Airframe Design and Construction

Bending stresses

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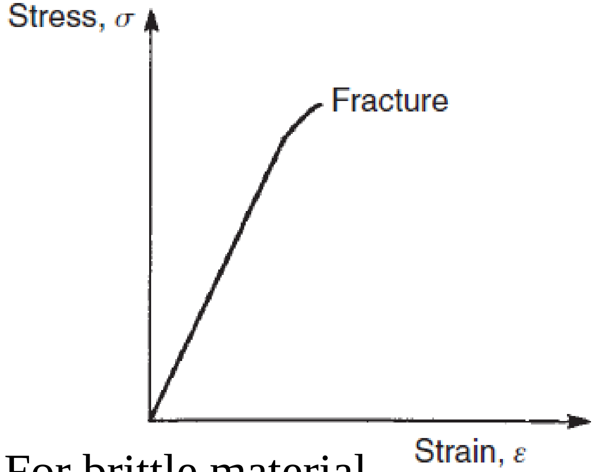
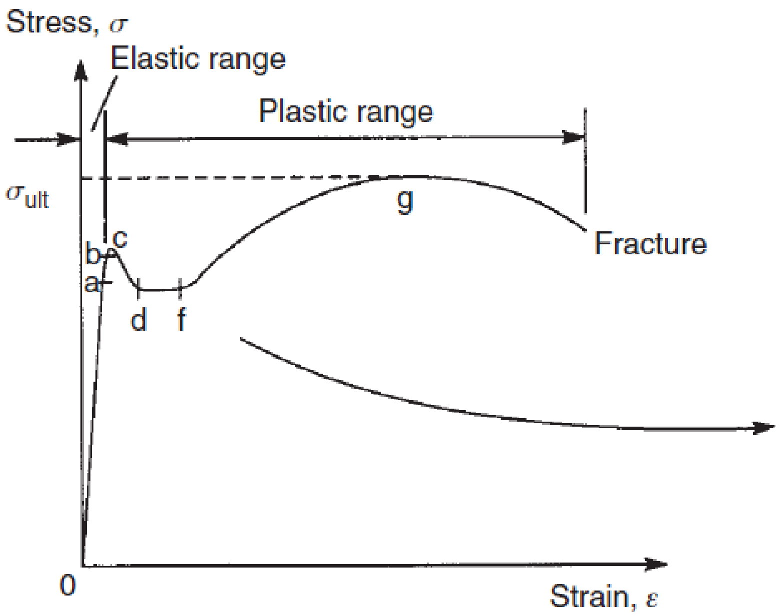
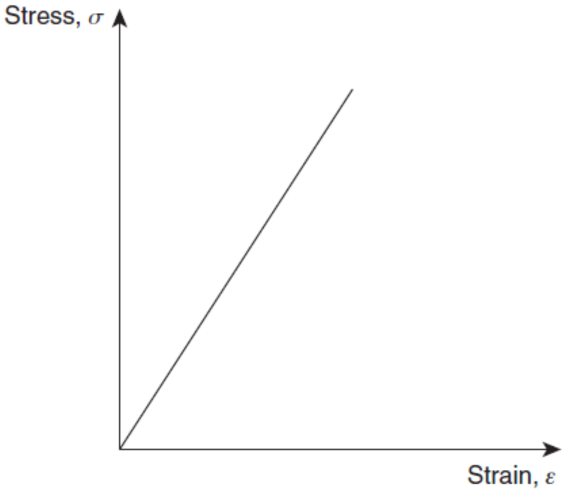
Material Properties



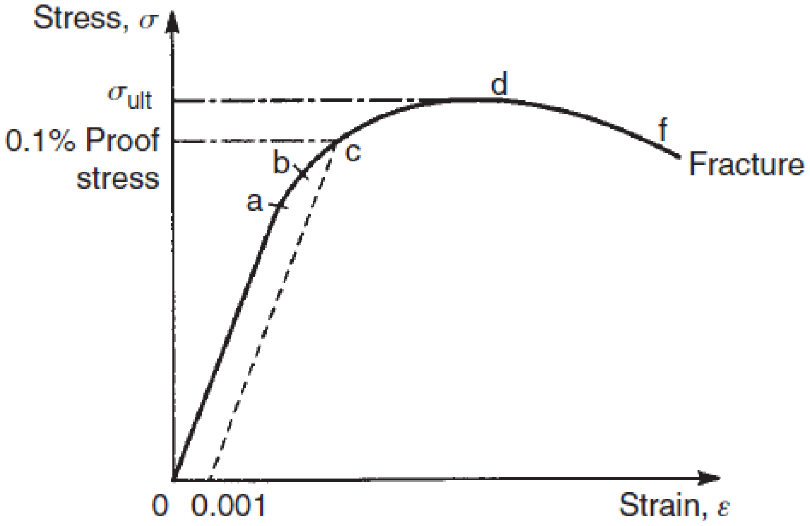
For brittle material



For steel



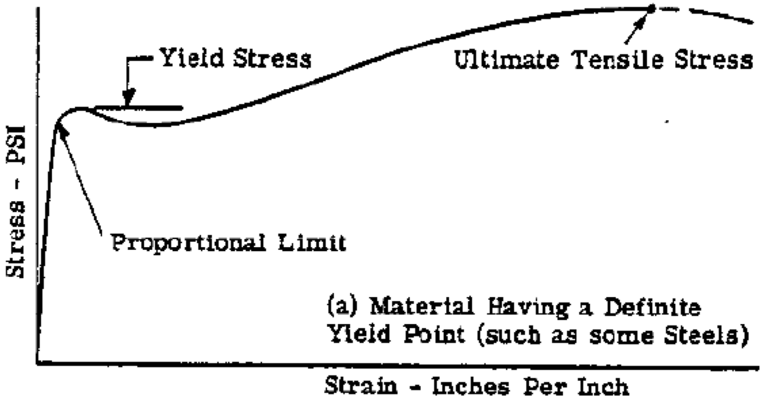
For fiber composite

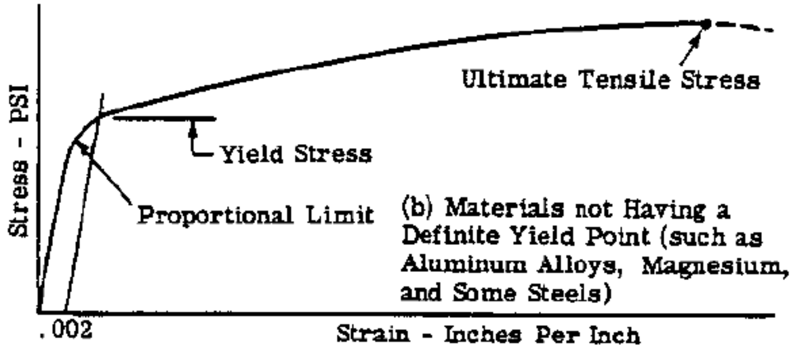


For aluminum

The static tension stress-strain diagram

- Modulus of elasticity (E): defines the material stiffness or the ratio between the stress and strain (slop of stress-strain diagram).
- Poisson's ratio (ν): defines the ratio of the strain in direction normal to the stress direction to the strain in direction of the stress.
- Tensile yield stress (σ_{ty}): the maximum stress that can be applied to the material before it starts to change shape permanently.
- Ultimate tensile stress (σ_{tu}): is the stress under the maximum load a material can carry.





Bending stress

- Plane sections remains planes after bending, but they rotate w.r.t each other.

- After applying stresses, the top fiber are shortened and the bottom fibers are elongated.

- At certain plane on the cross-section, the fiber suffer no deformation and no stresses, and this location is referred to as the *neutral axis*.

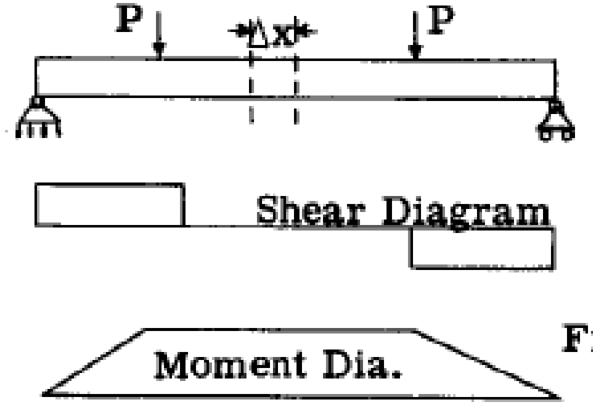


Fig. b

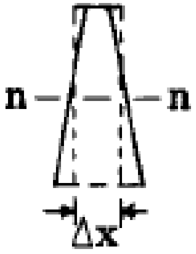


Fig. c

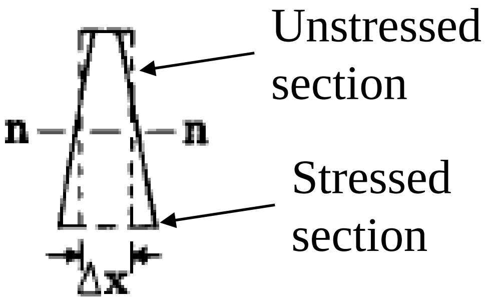


Fig. c

Location of Neutral axis

The neutral axis passes through the centroid of the cross-sectional area when the material follows Hooke's law and there is no axial force acting on the cross section.

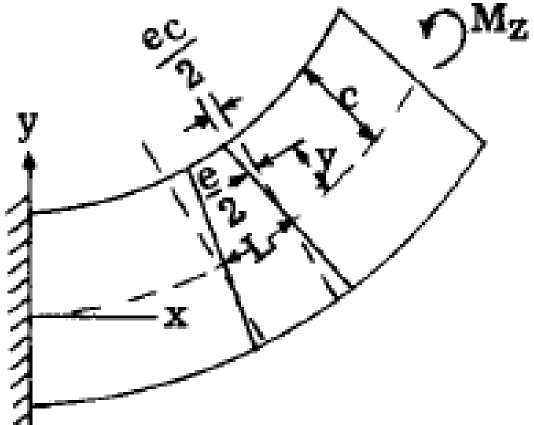
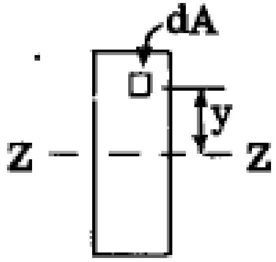


Fig. d



Beam Section

Equation of bending stresses

Assumptions:

- ➢Straight cantilever beam with constant cross-section.
- ➢The beam subject to pure bending such that no torsion moments applied.

It is required to determine the *neutral axis* direction and the bending stress at any point within the cross-section.

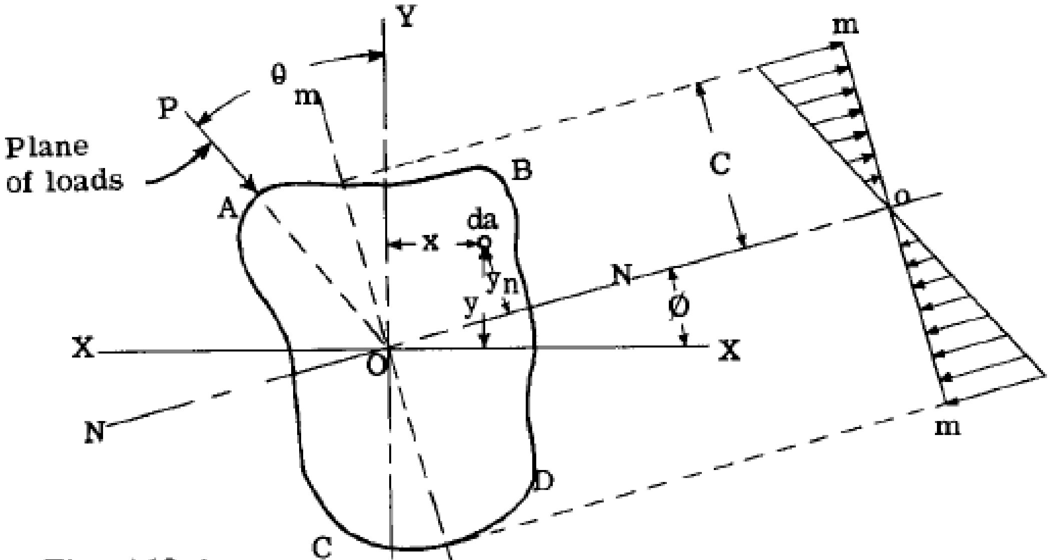


Fig. A13.1

Fig. A13.2

Equation of bending stresses

Let σ represent unit bending stress at any point a distance y_n from the neutral axis. Then the stress σ on da is

$$\sigma = ky_n \quad \text{-------------------} \quad (1)$$

where k is a constant. Since the position of the neutral axis is unknown, y_n will be expressed for convenience in terms of rectangular co-ordinates with respect to the axes X-X and Y-Y.

Thus, $y_n = (y - x \tan \varnothing) \cos \varnothing$ -------

$$= y \cos \varnothing - x \sin \varnothing \quad \text{-------------} \quad (2)$$

Then Eq. (1) becomes

$$\sigma = k\,(y \cos \varnothing - x \sin \varnothing) \quad \text{----------} \quad (3)$$

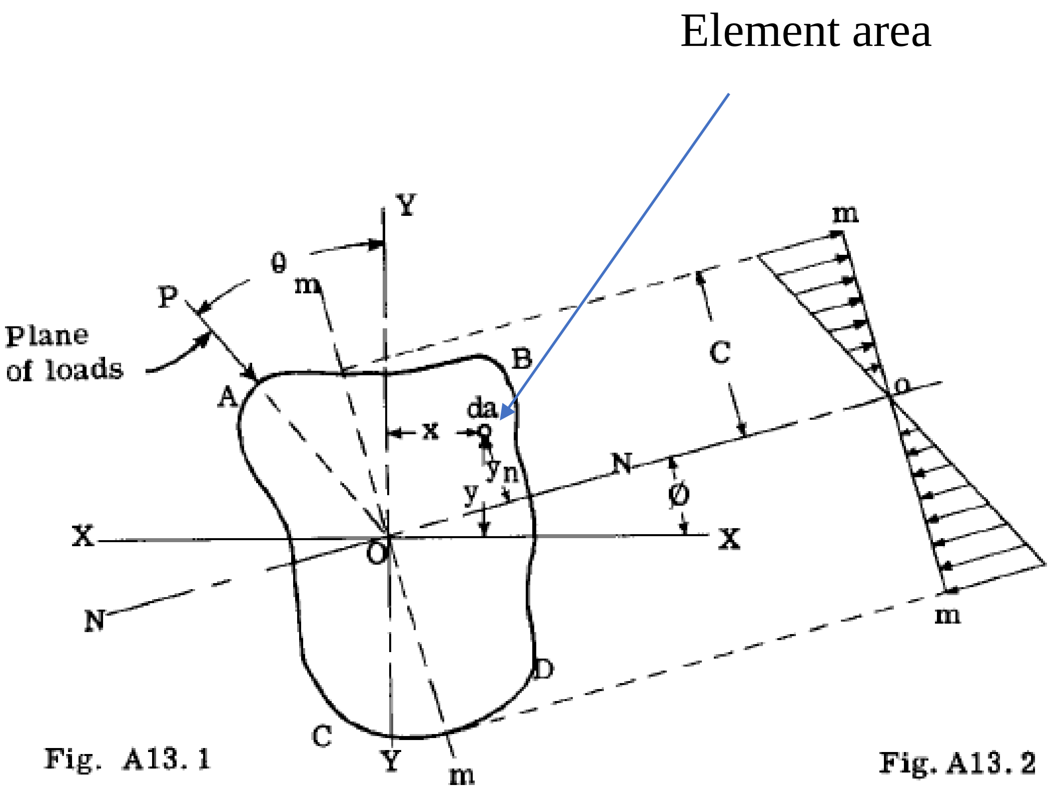


Fig. A13.1

Fig. A13.2

Equation of bending stresses

Let M represent the bending moment in the plane of the loads; then the moment about axis X-X and Y-Y is $M_x = M \cos \theta$ and $M_y = M \sin \theta$. The moment of the stresses on the beam section about axis X-X is $\int \sigma \, da \, y$. Hence, taking moments about axis X-X, we obtain for equilibrium,

$$M \cos \theta = \int \sigma \, da \, y$$

$$= \int k \, (\cos \phi \, y^2 da - \sin \phi \, xyda)$$

$$= k \cos \phi \int y^2 da - k \sin \phi \int xyda \quad - - - - \quad (4)$$

In similar manner, taking moments about the Y-Y axis

$$M \sin \theta = \int \sigma \, da \, x$$

whence

$$M \sin \theta = - k \sin \phi \int x^2 da + k \cos \phi \int xyda \quad (4a)$$

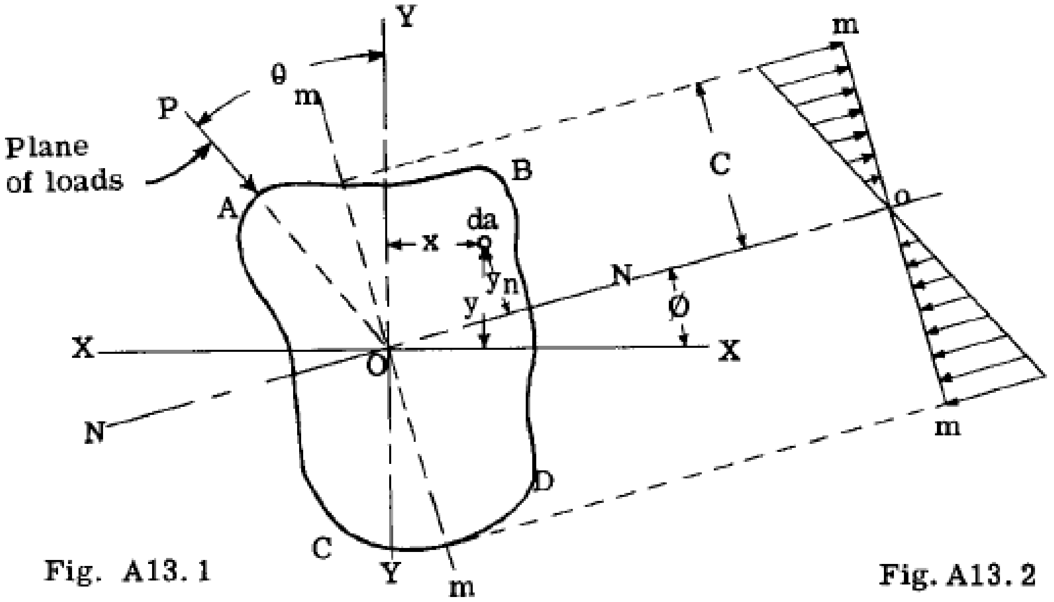


Fig. A13.1

Fig. A13.2

Equation of bending stresses

$$M \cos \theta = \int \sigma \, da \, y$$

$$= \int k (\cos \phi \, y^2 da - \sin \phi \, xyda)$$

$$= k \cos \phi \int y^2 da - k \sin \phi \int xyda \quad ---- \quad (4)$$

The fiber stresses can be found without resort to principal axes or to the neutral axis.

Equation (4) can be written:

$$M_X = k \cos \phi \, I_X - k \sin \phi \, I_{XY} \quad ------ \quad (11)$$

where $I_X = \int y^2 da$ and $I_{XY} = \int xyda$, and $M_X = M \cos \theta$.

In like manner,

$$M_Y = - k \sin \phi \, I_Y + k \cos \phi \, I_{XY} \quad ----- \quad (12)$$

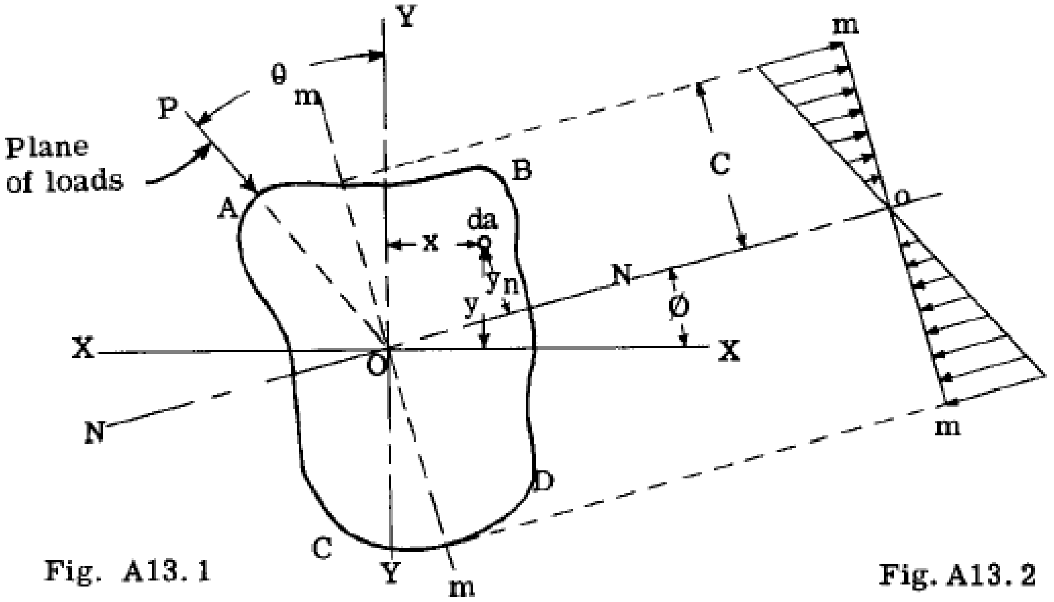


Fig. A13.1

Fig. A13.2

Equation of bending stresses

Solving equations (11) and (12) for sin Ø and cos Ø and substituting their values in equation (3), we obtain the following expression for the fiber stress σ_b: -

$$\sigma_b = -\frac{(M_y I_x - M_x I_{xy})}{I_x I_y - I_{xy}^2} x - \frac{(M_x I_y - M_y I_{xy}) y}{I_x I_y - I_{xy}^2} \quad - (13)$$

For simplification, let

$$K_1 = I_{xy}/(I_x I_y - I^2{}_{xy})$$

$$K_2 = I_y/(I_x I_y - I^2{}_{xy})$$

$$K_3 = I_x/(I_x I_y - I^2{}_{xy})$$

Substituting these values in Equation (13): -

$$\sigma_b = -(K_3 M_y - K_1 M_x)\, x - (K_2 M_x - K_1 M_y) y \quad - (14)$$

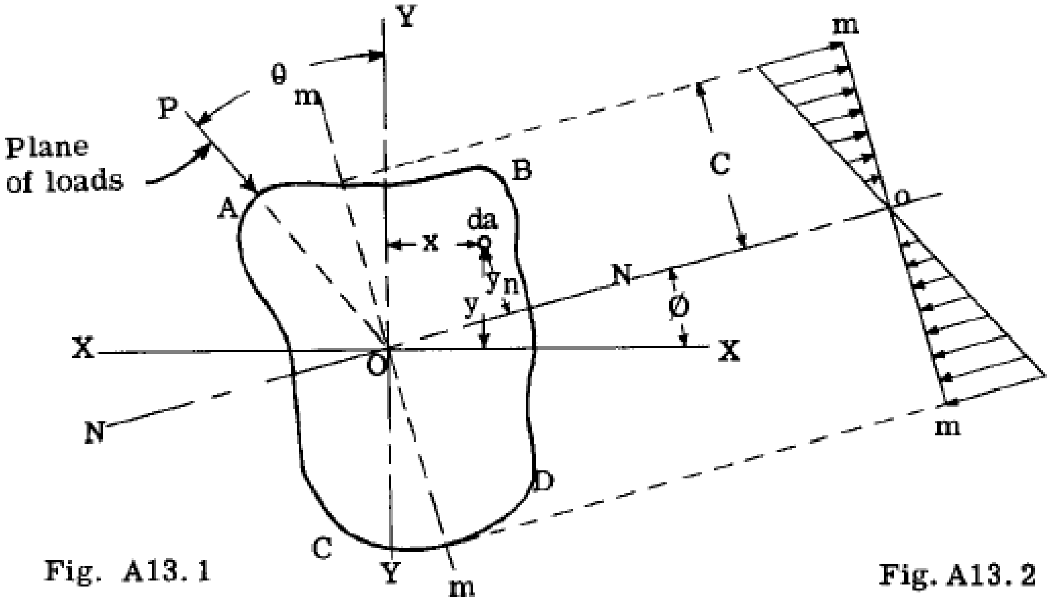


Fig. A13.1

Fig. A13.2

Equation of bending stresses

The bending stresses about symmetric *x* and *y* axes (Principal axes),

$$\sigma_b = -\frac{M_x y}{I_x} - \frac{M_y x}{I_y}$$

If the section is symmetric and $M_x = 0$, then

$$\sigma_b = -\frac{M_y x}{I_y}$$

If the section is symmetric and $M_y = 0$, then

$$\sigma_b = -\frac{M_x y}{I_x}$$

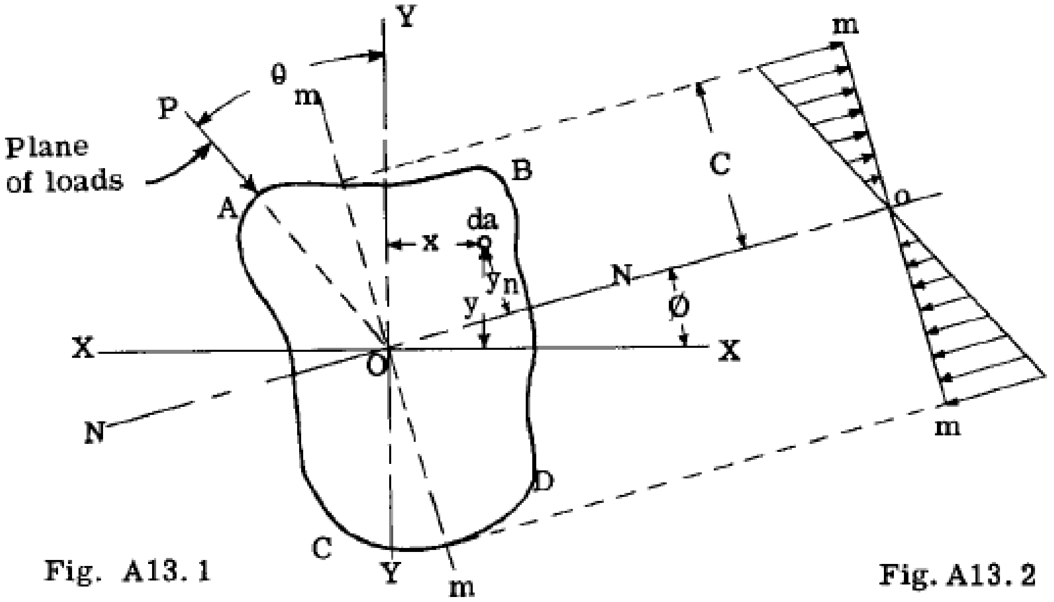


Fig. A13.1 Fig. A13.2

Process for calculating the Neutral Axis position

1. Define the stress equation to equal to zero.

$$\sigma_b = -\frac{(M_y I_x - M_x I_{xy})}{I_x I_y - I_{xy}^2}\,x - \frac{(M_x I_y - M_y I_{xy})y}{I_x I_y - I_{xy}^2}\,.$$

2. Then,

$$(K_3 M_y - K_1 M_x)x = -(K_2 M_x - K_2 M_y)y$$

Which applies to any point on the neutral axis.

3. The neutral axis equation is

$$\tan\phi = -\frac{(K_3 M_y - K_1 M_x)}{(K_2 M_x - K_1 M_y)} = \frac{y}{x}$$

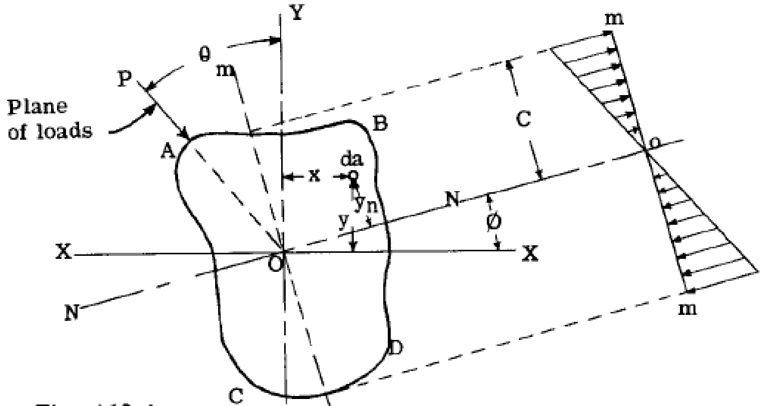


Fig. A13.1 Fig. A13.2

Neutral Axis and centroid

- Neutral axis passes through the centroid.
- Centroid depends only on the section geometry.

$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\sum_{i=1}^{N} x_i A_i}{\sum_{i=1}^{N} A_i}$$

- The neutral axis depends on the loading condition in addition to the section geometry

$$\tan \phi = - \frac{(K_3 M_y - K_1 M_x)}{(K_3 M_x - K_1 M_y)} = \frac{y}{x}$$

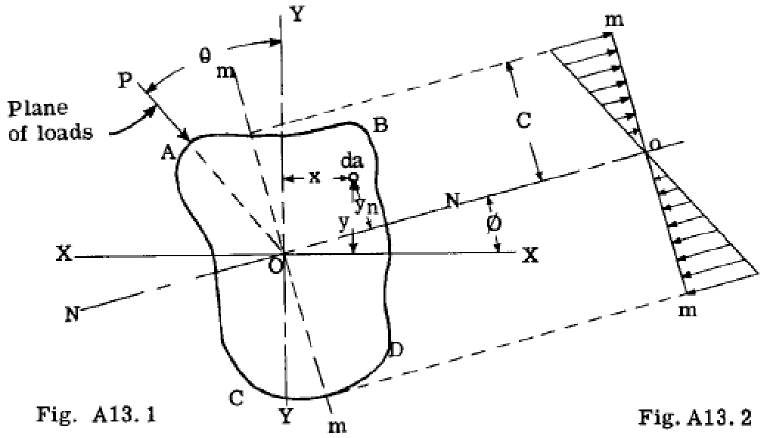


Fig. A13.1 Fig. A13.2

The centroid is important to determine the section moment of inertia and the neutral axis is important to determine the maximum stresses or the section stress distribution.

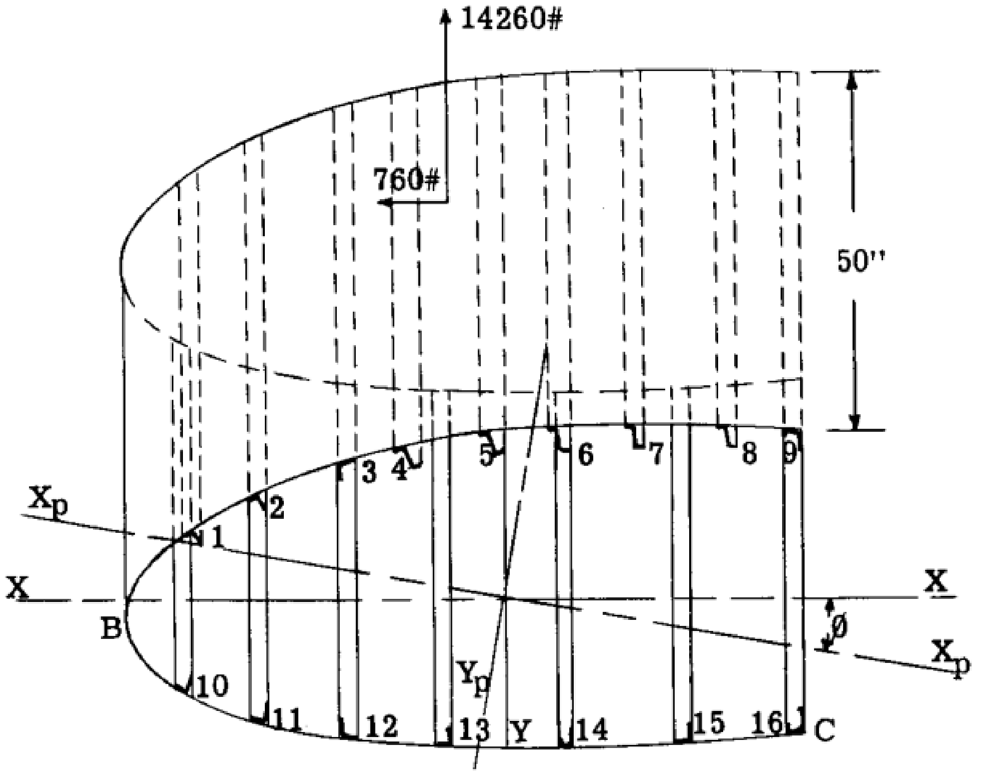
Exercise Problem – Bending stresses

For the wing portion shown in Figure. Calculate the bending stresses at stringer 1, 9, 12?

$$M_x = 713000''\#, \quad M_y = -38000''\#$$

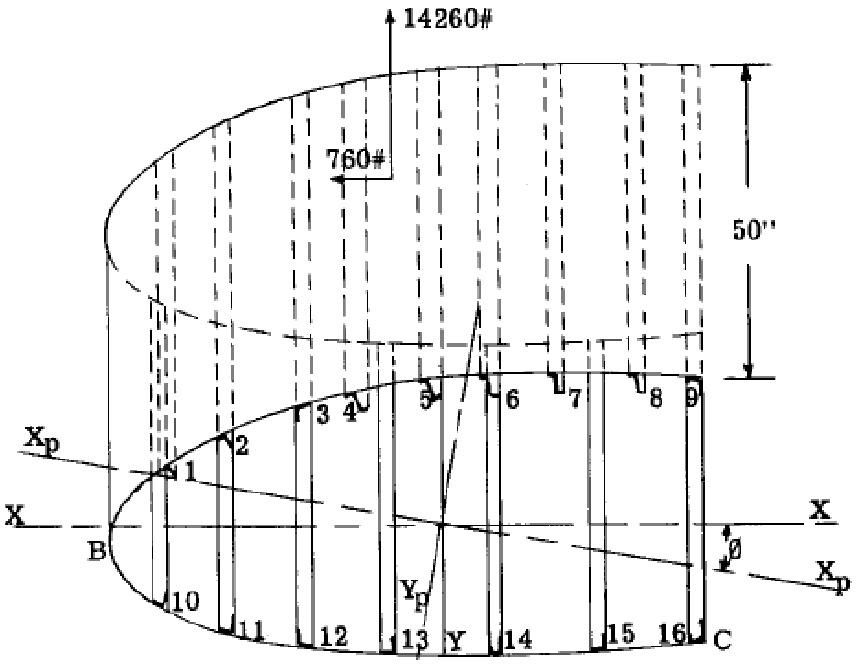
$$I_x = 186.46 \qquad I_y = 431.7 \qquad I_{xy} = -36.41$$

$$\sigma_b = -\frac{(M_y I_x - M_x I_{xy})}{I_x I_y - I_{xy}^2}\,x - \frac{(M_x I_y - M_y I_{xy})\,y}{I_x I_y - I_{xy}^2}$$



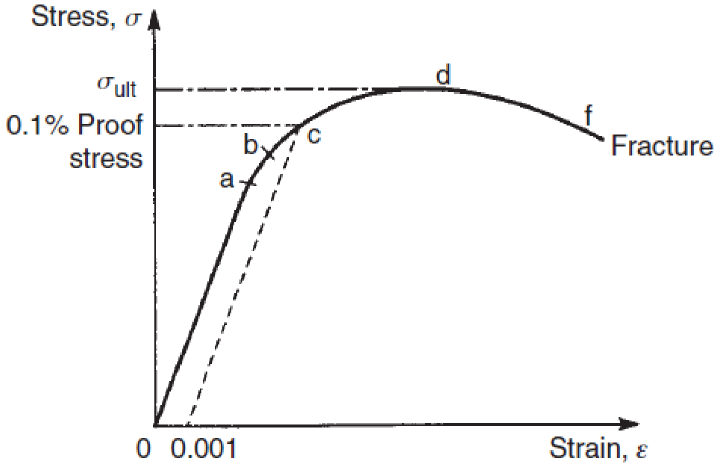
Exercise Problem – Bending stresses

NOTE: In the previous solution , the distance from the axis in question to the stringers 1, 9, and 12 have been taken to the centroid of each stringer unit. Thus, the stresses obtained are average axial stresses on the stringers. If the maximum stress is desired, the arm should refer to the most remote part of the stringer or the skin surface.



Stresses on beam above the elastic range

- In some cases, an airplane experience loading conditions that is above the elastic range of the aircraft material.
- In this range the material stress-strain curve is nonlinear, and the previous stress formulation is not applied.
- Experimental tests show that even the stress is nonlinear, plane sections after bending remains planes, i.e. *the strain is still linear*.



Example - Nonlinear stress

Portion (a) of Fig. A13.20 shows a solid round bar made from 24ST aluminum alloy material.

that the maximum failing compressive stress occurs at a strain of 0.01 in. per inch. The problem is to determine the ultimate resisting moment developed by this round bar and then compare the result with that obtained by using the beam bending stress formula based on linear variation of stress to strain.

Since the stress-strain diagram in tension is different from that in compression (See Fig. A13.19) the neutral axis will not coincide with the centroidal axis of the round bar regardless of the fact that it is a symmetrical shape.

Then the problem will be solved using trail and error Neutral axis is assumed 0.0375 above the centerline axis

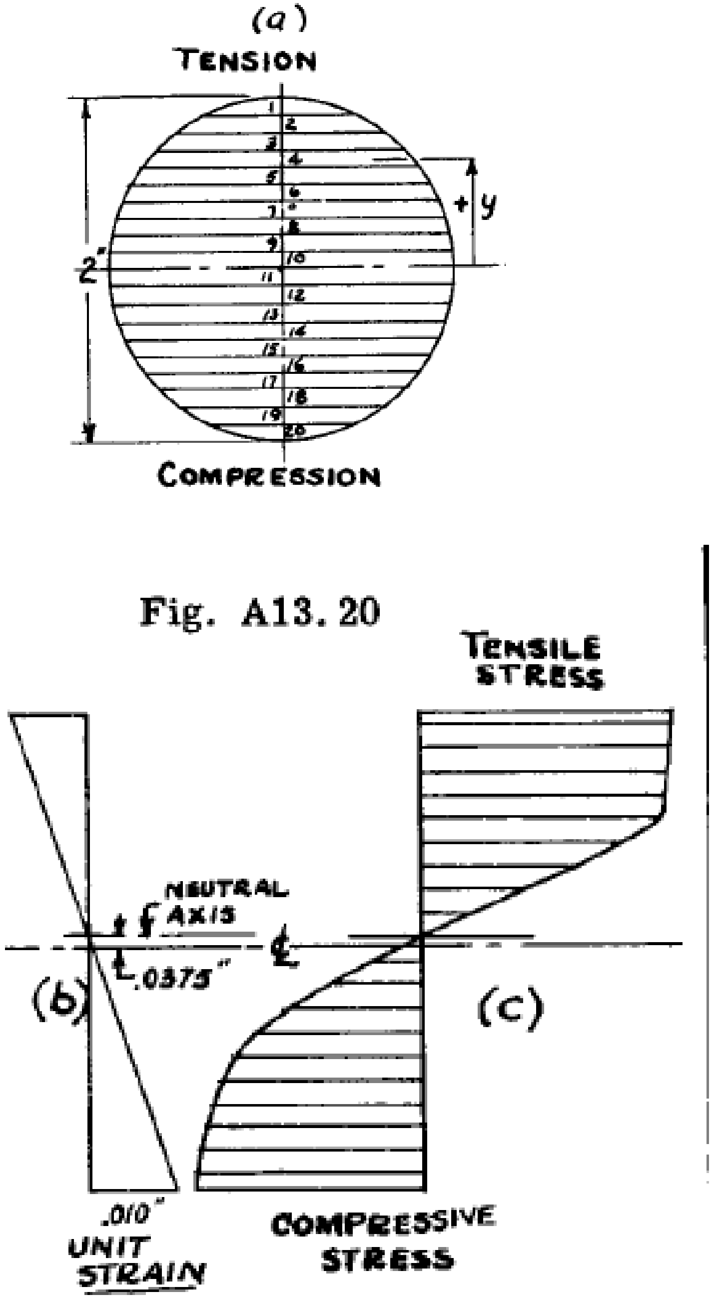


Fig. A13.20

Example - Nonlinear stress

1	2	3	4	5	6	7
Strip No.	Strip Area "A"	y	ε	Unit Stress σ	F = σA	Res. Moment M = Fr
1	.058	0.935	.00867	53000	3075	2760
2	.102	0.840	.00773	52500	5350	4300
3	.135	0.75	.00685	52100	7025	5040
4	.153	0.65	.00591	51500	7870	4820
5	.165	0.55	.00494	51000	8410	4310
6	.180	0.45	.00398	43000	7740	3200
7	.185	0.35	.00302	33200	6140	1920
8	.195	0.25	.00205	22800	4450	945
9	.197	0.15	.00108	12500	2460	280
10	.200	0.05	.00012	3200	640	10
11	.200	-0.05	-.00084	- 7250	-1450	130
12	.197	-0.15	-.00181	-17800	-3510	660
13	.195	-0.25	-.00276	-29500	-5750	1650
14	.185	-0.35	-.00374	-35500	-6560	2540
15	.180	-0.45	-.00470	-40000	-7200	3510
16	.165	-0.55	-.00566	-43000	-7100	4170
17	.153	-0.65	-.00663	-44800	-6850	4710
18	.135	-0.75	-.00759	-46000	-6210	4880
19	.102	-0.84	-.00846	-47200	-4810	4210
20	.058	-0.935	-.00937	-48000	-2780	2690
Total	3.140				740	56735

Col. 1 Rod divided into 20 strips .1" thick.
Col. 3 y = distance from centerline to strip c.g.
Col. 4 ε = strain at midpoint = (y - .0375)/103.75
Col. 5 Unit stress for ε strain from Fig. A13.19
Col. 6 Total stress on strip.
Col. 7 Moment about neutral axis. M = (y - .0375)F.

The summation of column (6) should be zero. Since the discrepancy is 740 lbs., it means that the assumed position for the neutral axis is a little too high, however the discrepancy is negligible. The total internal resisting moment is 56735 in. lbs. (Col. 7).

Example - Nonlinear stress

If we take a maximum unit compressive strain of 0.01 we find the corresponding stress from Fig.A13.19 to be 48500 psi. If this stress is used as the failing stress in the beam formula $M = \frac{\sigma I}{c}$ we obtain,

$$M = 48500 \times 0.785 = 38000 \text{ in. lbs.}$$

$$\left(0.785 = \frac{I}{c} \text{ of round bar} = \pi r^3/4\right)$$

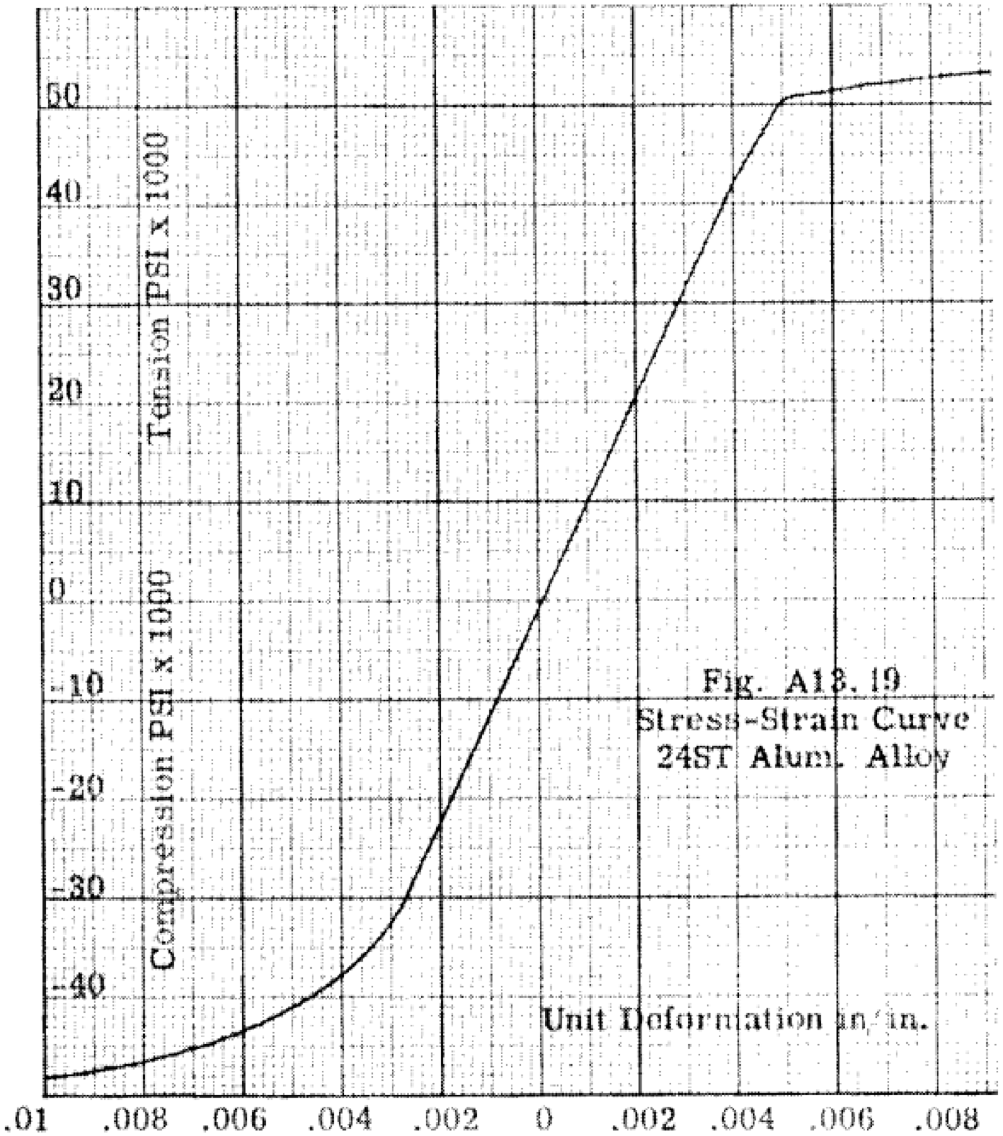


Fig. A13.19
Stress-Strain Curve
24ST Alum. Alloy