Airframe Design and Construction

Section properties - revision

Instructor: Mohamed Abdou Mahran Kasem, Ph.D.

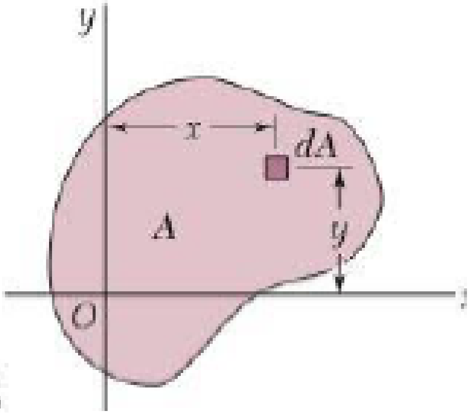
Aerospace Engineering Department

Cairo University

First Moment of area

Consider an area A located in the xy plane (Fig. A.1). Denoting by x and y the coordinates of an element of area dA, we define the *first moment of the area A with respect to the x axis* as the integral

$$Q_x = \int_A y\, dA$$



Similarly, the *first moment of the area A with respect to the y axis* is defined as the integral

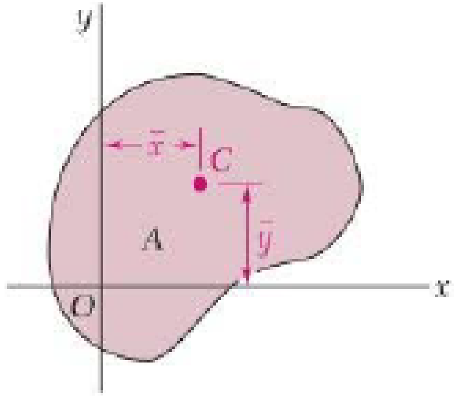
$$Q_y = \int_A x\, dA$$

First moment of area units is $[m^3]$ in SI system and $[in^3\ or ft^3]$ in U.S system.

Centroid of an area

The *centroid of the area A* is defined as the point *C* of coordinates $\bar{x}$ and $\bar{y}$ (Fig. A.2), which satisfy the relations

$$\int_A x\,dA = A\bar{x} \qquad \int_A y\,dA = A\bar{y}$$



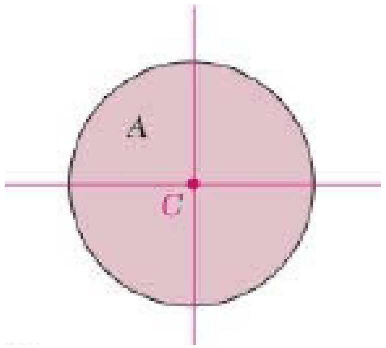
Then the centroid position can be calculated from the relation

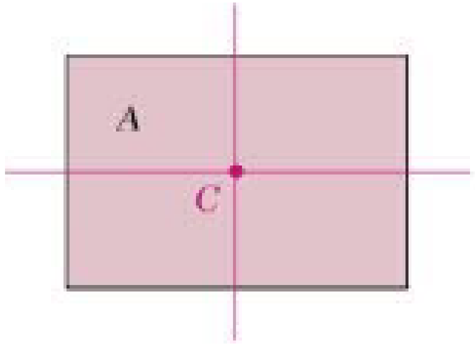
$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\sum_{i=1}^{N} x_i A_i}{\sum_{i=1}^{N} A_i} \qquad \bar{y} = \frac{\int y\,dA}{A} = \frac{\sum_{i=1}^{N} y_i A_i}{\sum_{i=1}^{N} A_i}$$

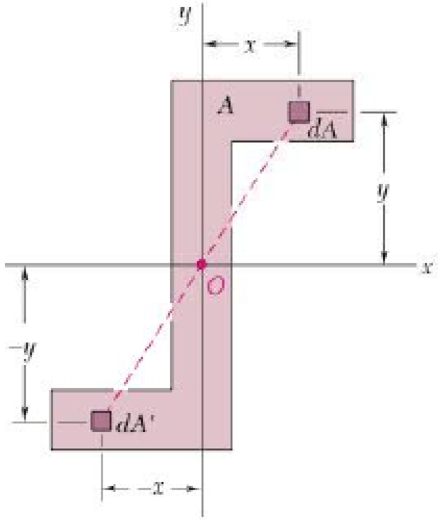
Symmetric sections

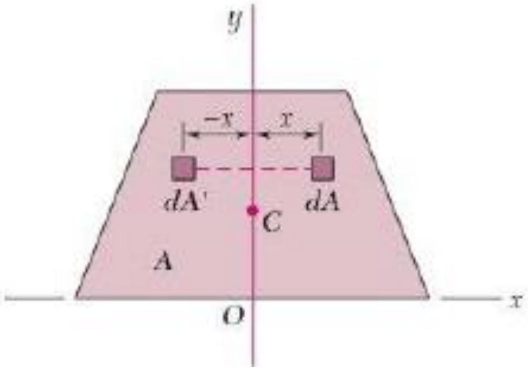
If an area A Possesses an axis of symmetry, its centroid C is located on that axis. Because the first moment of area will vanish (i.e. $Q_x = 0, or Q_y = 0$).

And if an area possesses a center of symmetry O, the first moment of area about any axis through O is zero.







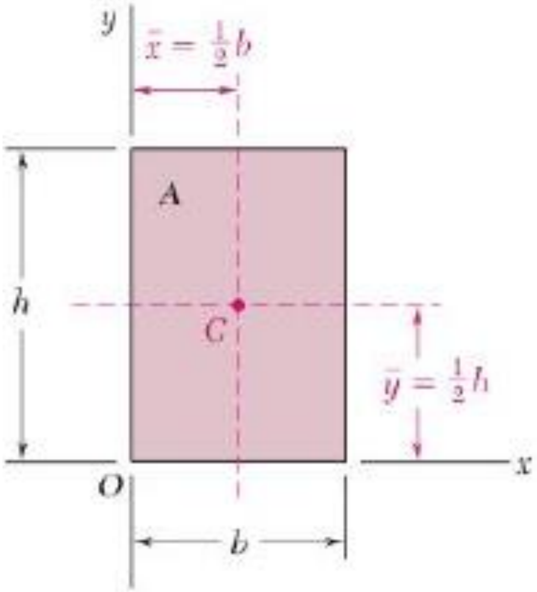


Symmetric sections

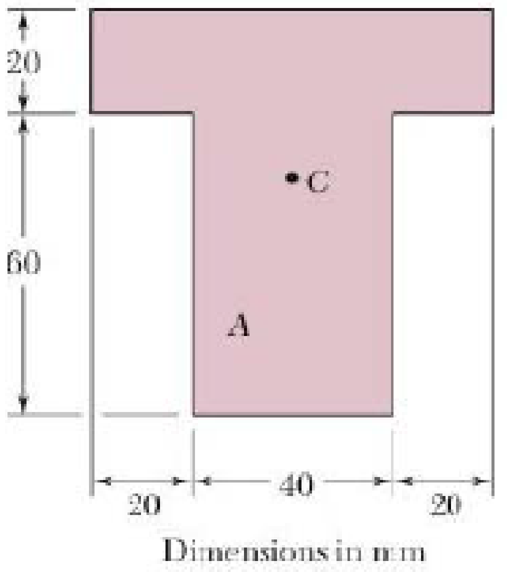
If the centroid is located by symmetry, then the first moment of area with respect to any axis can directly be obtained using the relations.

$$Q_x = A\bar{y} = (bh)(\tfrac{1}{2}h) = \tfrac{1}{2}bh^2$$

$$Q_y = A\bar{x} = (bh)(\tfrac{1}{2}b) = \tfrac{1}{2}b^2h$$



Example 1 – T section

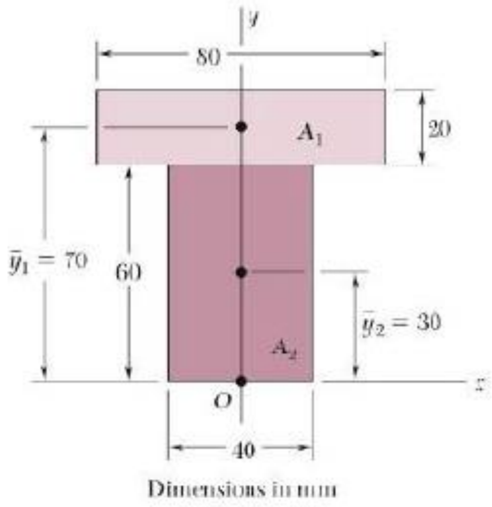


Selecting the coordinate axes shown in Fig. A.11, we note that the centroid C must be located on the y axis, since this axis is an axis of symmetry; thus, $\bar{X} = 0$.

Dividing A into its component parts A_1 and A_2, we use the second of Eqs. (A.6) to determine the ordinate $\bar{Y}$ of the centroid. The actual computation is best carried out in tabular form.

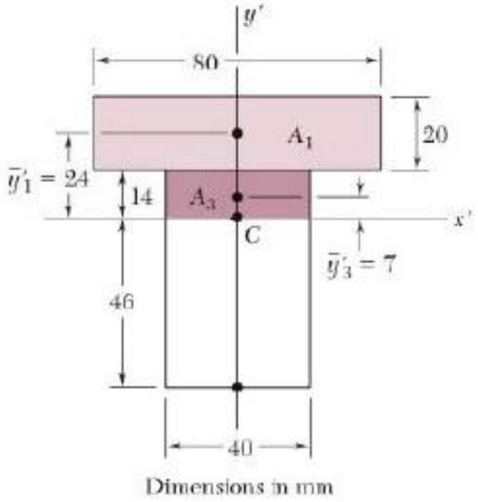
	Area, mm²	$\bar{y}_i$, mm	$A_i\bar{y}_i$, mm³
A_1	$(20)(80) = 1600$	70	112×10^3
A_2	$(40)(60) = 2400$	30	72×10^3
	$\sum_i A_i = 4000$		$\sum_i A_i\bar{y}_i = 184 \times 10^3$

$$\bar{Y} = \frac{\sum_i A_i\bar{y}_i}{\sum_i A_i} = \frac{184 \times 10^3 \text{ mm}^3}{4 \times 10^3 \text{ mm}^2} = 46 \text{ mm}$$

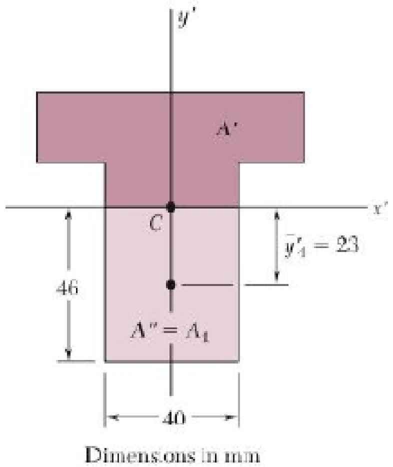


Example 1 – T section

The first moment of area can be calculated w.r.t. the axis *x'*



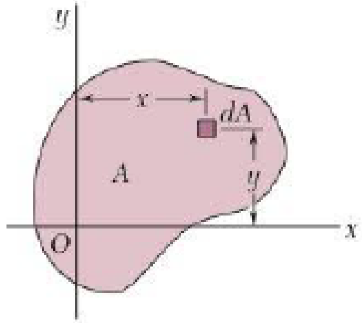
$$Q'_{x'} = A_1\bar{y}'_1 + A_3\bar{y}'_3$$
$$= (20 \times 80)(24) + (14 \times 40)(7) = 42.3 \times 10^3 \text{ mm}^3$$



Second moment of area

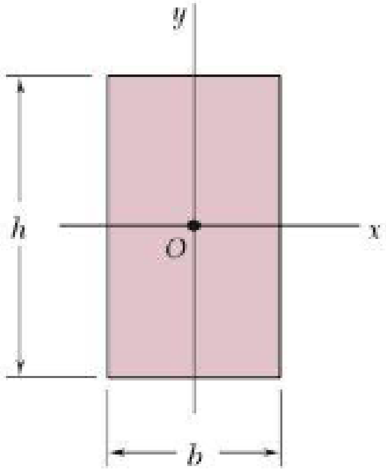
The second moments of area (moment of inertia)of A with respect to x-axis and y-axis are

$$I_x = \int_A y^2 \, dA \qquad I_y = \int_A x^2 \, dA$$



Example 2 – rectangular section

Determine the moment of inertia around the x-axis for the given rectangular section?



$$dI_x = y^2\, dA = y^2(b\, dy)$$

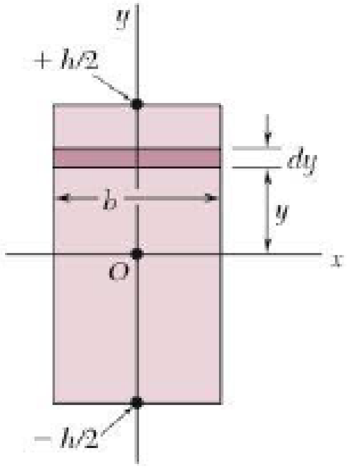
Integrating from $y = -h/2$ to $y = +h/2$, we write

$$I_x = \int_A y^2\, dA = \int_{-h/2}^{+h/2} y^2(b\, dy) = \tfrac{1}{3}b[y^3]_{-h/2}^{+h/2}$$

$$= \tfrac{1}{3}b\left(\frac{h^3}{8} + \frac{h^3}{8}\right)$$

or

$$I_x = \tfrac{1}{12}bh^3$$



Parallel-axis theorem

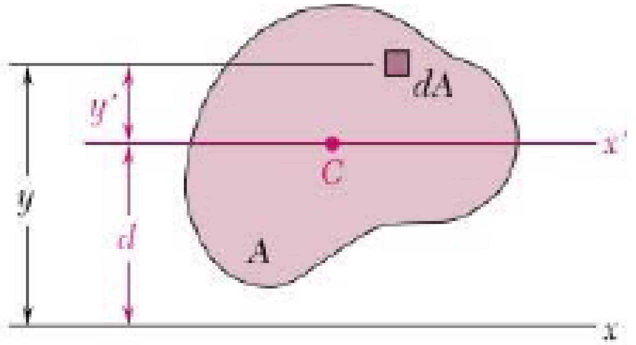
Consider the moment of inertia I_x of an area A with respect to an arbitrary x axis (Fig. A.20). Denoting by y the distance from an element of area dA to that axis, we recall from Sec. A.3 that

$$I_x = \int_A y^2\, dA$$

Let us now draw the *centroidal x′ axis*, i.e., the axis parallel to the x axis which passes through the centroid C of the area. Denoting by y' the distance from the element dA to that axis, we write $y = y' + d$, where d is the distance between the two axes. Substituting for y in the integral representing I_x, we write

$$I_x = \int_A y^2\, dA = \int_A (y' + d)^2 dA$$

$$I_x = \int_A y'^2\, dA + 2d \int_A y'\, dA + d^2 \int_A dA$$



$$I_x = \bar{I}_{x'} + Ad^2$$

Example 3 – T section

Rectangular Area A_1.

$$(\bar{I}_{x'})_1 = \tfrac{1}{12}bh^3 = \tfrac{1}{12}(80\text{ mm})(20\text{ mm})^3 = 53.3 \times 10^3\text{ mm}^4$$

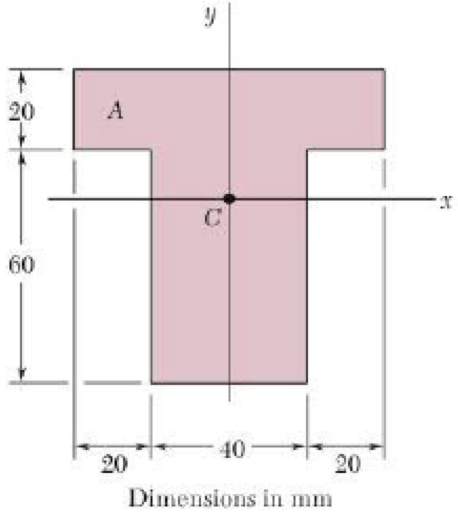
$$(I_x)_1 = (\bar{I}_{x'})_1 + A_1 d_1^2 = 53.3 \times 10^3 + (80 \times 20)(24)^2$$
$$= 975 \times 10^3\text{ mm}^4$$

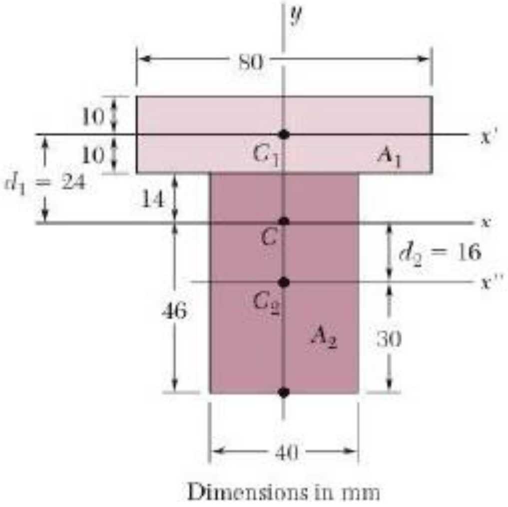
Rectangular Area A_2.

$$(\bar{I}_{x''})_2 = \tfrac{1}{12}bh^3 = \tfrac{1}{12}(40)(60)^3 = 720 \times 10^3\text{ mm}^4$$
$$(I_x)_2 = (\bar{I}_{x''})_2 + A_2 d_2^2 = 720 \times 10^3 + (40 \times 60)(16)^2$$
$$= 1334 \times 10^3\text{ mm}^4$$

Entire Area A.

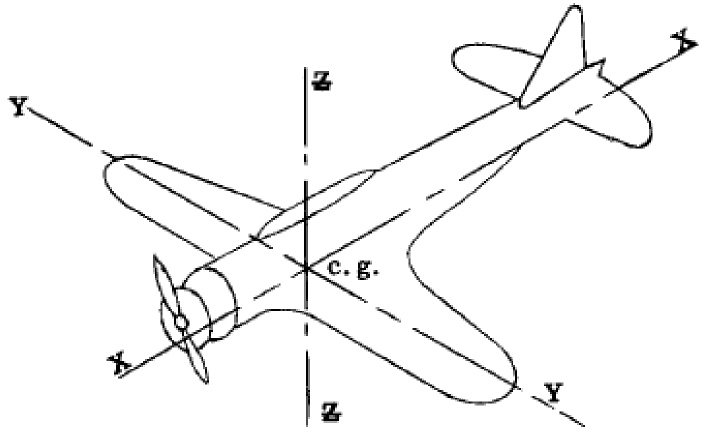
$$\bar{I}_x = (I_x)_1 + (I_x)_2 = 975 \times 10^3 + 1334 \times 10^3$$
$$\bar{I}_x = 2.31 \times 10^6\text{ mm}^4$$





Airplane moments of inertia

To determine the airplane inertia forces and subsequently calculate the airplane stresses, it is important to determine the airplane moment of inertia.



The mass moments of inertia of the airplane about the coordinate X, Y and Z axes through the center of gravity of the airplane can be expressed as follows:

$$I_x = \Sigma wy^2 + \Sigma wz^2 + \Sigma\Delta I_x$$

$$I_y = \Sigma wx^2 + \Sigma wz^2 + \Sigma\Delta I_y$$

$$I_z = \Sigma wx^2 + \Sigma wy^2 + \Sigma\Delta I_z$$

where I_x, I_y, and I_z are generally referred to as the rolling, pitching and yawing moments of inertia of the airplane.

w = weight of the items in the airplane

x, y and z equal the distances from the axes thru the center of gravity of the airplane and the weights w. The last term in each equation is the summation of the moments of inertia of the various items about their own X, Y and Z centroidal axes.

Example 6 – Airplane moment of inertia

Determine the gross weight

center of gravity of the airplane shown in Fig. A3.3. The airplane weight has been broken down into the 10 items or weight groups, with their individual c.g. locations denoted by the symbol +.

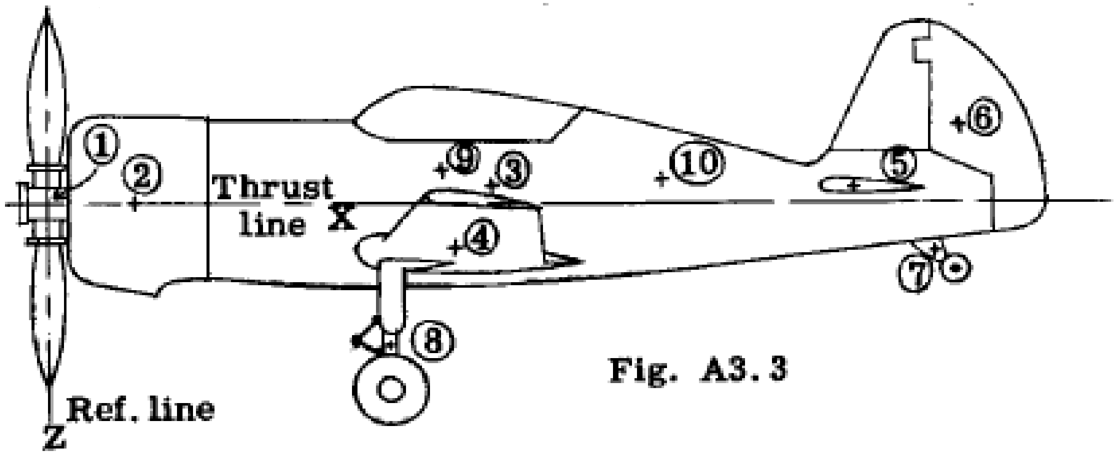


Fig. A3.3

Example 6 – Centroid

Solution. The airplane center of gravity will be located with respect to two rectangular axes. In this example, a vertical axis thru the center-line of the propeller will be selected as a reference axis for horizontal distances, and the thrust line as a reference axis for vertical distances. The general expressions to be solved are:-

$\bar{x} = \frac{\Sigma wx}{\Sigma w}$ = distance to airplane c.g. from ref. axis Z-Z

$\bar{y} = \frac{\Sigma wy}{\Sigma w}$ = distance to airplane c.g. from ref. axis X-X

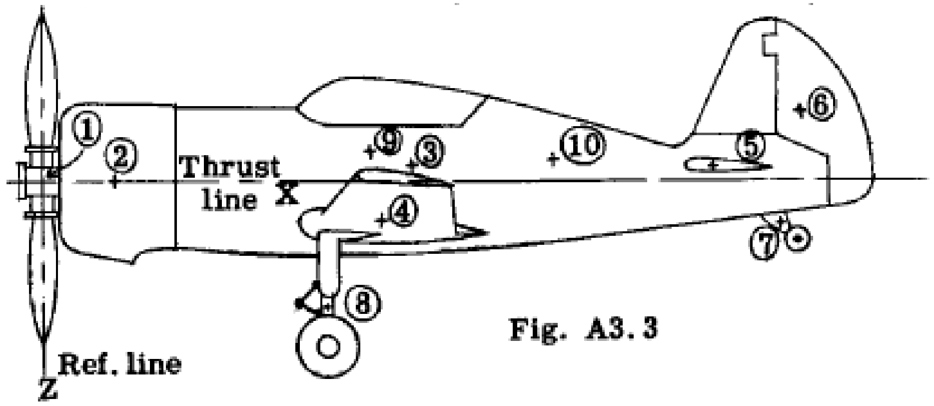


Fig. A3.3

Example 6 – Airplane Centroid

Item No.	Item Name	Weight W#	Horizontal Arm = x	Horizontal Moment = Wx	Vertical Arm = y	Vertical Moment = Wy
1	Propeller	180	0 in.	0	0	0
2	Engine Group	820	46	37720	0	0
3	Fuselage Group	800	182	145600	4	3200
4	Wing Group	600	158	94800	-18	-10800
5	Hori. Tail	60	296	17760	8	480
6	Vert. Tail	40	335	13400	26	1040
7	Tail Wheel	50	328	16400	-20	-1000
8	Front Land.Gear	300	115	34500	-30	- 900
9	Pilot	200	165	33000	10	2000
10	Radio	100	240	24000	5	500
Totals		3150		417180		-5480

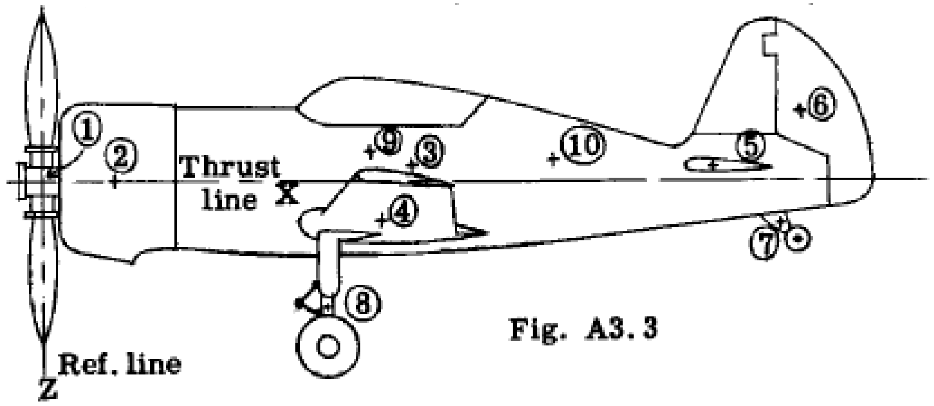


Fig. A3.3

$\bar{x} = \frac{417180}{3150} = 133.3''$ aft of ℄ propeller

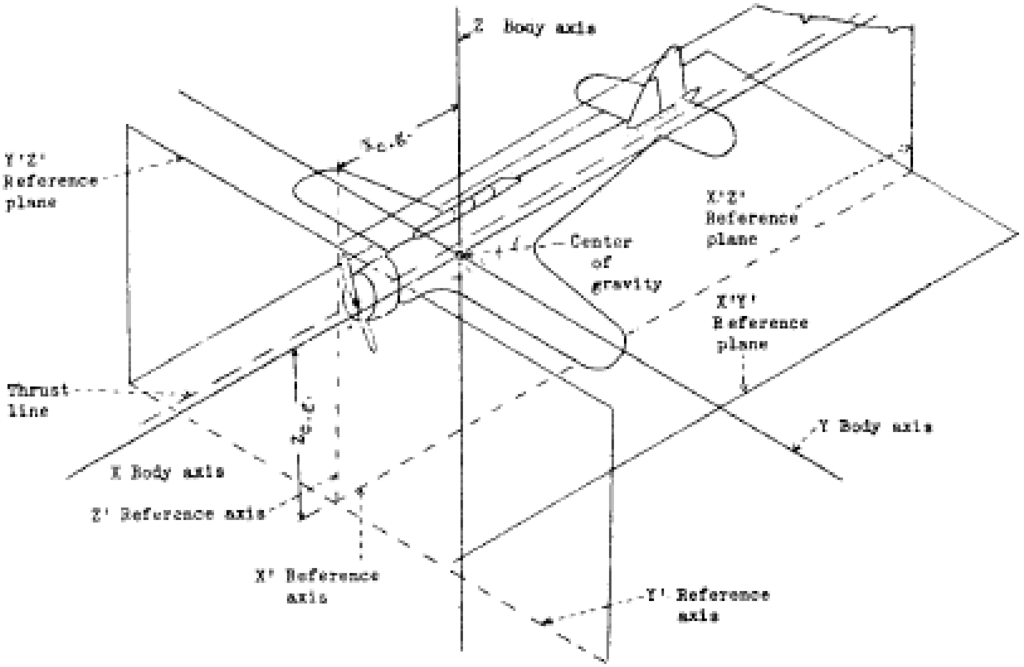
$\bar{y} = \frac{5480}{3150} = -1.74''$ (below thrust line)

Example 7 – Airplane moment of inertia

$$I_y = \Sigma wx^2 + \Sigma wz^2 + \Sigma \Delta I_y = 26,691,595 + 999,035 + 3,120,384 = 30,804,014 \text{ lb. in}^2$$

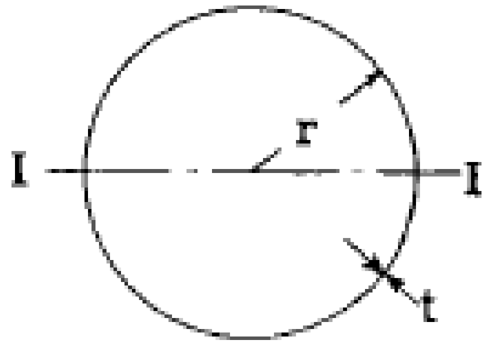
$$I_x = \Sigma wy^2 + \Sigma wz^2 + \Sigma \Delta I_x = 10,287,522 + 992,023 + 2,899,470 = 14,179,027 \text{ lb. in.}^2$$

$$I_z = \Sigma wy^2 + \Sigma wz^2 + \Sigma \Delta I_z = 10,287,522 + 26,691,595 + 5,157,186 = 42,136,303 \text{ lb. in.}^2$$



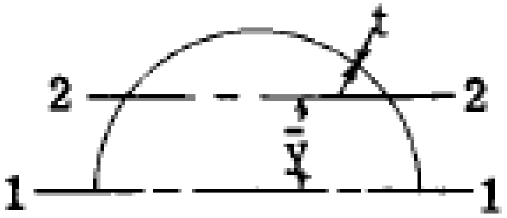
1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.	14.
Item	Weight	x	y	z	wx	wz	wx^2	wy^2	wz^3	ΔI_X	ΔI_Y	ΔI_Z	wxz
Center section nose assembly	108.8	102	–	57	11,098	6,202	1,131,955	–	353,491	261,239	–	261,229	632,563
Center section beam, etc.	204.6	121	–	57	24,757	11,662	2,995,549	–	664,745	491,245	–	491,245	1,411,136
Center section ribs, etc.	84.2	148	–	55	12,462	4,631	1,844,317	–	254,705	202,164	33,680	235,844	685,388
Flap	22.0	180	–	53	3,960	1,166	712,800	–	61,798	48,598	–	48,598	209,880
Outer panel nose	104.6	105	156	65	10,983	6,799	1,153,215	2,545,546	441,935	184,514	–	184,514	713,895
Outer panel beam	155.6	120	156	65	18,672	10,114	2,240,640	3,786,682	657,410	274,478	–	274,478	1,213,680
Outer panel ribs	89.8	139	156	64	12,482	5,747	1,735,026	2,185,373	367,821	158,407	17,601	176,008	798,861
Ailerons	31.4	172	156	62	5,401	1,947	928,938	764,150	120,702	55,390	–	55,390	334,850

Section properties of thin sheets



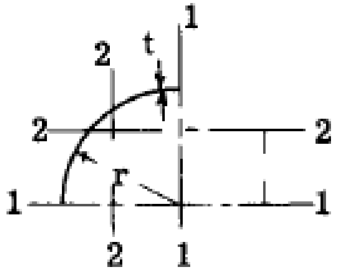
Area = $2 \pi rt$

$I_{1-1} = \pi r^3 t$



$A = \pi rt$

$\bar{y} = .6366\ r$

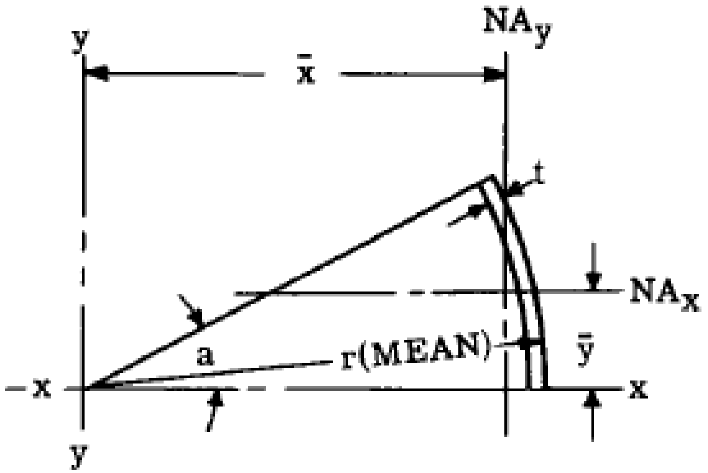


Area = $\frac{\pi rt}{2}$

$\bar{y} = .6366\ r$

Circular arc section

CIRCULAR ARC



Area = $\alpha r t$ $\qquad$ α in Radians

$$\bar{x} = \frac{r \sin \alpha}{\alpha}, \quad (M_{yy} = A\,\bar{x} = r^2 t \sin \alpha)$$

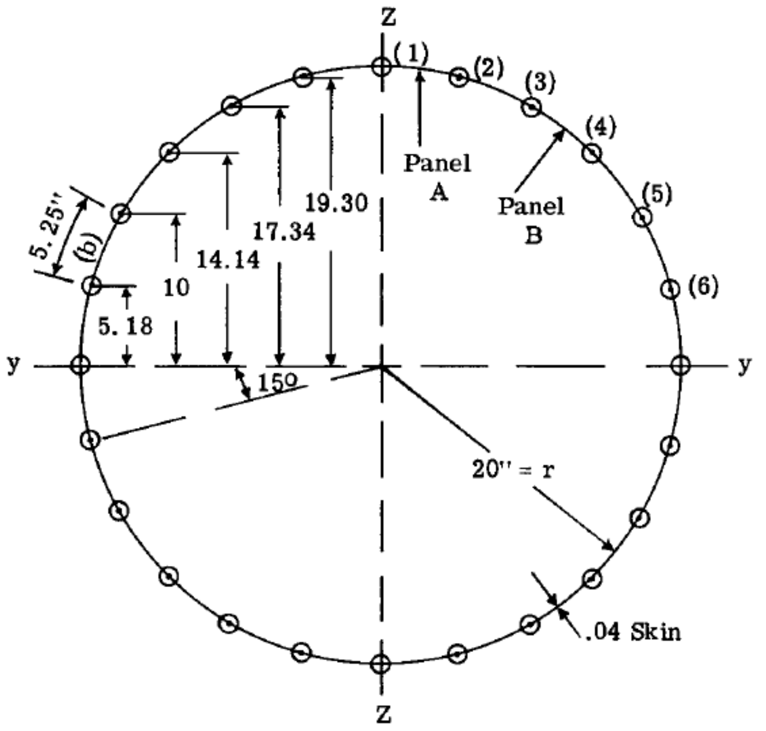
$$I_{yy} = \frac{r^3 t}{2}\left(\alpha + \frac{\sin 2\alpha}{2}\right)$$

$$\bar{y} = r\left(\frac{1-\cos \alpha}{\alpha}\right), \quad M_{xx} = A\bar{y} = r^2 t\,(1-\cos \alpha)$$

$$I_{xx} = \frac{r^3 t}{2}\left(\alpha - \frac{\sin 2\alpha}{2}\right)$$

Example 4 – Circular fuselage

Fig. C9.6 illustrates a circular fuselage section with longitudinal stringers represented by the small circles. The area of each stringer is .15 sq. in. The skin thickness is .04 inches. All material is aluminum alloy



Find the Fuselage centroid position and the fuselage second moment of area about the y-axis?

Example 5 – Idealized Circular Fuselage

Fig. A20.3 shows the cross-section of a circular fuselage. The Z stringers are arranged symmetrically with respect to the center line Z and X axes.

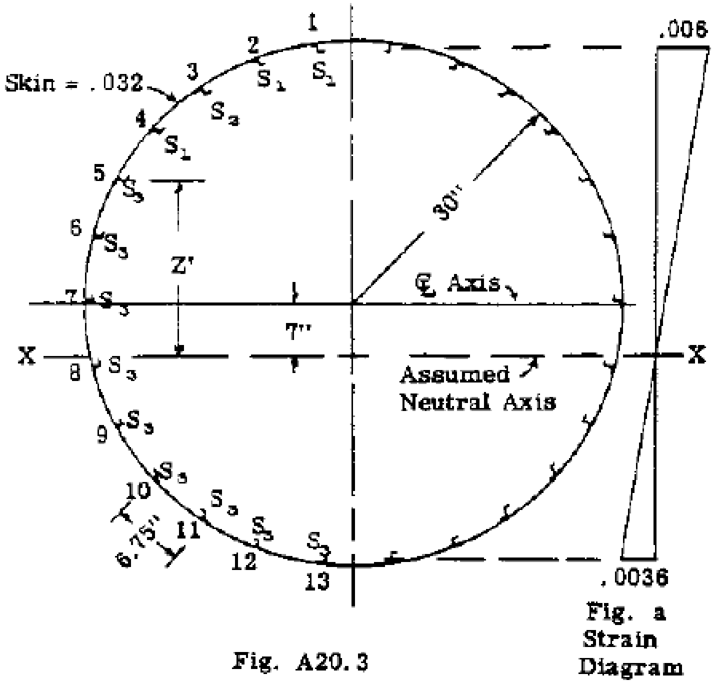


Fig. A20.3

To determine the fuselage section properties, it is more suitable to work with an idealized section in which the stringers and effective skin areas are collected at the stringer centroids.

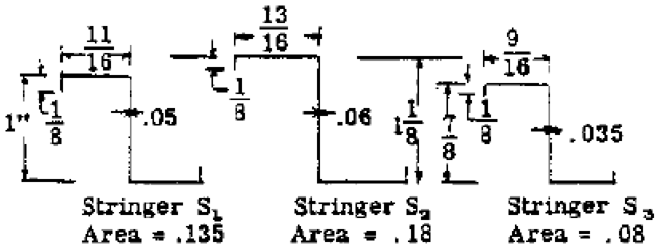


Fig. A20.4

In the present example, initially we will neglect the skin effect.

Example 5 – Idealized Circular Fuselage

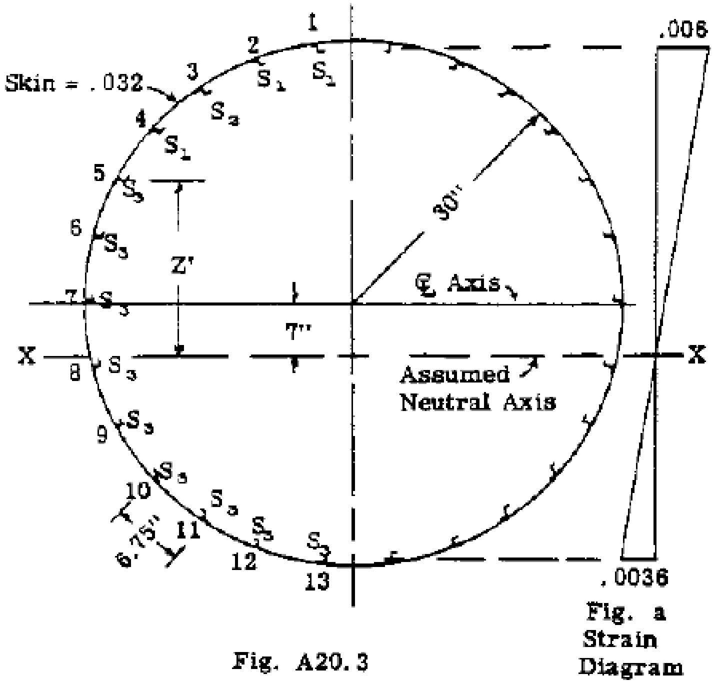


Fig. A20.3

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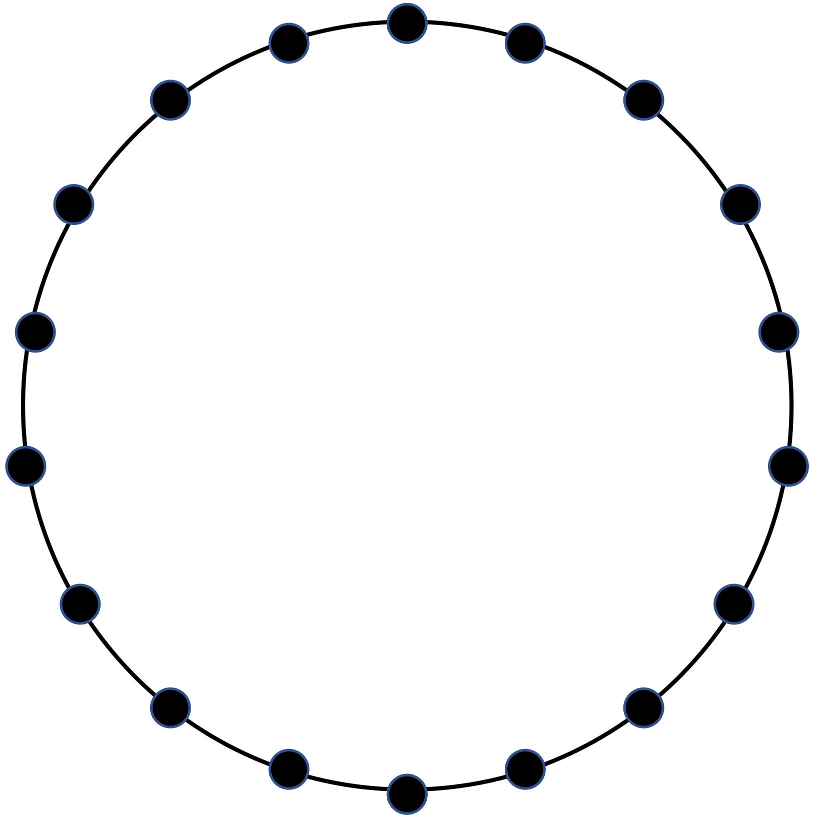


Example 5 – Idealized Circular Fuselage

Solution steps:

1. List the area of each stringer.
2. Select a reference center point.
3. Calculate the centroid position of each stringer w.r.t initial axes (Z').
4. Calculate the first moment of area ($\sum AZ'$)
5. Determine the centroid position, where

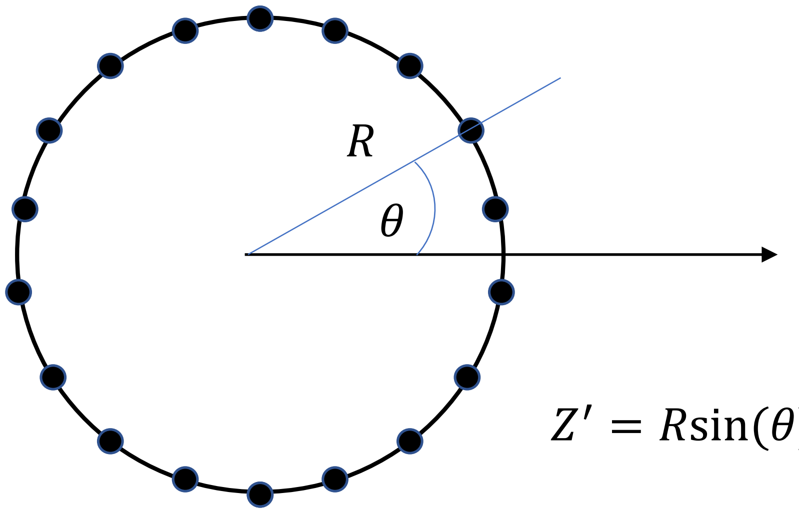
$$\bar{Z} = \frac{\sum AZ'}{\sum A}$$

1. Correct the stringers centerline position.

$$Z = Z' - \bar{Z}$$

1. Determine the second moment of area.

$$I_{xx} = \sum AZ^2$$



$Z' = R\sin(\theta)$