers or transverse intermediate frames. Those cylinders can be classified to be:

- **Short cylinders**: behaves like flat plate columns. They develop buckling as sinusoidal wave similar to flat plate. The end fixity conditions are important.
- **Intermediate cylinders**: buckling includes a mixed pattern between sinusoidal and diamond shapes. The end fixity condition less important.
- **Long Cylinders**: buckle in diamond shape pattern. The end fixity conditions has minor importance.
- **Very long cylinders**: buckle as Euler type column buckling.

Long cylinders have \( \frac{L^2}{rt} > 100 \)
Buckling of Monocoque cylinders under compression

The buckling equation takes the form

\[ \frac{F_{cr}}{\eta} = \frac{K_c \pi^2 E}{12(1 - \nu^2)} \left( \frac{t}{L} \right)^2 \]

where, \( t \) = wall thickness, \( L \) is the cylinder length and \( \nu \) is Poisson's ratio. The term \( \eta \) is the plasticity correction factor and equals 1.0 for elastic buckling.

These curves are for short and intermediate cylinders with 99% probability and 95% level of confidence. The curve C8.8b is for 90% probability and 95% level of confidence.

The two curves are for clamped cylinders. Fig. C8.8a
Buckling of Monocoque cylinders under compression

The buckling equation takes the form

\[ \frac{F_{cr}}{\eta} = \frac{K_c \pi^2 E}{12(1-\nu^2)} \left( \frac{t}{L} \right)^2 \]

where, \( t \) = wall thickness, \( L \) is the cylinder length and \( \nu \) is Poisson's ratio. The term \( \eta \) is the plasticity correction factor and equals 1.0 for elastic buckling.

\( \mu \) in this curve means \( \nu \).

All these curves are based on experimental test of aluminum and steel sheets.
Buckling of Monocoque cylinders under bending

Unpressurized bending buckling strength

This curve is has 99% probability and 95% level of confidence.
The curve C8.13a is for 90% probability and 95% level of confidence.
Effect of internal pressure

The internal pressure usually increase the value of the sheet buckling strength.

Increase in Bending Buckling Stress Coefficients Due to Internal Pressure.
Buckling under external hydrostatic pressure

\[ Z_L = \frac{L^2}{rt} \sqrt{1 - \nu_e^2} \]

Fig. C8.15 Buckling Under External Hydrostatic Pressure. \( F_{ccr} = \frac{k_p n^a E}{12 (1 - \nu_e^a) \left( \frac{t}{L} \right)^a} \).
Buckling due external radial pressure

\[ Z_L = \frac{L^2}{rt} \sqrt{1 - \nu_e^2} \]

Fig. C8.16 Buckling Under External Radial Pressure. \[ F_{cr} = \frac{k_y \pi^2 E}{12 (1 - \nu_e^2)} \left( \frac{t}{L} \right)^2 \].
Buckling under pure Torsion

This curve is has 99% probability and 95% level of confidence. The curve C8.21 is for 90% probability and 95% level of confidence.

Fig. C8.20
Buckling under shear

Buckling under shear can be solved as buckling under torsion but with $1.25 \, K_t$
Buckling under torsion with internal pressure

\[ R_{st}^a + R_p = 1 \]

where,

\[ R_{st} = \text{ratio of } \frac{\text{applied torsion shear stress}}{\text{allowable torsion shear stress}} \]

\[ R_p = \text{ratio of } \frac{\text{applied internal pressure}}{\text{external hydrostatic buckling pressure}} \]

Note that \( R_p \) has a negative sign. The value of the external hydrostatic buckling pressure can be determined by use of Fig. C8.15. Fig. C8.22 shows a plot from Equation C8.10 and its comparison with test data.
Buckling under transverse shear and internal pressure

\[ R_s + R_p = 1 \]

\[ R_s = \frac{\text{applied transverse shear stress}}{\text{allowable transverse shear stress}} \]

\[ R_p = \text{ratio of} \frac{\text{applied internal pressure}}{\text{external hydrostatic buckling pressure}} \]
### Table C8.1

**Summary of Interaction Equations for Buckling of Pressurized and Unpressurized Circular Cylinders**

<table>
<thead>
<tr>
<th>Combined Loading Condition</th>
<th>Interaction Equation</th>
<th>Equation For Margin of Safety</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal Compression and Pure Bending</td>
<td>$R_c + R_b = 1$</td>
<td>$M.S. = \frac{1}{R_c + R_b} - 1$</td>
</tr>
<tr>
<td>Longitudinal Compression and Torsion</td>
<td>$R_c + R_{st}^a = 1$ (See Fig. C8.23)</td>
<td>$M.S. = \frac{2}{R_c + \sqrt{R_c^4 + 4R_{st}^4}} - 1$</td>
</tr>
<tr>
<td>Longitudinal Tension and Torsion</td>
<td>$R_{st}^a - (R_t) = 1$ (See Note 1)</td>
<td></td>
</tr>
<tr>
<td>Pure Bending and Torsion</td>
<td>$R_b^{\frac{1}{2} + 0} + R_{st}^a = 1$ (See Fig. C8.20)</td>
<td></td>
</tr>
<tr>
<td>Pure Bending and Transverse Shear</td>
<td>$R_b^3 + R_{st}^a = 1$ (See Fig. C8.23)</td>
<td></td>
</tr>
<tr>
<td>Longitudinal Compression, Pure Bending and Transverse Shear</td>
<td>$R_c + \sqrt{R_{st}^a + R_b^a} = 1$</td>
<td></td>
</tr>
<tr>
<td>Longitudinal Compression, Pure Bending and Torsion</td>
<td>$R_c + R_b + R_{st}^a = 1$</td>
<td>$M.S. = \frac{2}{R_c + R_b + \sqrt{(R_c + R_b)^a + 4R_{st}^a}} - 1$</td>
</tr>
<tr>
<td>Longitudinal Compression, Pure Bending, Transverse Shear and Torsion</td>
<td>$R_c + R_{st}^a + \sqrt{R_{st}^a + R_b^a} = 1$</td>
<td></td>
</tr>
</tbody>
</table>

**NOTE 1.** $R_t = \frac{\text{applied tensile stress}}{\text{compression buckling allowable}}$

$R_t < 0.8$
Example 1 - Buckling of circular Monocoque cylinders

A circular cylinder has a radius \( r = 50 \) inches, a length \( L = 75 \) inches, and a wall thickness \( t \) of \( .05 \) inches. The material is aluminum alloy 2024-T3, for which \( E_c = 10,700,000 \) psi.

Question 1: what compressive load will it carry?
Question 2: what bending moment will buckle the cylinder?
Question 3: what torsional moment this cylinder can develop?
Question 4: if this cylinder is subject to compressive load \( P = 10700 \) Ibs, and a bending moment \( M = 282000 \) in. Ibs. What is the margin of safety under this combined loading?
Example 1 – Question 1

A circular cylinder has a radius \( r = 50 \) inches, a length \( L = 75 \) inches, and a wall thickness \( t \) of \( 0.05 \) inches. The material is aluminum alloy 2024-T3, for which \( E_c = 10,700,000 \) psi.

Question 1: what compressive load will it carry?

Solution

\[ r/t = 50/0.05 = 1000, \quad L/r = 75/50 = 1.5 \]

\[
F_{cr} = \frac{k_c \frac{\pi^2 E}{12} (t)^2}{L} \quad \text{(See Eq. C8.2)}
\]

To find the buckling coefficient \( k_c \), we use Fig. C8.7.

\[
Z = \frac{L^2}{rt} \sqrt{1 - \nu_e^2}
\]

\[
Z = \frac{75^2}{50 \times 0.05} \sqrt{1 - 0.3^2} = 2140
\]

From Fig. C8.7 using \( Z = 2140 \) and \( r/t = 1000 \), we read \( k_c = 280 \), then

\[
F_{cr} = \frac{280 \frac{\pi^2 \times 10,700,000}{12} (0.05)^2}{75} = 1210 \text{ psi}
\]

The buckling axial compressive load

\[
P = 2\pi rt F_{cr} = 2\pi \times 50 \times 0.05 \times 1205 = 18950 \text{ lb.}
\]
Example 1 – Question 1

A circular cylinder has a radius \( r = 50 \) inches, a length \( L = 75 \) inches, and a wall thickness \( t \) of .05 inches. The material is aluminum alloy 2024-T3, for which \( E_c = 10,700,000 \) psi.

Question 1: what compressive load will it carry?

Solution

If we use the design curves of Fig. C8.8b based on 90 percent probability and 95 percent confidence, we read for \( r/t = 1000 \) and \( L/r = 1.5 \), that \( F_{c_{cr}}/E = .000121 \). Thus \( F_{c_{cr}} = 10,700,000 \times .000121 = 1295 \) psi. If we multiply this value by the .95 confidence value, we obtain 1230 psi which is practically the same as obtained above using Fig. C8.7.

\[ P_a = 1295 \times 2\pi \times .05 \times 50 = 20350 \text{ lbs.} \]

If we require 99 percent probability and 95 percent confidence, we use Fig. C8.8a. For \( r/t = 1000 \) and \( L/r = 1.5 \), we read \( F_{s_{cr}}/E = .000082 \) or \( F_{s_{cr}} = 10,700,000 \times .000082 = 877 \) psi, which would give an axial buckling load of 12,700 lbs. Thus requiring a 99 percent probability decreases the buckling load considerably.

As the probability increase the buckling load will decrease and subsequently increase the aircraft weight.
Example 1 – Question 2

A circular cylinder has a radius $r = 50$ inches, a length $L = 75$ inches, and a wall thickness $t$ of .05 inches. The material is aluminum alloy 2024-T3, for which $E_c = 10,700,000$ psi.

Question 2: what bending moment will buckle the cylinder?

Solution. The curve in Fig. C8.12 will be used. For $r/t = 1000$ and $L/r = 1.5$, we read from Fig. C8.12 that the bending buckling coefficient $C_b = .16$.

$$F_{bc} = C_b E_t / r \quad \text{(See Eq. C8.5)}$$

$$= .16 \times 10,700,000 \times .05/50 = 1710 \text{ psi}$$
Example 1 – Question 2

A circular cylinder has a radius $r = 50$ inches, a length $L = 75$ inches, and a wall thickness $t$ of .05 inches. The material is aluminum alloy 2024-T3, for which $E_c = 10,700,000$ psi.

Question 2: what bending moment will buckle the cylinder?

The bending moment developed at this buckling stress is,

$$M = F_{b_{cr}} \frac{I}{r} = F_{b_{cr}} \pi r^4 t$$

$$= 1710 \pi \times 50^4 \times .05 = 670,000 \text{ in. lb.}$$

If we use Fig. C8.13a based on 90 percent probability and 95 percent confidence, we read for $r/t = 1000$ and $L/r = 1.5$ that $F_{b_{cr}}/E$ is .000160. Thus $F_{b_{cr}} = 10,700,000 \times 0.000160 = 1710$ psi. Since it is difficult to read Fig. C8.12, it is recommended that Fig. C8.13a be used in design.

If we require 99 percent probability, we use Fig. C8.13 and obtain $F_{b_{cr}}/E = .000092$, which gives $F_{b_{cr}} = 10,700,000 \times .000092 = 985$ psi as against 1710 for 90 percent probability.
Example 1 – Question 3

A circular cylinder has a radius \( r = 50 \) inches, a length \( L = 75 \) inches, and a wall thickness \( t \) of \( .05 \) inches. The material is aluminum alloy 2024-T3, for which \( E_c \) 10,700,000 psi.

Question 3: what torsional moment this cylinder can develop?

Solution. Using design curve in Fig. C8.18

\[
Z = \frac{L^2}{rt} \sqrt{1 - \nu_e^2} = \frac{75^2}{50 \times .05} \sqrt{1 - .3^2} = 2140
\]

For this value of \( Z \), we read the torsional buckling coefficient \( k_t \) from Fig. C8.18 to be 170.

\[
F_{stcr} = \frac{k_t \pi^2 E}{12 (1 - \nu_e^2) \left( \frac{t}{L} \right)^2} \quad (\text{See Eq. C8.9})
\]

\[
F_{stcr} = \frac{170 \pi^2 \times 10,700,000}{12 (1 - .3^2)} \left( \frac{.05}{75} \right)^2 = 735 \text{ psi}
\]

The torsional moment developed by this buckling stress is,

\[
T = F_{stcr} \frac{J}{r}. \quad J/r = 2\pi^2 t/r = 2\pi^2 t
\]

whence,

\[
T = F_{stcr} 2\pi^2 t
\]

\[
T = 735 \times 2\pi \times 50^2 \times .05 = 578000 \text{ in.\text{lbs.}}
\]
Example 1 – Question 3

A circular cylinder has a radius \( r = 50 \) inches, a length \( L = 75 \) inches, and a wall thickness \( t \) of \( 0.05 \) inches. The material is aluminum alloy 2024-T3, for which \( E_c = 10,700,000 \) psi.

Question 3: what torsional moment this cylinder can develop?

The result will also be calculated using Figs. C8.20 and C8.21 based on 99 percent and 90 percent probability respectively and 95 percent confidence. Using Fig. C8.21:-

For \( r/t = 1000 \) and \( L/r = 1.5 \), we read \( F_{st/E} = 0.000082 \). Then \( F_{st} = 0.000082 \times 10,700,000 = 876 \) psi. \( T = 876 \times 2\pi \times 50^\circ \times 0.05 \)

\[ = \frac{668000}{\text{in. lbs.}} \]

Using Fig. C8.20 which is for 99 percent proba-

\[ = \frac{668000}{\text{in. lbs.}} \]
Example 1 – Question 4

A circular cylinder has a radius $r = 50$ inches, a length $L = 75$ inches, and a wall thickness $t$ of .05 inches. The material is aluminum alloy 2024-T3, for which $E_c = 10,700,000$ psi.

Question 4: if this cylinder is subject to compressive load $P = 10700$ Ibs, and a bending moment $M = 282000$ in. Ibs. What is the margin of safety under this combined loading?

Solution. The interaction equation for this type of combined loading from Table C8.1 is,

$$R_c + R_b = 1$$

$$R_c = \frac{P}{P_a} \quad (P_a \text{ from Problem 1 solution is } 20350 \text{ lb.})$$

$$R_c = \frac{10700}{20350} = .527$$

$$R_b = \frac{M}{M_a} \quad (\text{From Problem 2 results: } M_a = 670,000 \text{ in.lbs.})$$

$$R_b = \frac{282,000}{670,000} = .421$$

$$R_c + R_b = .527 + .421 = .948. \text{ Since the result is less than } 1.0, \text{ we have a small margin of safety.}$$

$$M.S. = \frac{1}{R_c + R_b} - 1 = \frac{1}{.948} - 1 = .055$$
Buckling strength of curved-stiffened sheets
Buckling compression strength of curved sheets

\[ r = \frac{r_{\text{station1}} + r_{\text{station2}}}{2}, \quad Z = \frac{b^2}{r \times t} \sqrt{1 - v^2}, \quad \sigma_{cr} = \frac{k_c \pi^2 E}{12(1 - v^2)} \left( \frac{t}{b} \right)^2 \]
Shear buckling strength

The general buckling equation under shear load takes the form,

\[ \sigma_s = \frac{K_s \pi^2 E}{12(1 - \nu^2)} \left( \frac{t}{b} \right)^2 \]

- \( K_s \): Buckling coefficient under shear
- \( E \): Young’s Modulus
- \( \nu \): Poisson ratio
- \( t \): plate thickness
- \( b \): panel width
Shear buckling of curved sheets

For long clamped curved plates.

\[ Z = \frac{b^2}{r^*t}\sqrt{1-v^2} \]

(From Ref. 2)
Shear buckling of curved sheets

For wide clamped curved plates.

\[ Z = \frac{b^2}{r \cdot t} \sqrt{1 - v^2} \]

Fig. C9.3
Shear buckling of curved sheets

**Fig. C8.5** Shear Buckling Coefficient for Wide Simply Supported Curved Plates.

\[ F_{scr} = \frac{K_s \pi^2 E}{12 (1 - \nu^2)} \left( \frac{t}{b} \right)^3 \]

\[ Z_b = \frac{b^3}{t} \left( 1 - \nu^2 \right)^{1/3} \]

*(From Ref. 2)*

**Fig. C9.4** Shear Buckling Coefficient for Long Simply Supported Curved Plates.

\[ F_{scr} = \frac{K_s \pi^2 E}{12 (1 - \nu^2)} \left( \frac{t}{b} \right)^3 \]

\[ Z_b = \frac{b^3}{rt} \left( 1 - \nu^2 \right)^{2/3} \]

*(From Ref. 2)*
Buckling of curved sheets under combined axial compressive and shear

\[ R_s^2 + R_L = 1 \]

where, \( R_s = \frac{f_s}{F_{scr}} \), \( R_L = \frac{f_c}{F_{ccr}} \)

\[ M.S. = \frac{2}{R_L + \sqrt{R_L^2 + 4R_s^2}} - 1 \]
Compressive buckling of curved sheets with internal pressure

\[ R_c^a + R_p = 1 \]

where \( R_c = \frac{f_c}{F_{c_{cr}}} \)

\[ R_p = \text{ratio of } \left[ \frac{\text{applied internal pressure}}{\text{external inward pressure that would buckle the cylinder of which the curved panel is a section.}} \right] \]

Found by use of Fig. C8.16 in Chapter C8.

Fig. C8.16 Buckling Under External Radial Pressure. \( F_{c_{cr}} = \frac{k_y n^a E}{12 \left( 1 - \nu_e^2 \right)} \left( \frac{L}{L} \right)^a. \)
Shear buckling of curved sheets with internal pressure

\[ R_s^a + R_p = 1 \]

where \( R_s = \frac{f_s}{F_{SCR}} \)

\[ R_p = \text{ratio of } \left[ \frac{\text{applied internal pressure}}{\text{external inward pressure that would buckle the cylinder of which the curved sheet panel is a section}} \right] \]

Fig. C8.16 Buckling Under External Radial Pressure.  \( F_{cr} = \frac{k \pi^2 E}{12 (1 - \nu_e^2) \left( \frac{L}{L} \right)^4} \).
Example 2: Buckling of curved – stiffened sheets

Fig. C9.6 illustrates a circular fuselage section with longitudinal stringers represented by the small circles. The area of each stringer is .15 sq. in. The skin thickness is .04 inches. All material is aluminum alloy with $E_c = 10,700,000$. The fuselage frame spacing (a) is 15.75 inches. The fuselage section is subjected to the following load system:

$M_y = 600,000$ in. lb. (causing compression on top half)
$V_z = 5175$ lbs. (acting up)
$T = $ Torsional moment $= 210,000$ in. lbs. (acting counterclockwise)

Question 1: Determine whether skin panels (A) and (B) will buckle under the given combined load?
Question 2: what is the compressive buckling stress, if the fuselage is subject to internal outward pressure of 5 psi?
Example 2: Question 1

To find the bending stresses, the moment of inertia of the cross section about axis y-y is necessary, which axis is also the neutral axis since all material is effective. The moment of inertia will equal 4 times that due to material in one quadrant.

\[ I_y \text{ due to stringers is,} \]

\[ I_y = 4 \left( 0.075 \times 20^2 + 0.15 \times (19.30^2 + 17.34^2 + 14.14^2 + 10^2 + 5.18^2) \right) = 720 \text{ in.}^4 \]

Due to skin:--

\[ I_y = \pi r^3 t = \pi \times 20^3 \times 0.04 = 1005 \text{ in.}^4. \]

Total \( I_y = 720 + 1005 = 1725 \text{ in.}^4. \)
Consider Skin Panel (A):

\[ \frac{r}{t} = 20/0.04 = 500, \quad \frac{a}{b} = \frac{15.75}{5.25} = 3.0 \]

To determine the compressive buckling stress, use will be made of Fig. C9.1 to find the buckling coefficient \( K_c \).

\[ Z = \frac{b^2}{rt} (1 - \nu e^{2})^{1/2} \]

\[ = \frac{5.25^2}{20 \times 0.04} (1 - 0.3^2)^{1/2} = 32.9 \]

From Fig. C9.1, \( K_c = 14 \)
Example 2: Question 1

\[ F_{cr} = \frac{K_c \pi^2 E}{12 \left(1 - \nu_e^2\right)} \left(\frac{t}{b}\right)^2 \]

\[ = \frac{14 \pi^2 \times 10,700,000}{12 \left(1 - .3^2\right)} \left(\frac{.04}{5.25}\right)^2 = 7850 \text{ psi} \]

To find the shear buckling stress, we use Fig. C9.2. Z is the same as calculated above or 32.9. Thus from Fig. C9.2 we read for \(\frac{a}{b} = 3\), that \(K_s = 20\).

\[ F_{scr} = \frac{20 \pi^2 \times 10,700,000}{12 \left(1 - .3^2\right)} \left(\frac{.04}{5.25}\right)^2 = 11,200 \text{ psi} \]

The bending stress at midpoint of Panel (A) will be calculated:

\[ f_c = \frac{Mz}{I_y} = \frac{600,000 \times 19.7}{1725} = 6850 \text{ psi} \]

Thus stress ratio \(R_c = \frac{f_c}{F_{cr}} = 6850/7850\)

\[ = .874 \]
Example 2: Question 1

Shear stress on Panel (A) due to torsion is,

\[ f_s = \frac{T}{2At}, \text{ where } A \text{ is inclosed area of fuselage cell.} \]

\[ f_s = \frac{210,000}{2} \times \pi \times 20^a \times .04 = 2090 \text{ psi.} \]

The panel is also subjected to shear stress due to transverse shear of \( V_z = 5175 \text{ lbs.} \)

The shear flow equation is,

\[ q = \frac{-V_z}{I_y} \sum ZA = \frac{-5175}{1725} \sum ZA = -3 \sum ZA. \]
Example 2: Question 1

The shear flow will be zero on Z axis. The shear flow at top edge of Panel (A) will be due to effect of one-half the area of stringer (1).

\[ q_{a-1} = -3 \times 0.075 \times 20 = -4.5 \text{ lb./in.} \]

Area of skin between stringers (1) and (2) is \( 5.25 \times 0.04 = 0.21 = \text{ft.} \).

Distance from centroid of Panel (A) from neutral axis is \( Z = r \sin a/a \). \( a = 15^\circ \), which gives \( Z = 19.8 \text{ in.} \)

\[ Z = R \cos(\theta) = 20 \cos \left( \frac{15}{2} \right) \]

Thus \( q_{a-1} = -4.5 - 3 \times 0.21 \times 19.8 = -17 \text{ lb./in} \)

Then average shear flow on panel is \( (17 + 4.5)/2 = 10.75 \).

Then shear stress = \( q/t = 10.75/0.04 = 269 \text{ psi} \).

This shear stress has the same sense or direction as the torsional shear stress so we add the two to obtain the true shear or:
Example 2: Question 1

\[ f_s(\text{total}) = 269 + 2090 = 2359 \text{ psi}. \]

Then shear stress ratio \( R_s = \frac{f_s}{F_{scr}} = \frac{2359}{11200} = .21. \)

The interaction equation for combined compression and shear is,

\[ R_c + R_s^2 = 1 \]

\[ .874 + .21^2 = .918. \] This is less than 1.0 so panel will not buckle.

\[
M.S. = \frac{2}{.874 + \sqrt{.874^2 + 4 \times .21^2} - 1} = .09
\]
Example 2: Question 1

Consider Skin Panel (B).

Arm $Z$ to midpoint of panel = 15.88 in.

$f_c = \frac{M_Z}{I} = 600,000 \times 15.88/1725 = 5520 \text{ psi}$

$R_c = \frac{5520}{7850} = .704$

The torsional shear stress is the same on all panels or $f_g = 2090 \text{ psi}$ as previously calculated.

Shear flow $q$ due to transverse shear load:

$q = \frac{V}{I} \Sigma Z A = 3 \Sigma Z A.$

Calculation of $\Sigma Z A$ at upper edge of panel:

For stringers = $.075 \times 20 + .15 (19.3 + 17.34) = 7.0$
Example 2: Question 1

For skin: Area $= 2 \times 5.25 \times .04 = .42$.

Vertical distance $\bar{Z}$ to centroid of skin portion $= r \sin \alpha / \alpha$. $\alpha = 30^\circ$. The result is $\bar{Z} = 19.1$ in. The $\bar{Z}A = 19.1 \times .42 = 8.03$. Total $\Sigma ZA = 8.03 + 7.0 = 15.03$. Then $q = 3 \Sigma ZA = 3 \times 15.03 = 45.09$ lb./in.

$$Z = R \cos(\theta)$$

A similar calculation for shear flow at lower edge of Panel (B) would give $q$ equal 55.0. Thus average shear flow on panel is $(55 + 45.09)/2 = 50.04$. Then $f_s = q/t = 50.14/.04 = 1251$ psi. The total shear stress $f_s$ on panel then equals $1251 + 2090 = 3341$ psi.

$$R_s = f_s / F_{scr} = 3341/11200 = .299$$
Example 2: Question 1

Substituting in interaction equation $R_c + R_s^2 = .704 + .293^2 = .793$. The result is less than 1.0, thus panel will not buckle.

The student should check other panels for buckling and compare their margins of safety.

In general the compressive stress is the dominant factor in causing the panel buckling. Thus to increase the buckling stress of the panels and also to give a more effective stringer arrangement to carry the bending moment, the stringers should be spaced closer in the top and bottom regions of the cross-section and with increased spacing as the neutral axis is approached.
Example 2: Question 2

From Art. C9.6, the interaction equation is,

$$R_c^2 + R_p = 1, \quad R_c = \frac{f_c}{F_{c\text{cr}}}$$

From Problem 1, the compressive buckling stress \( F_{c\text{cr}} = 7850 \text{ psi} \).

To evaluate \( R_p \), the external inward radial acting pressure that would cause buckling of a circular cylinder having a radius equal to that of the curved sheet panel must be determined. Use is made of Fig. C8.16 of Chapter C8 to determine the backing stress under such a loading. The lower scale parameter for Fig. C8.16 is,
Example 2: Question 1

\[ Z_L = \frac{L^2}{rt} \sqrt{1 - \nu_e^2} \]

\[ = \frac{15.75^2}{20 \times 0.04} \sqrt{1 - 0.3^2} = 296 \]

From Fig. 8.16, we read \( K_y = 19 \)

\[ F_{c_{cr}} = \frac{K_y \pi^2 E}{12 (1 - \nu_e^2)} \left( \frac{t}{L} \right)^2 \]

\[ = \frac{19 \pi^2 \times 10,700,000}{12 (1 - 0.3^2)} \left( \frac{0.04}{15.75} \right)^2 = 1215 \text{ psi} \]

The external radial pressure to produce this buckling stress is,

\[ p = F_{c_{cr}} \frac{t}{r} = 1215 \times 0.04/20 = 2.43 \text{ psi} \]

\[ R_p = \frac{5.0}{2.43} = 2.06 \]

Subt. in interaction equation with a minus sign for \( R_p \),
Example 2: Question 1

\[
\frac{f_c^2}{F_{ccr}} - R_p = 1
\]

\[
\frac{f_c}{7850^2} - 2.06 = 1, \text{ or } f_c = 13750 \text{ psi}
\]

Thus the internal outward pressure of 5 psi increases the axial compressive buckling stress from 7850 to 13750 psi.