Airframe Design and Construction

Shear Flow

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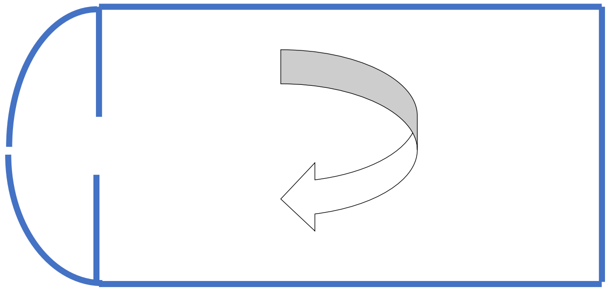
Shear flow analysis – 1-cell cantilevered wing

Equilibrium equations

$$M_{ext} + \sum mq_{avg} + 2A_1 q_1 + M_{st} = 0$$



$$P = \sigma_{true} * A_{true} = \sigma_{cal} * A_{cal}$$

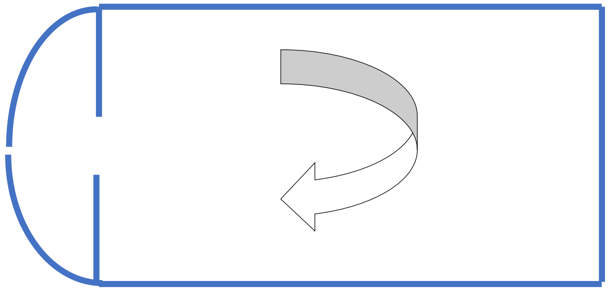
$$q_{open} = \sum -\frac{\Delta p}{\Delta y} = \sum \frac{p(20) - p(0)}{\Delta y} \,, \qquad q_{avg} = q_{open} * TR$$

$$M_{ext} = M_y \text{ (twisting moment)}, \qquad M_{st} = \sum p_x z' - \sum p_z x' \,, p_x = p\frac{\Delta x}{\Delta y} \,,$$

$$p_z = p\frac{\Delta z}{\Delta y} \,, \Delta x = x'(0) - x'(20)$$

Shear flow analysis – 2-cell cantilevered wing

Equilibrium equations



$$M_{ext} + \sum m q_{avg} + 2A_1 q_1 + 2A_2 q_2 + M_{st} = 0 \qquad (1)$$

$$\left[\Sigma_1 \frac{L}{t} + \frac{A_1}{A_2}\frac{L}{t_{1-6}}\right] q_1 - \left[\frac{L}{t_{1-6}} + \frac{A_1}{A_2}\Sigma_2 \frac{L}{t}\right] q_2 = \frac{A_1}{A_2}\sum q_{avg} \qquad (2)$$

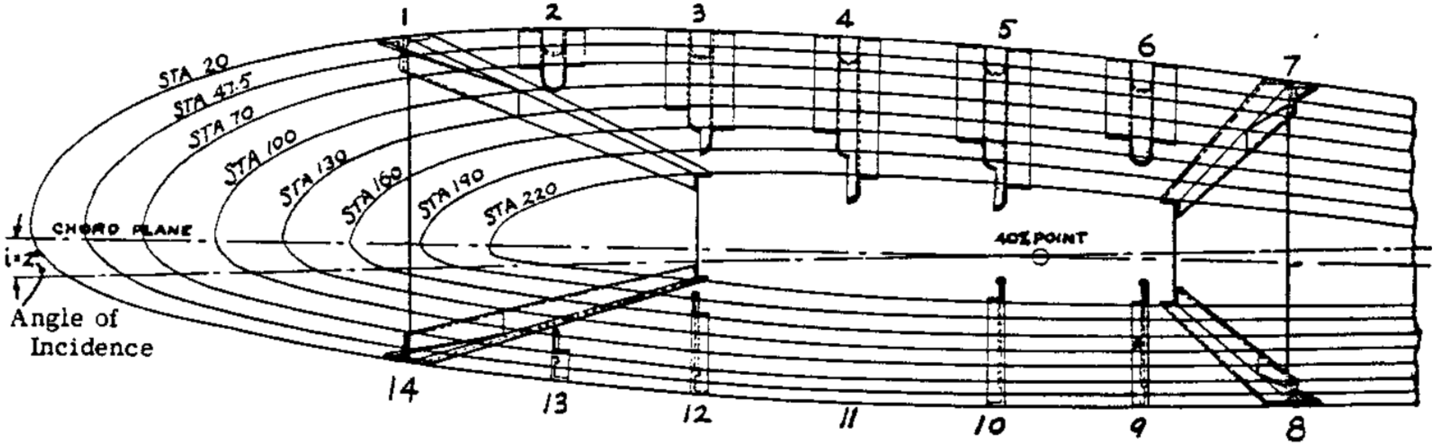
$$P = \sigma_{true} * A_{true} = \sigma_{cal} * A_{cal}$$

$$q_{open} = \sum -\frac{\Delta p}{\Delta y} = \sum \frac{p(20) - p(0)}{\Delta y}\ , \qquad q_{avg} = q_{open} * TR$$

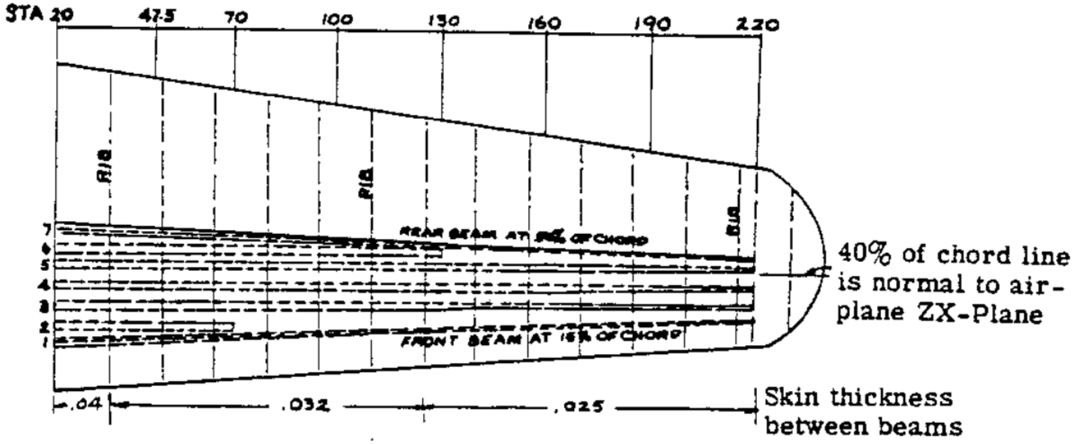
$$M_{ext} = M_y\ , \qquad M_{st} = \sum p_x z' - \sum p_z x'\ , p_x = p\frac{\Delta x}{\Delta y}\ , \qquad p_z = p\frac{\Delta z}{\Delta y}\ , \Delta x = x'(0) - x'(20)$$

Shear flow analysis

WING BODY SECTION PLAN VIEW LOOKING INBOARD FROM TIP

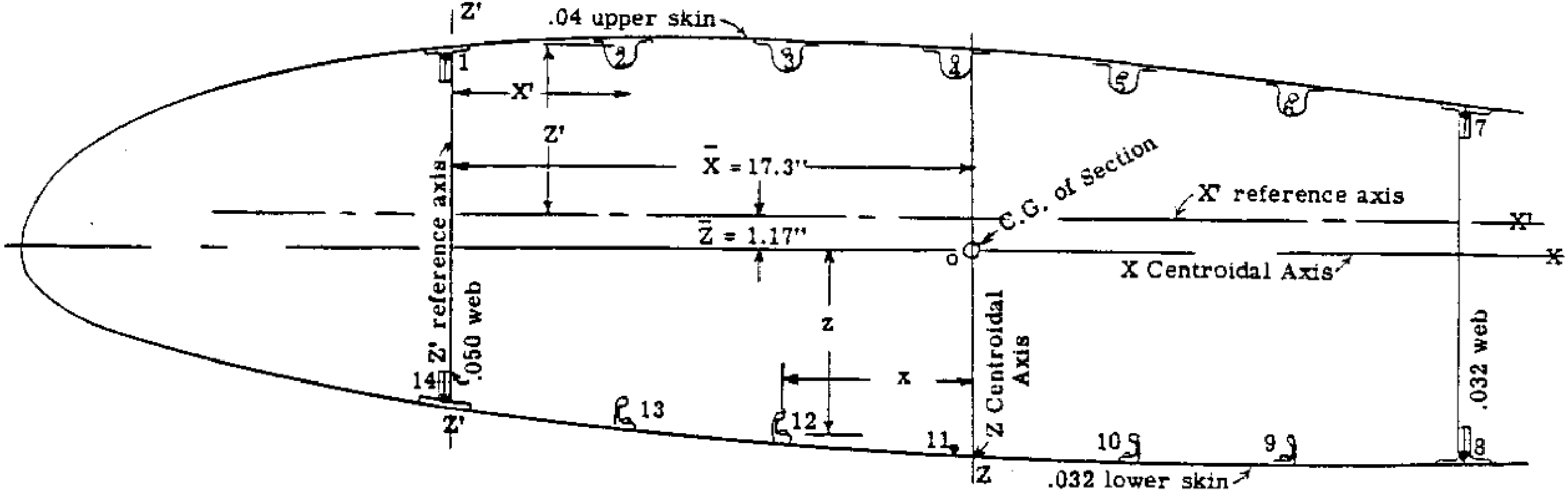


UPPER SURFACE STRINGER AND SKIN ARRANGEMENT



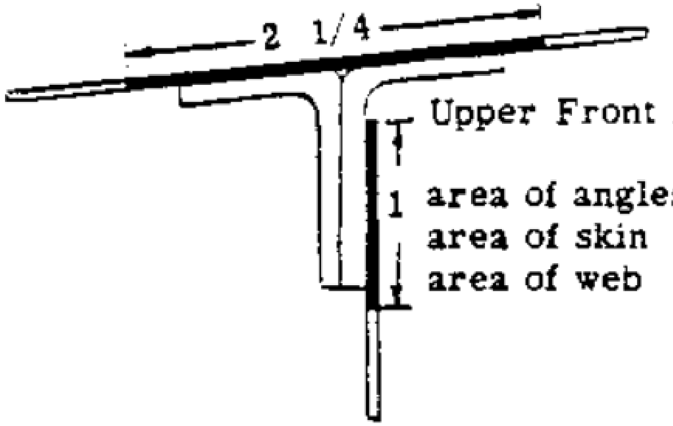
Shear flow analysis

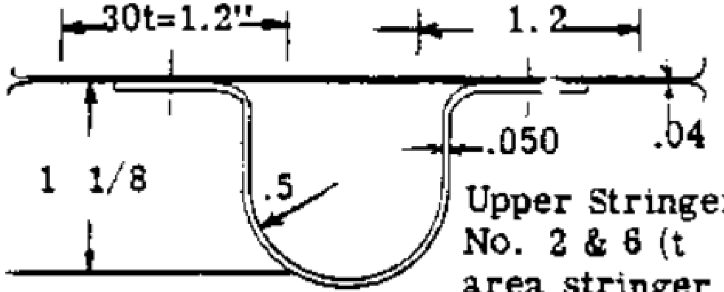
Wing section at Station 20

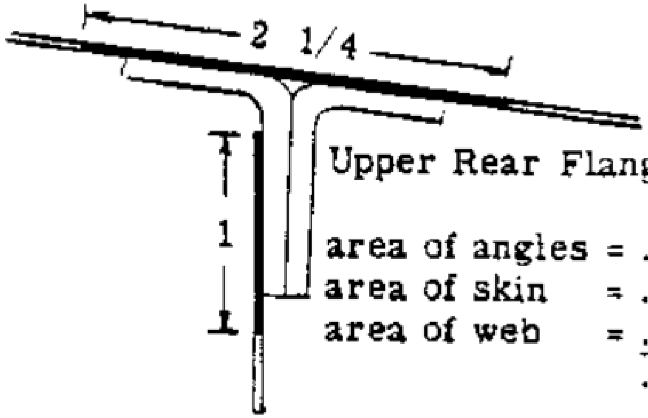


Shear flow analysis

Upper Stringers of the Wing section at Station 20

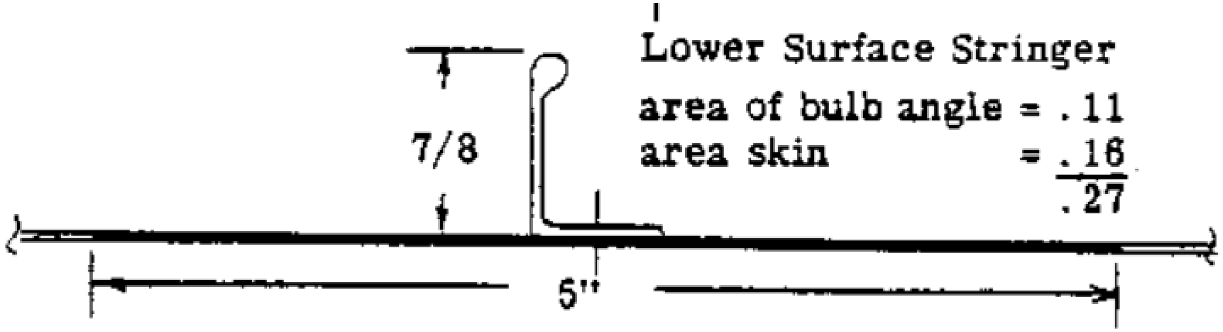


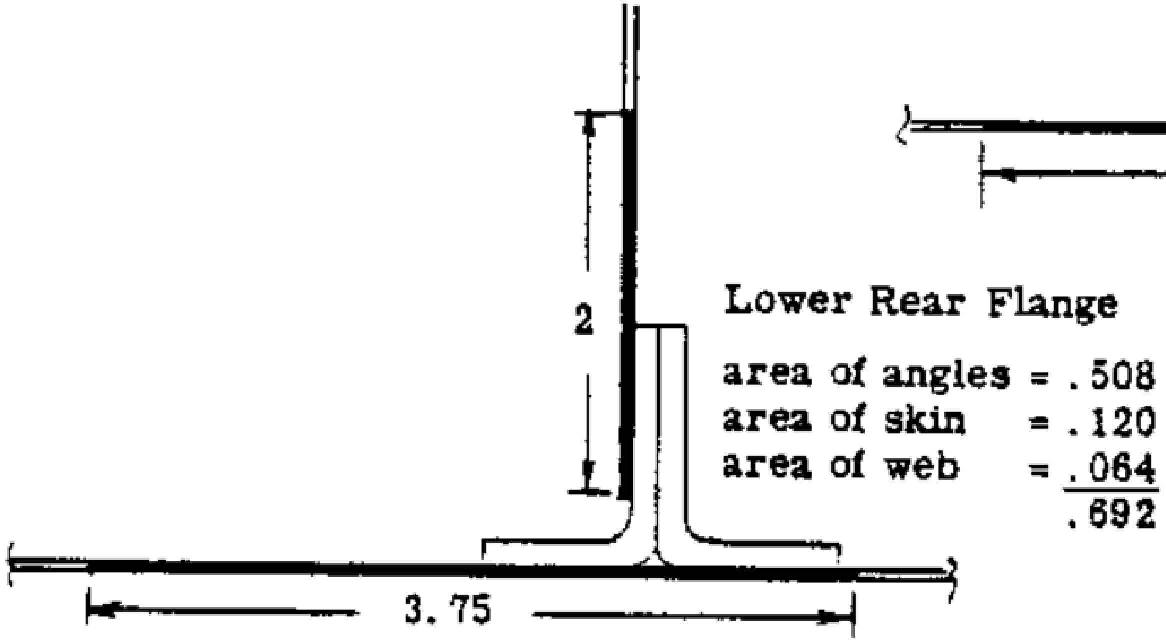


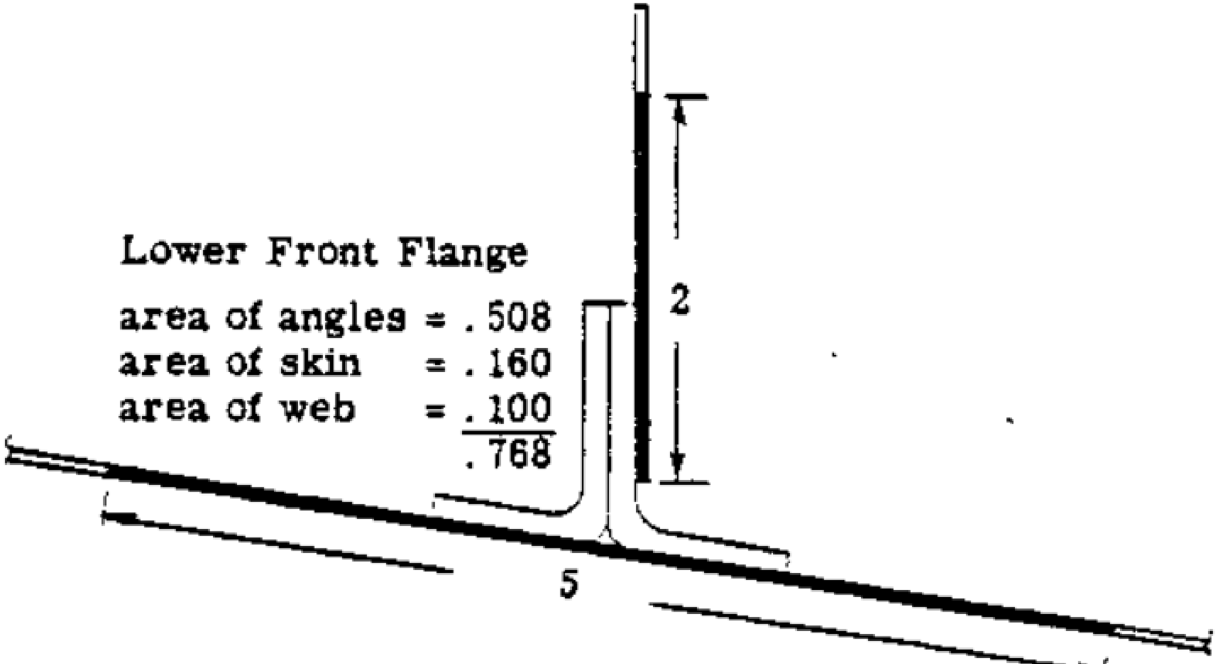


Shear flow analysis

Lower Stringers of the Wing section at Station 20







Shear flow analysis

Both the leading and trailing edges are assumed to be ineffective

The design bending moments will be assumed and are as follows: -

<u>Station 20</u> $M_X = 1,300,000$ in.lb.

$M_Z = -285,000$ in.lb.

<u>Station 47.5</u> $M_X = 1,000,000$ in.lb.

$M_Z = -215,000$ in.lb.

Shear flow analysis

The shear flow distribution will be calculated by using the change in axial load in the stringers between stations 20 and 47.5, a method commonly referred to as the ΔP method.

The shear flow in the y direction at a point n of the cell wall equals,

$$q_{y_n} = q_o - \sum_{o}^{n} \frac{\Delta P}{d} \quad (6)$$

where q_o is a known value of shear flow at some point o and the second term is the change in shear flow between points o and n.

Shear flow analysis

ΔP equals the change in stringer axial load over a distance d in the y direction.

Since the cell in our problem is closed the value q_o at any point is unknown. We assume it zero on web 1-14 by imagining that the web is cut as shown in Fig. A19.27. Equation (6) thus reduces to,

$$q_y = - \Sigma \frac{\Delta P}{27.5} \quad \text{- - - - - - - - - - - - - -} \quad (7)$$

1	2	3	4	5
Flange No.	P at Station 20	P at Station 47.5	$-\frac{\Delta P}{27.5}$	$q = \Sigma \frac{\Delta P}{27.5}$
1	-24390#	-18700	206.8	206.8
2	-14070	-12990	39.1	245.9
3	-11890	-10710	42.9	288.8
4	-11370	-10200	42.5	331.3
5	-10510	- 9320	43.3	374.6
6	-11090	-10000	39.6	414.2
7	-16700	-12880	139.0	553.2
8	28230	22770	-198.7	354.5
9	11180	10140	- 37.8	316.7
10	11170	10160	- 36.7	280.0
11	6430	5240	- 43.2	236.8
12	10390	9410	- 35.6	201.2
13	9600	8470	- 41.1	160.1
14	23020	18600	-160.3	0
1				

Shear flow analysis

moment of the shear flow q on any sheet element equal q times double the area of the triangle formed by joining the c.g. with lines going to each end of the sheet element. These double

areas are referred as m values (See Fig. A19.27). Column 6 of Table A19.4 records these double areas which were obtained by use of a planimeter. Column (7) gives the moment of each shear flow about the c.g. and the total of this column gives the moment about the c.g. of the complete shear flow system of Fig. A19.27 or a value of 256060 in.lb.

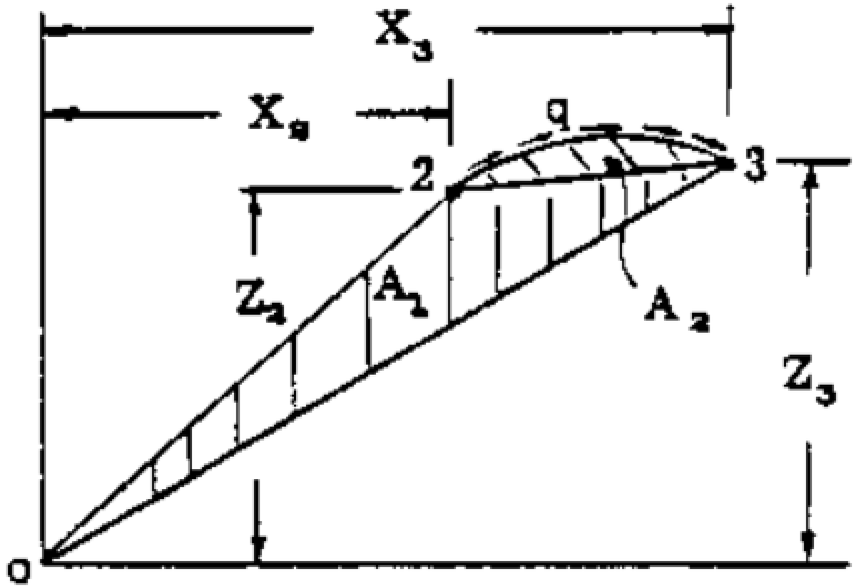
6	7
m sq. in.	mq
48.4	10000
43.8	10780
42.0	12120
42.6	14100
43.6	16340
42.0	17500
197.0	108900
47.4	16800
37.2	11800
40.8	11420
42.2	10000
38.0	7650
54.0	8650
204.0	0
	256060

Shear flow analysis

m = double the area enclosed

The double areas (m) can be found approximately as follows:

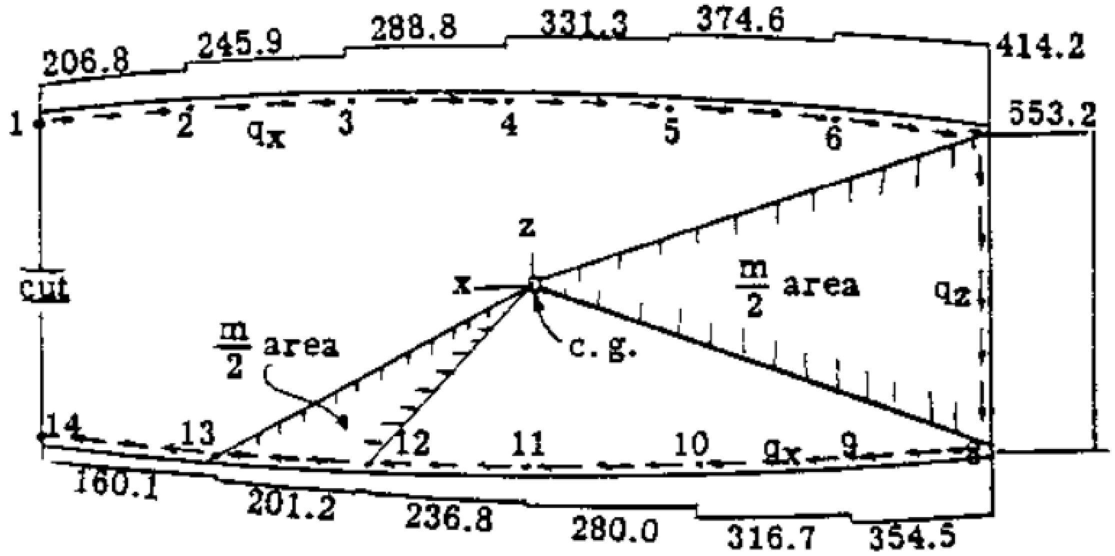
The moment of the shear flow q on the web (2-3) about point O equals q times twice the area ($A_1 + A_2$). In most cases, the area A_2 can be neglected.



By simple geometry, the area $A_1 = 1/2\ (X_3 Z_2 - X_2 Z_3)$. The moment of the shear flow q on web (2-3) thus equals q $(X_3 Z_2 - X_2 Z_3)$. Since values of X and Z for all flange points with reference to section (c.g.) are given in the Table A19.2, it is unnecessary to use the planimeter except for regions of sharp curvature.

Shear flow analysis

5	6	7
$q = \Sigma \frac{\Delta P}{27.5}$	m sq. in.	mq
206.8	48.4	10000
245.9	43.8	10780
288.8	42.0	12120
331.3	42.6	14100
374.6	43.6	16340
414.2	42.0	17500
553.2	197.0	108900
354.5	47.4	16800
316.7	37.2	11800
280.0	40.8	11420
236.8	42.2	10000
201.2	38.0	7650
160.1	54.0	8650
0	204.0	0



Shear flow analysis

Shear correction

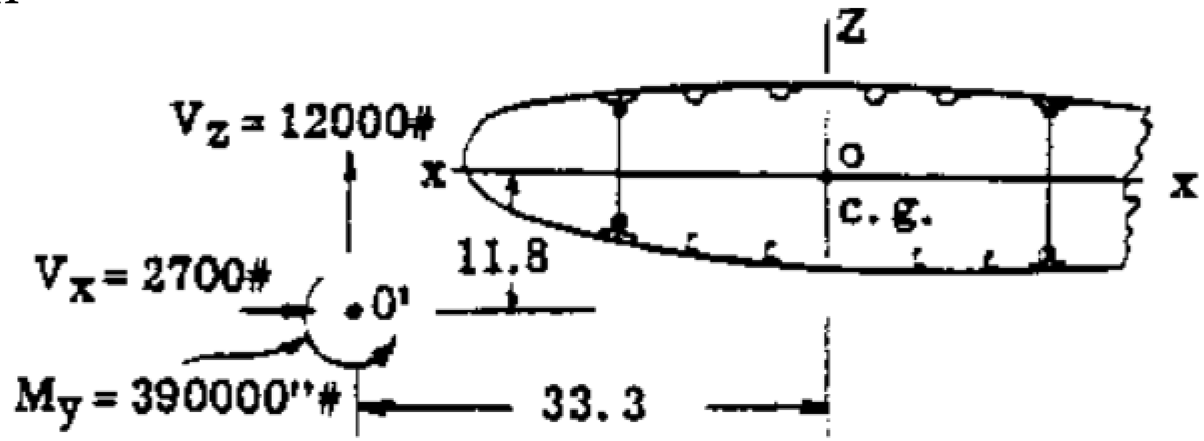


Fig. A19.28

Therefore moment of external loads about c.g. is,

$$\Sigma M_{c.g.} = 12000 \times 33.3 - 2700 \times 11.8 - 390000$$

$$= 41800 \text{ in.lb.}$$

Shear flow analysis

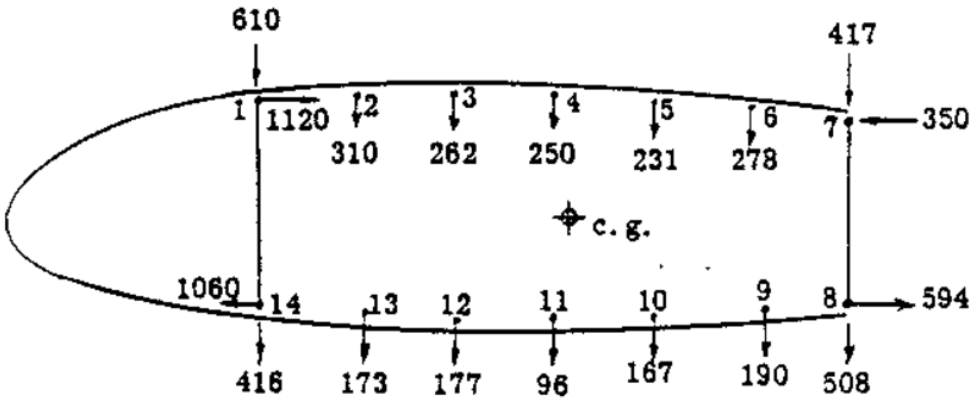
Shear correction

Since the flange members in general are not normal to the beam sections, the flange loads

have components in the Z and X directions. Columns (4) and (7) of the Table A19.5 give the values of these in plane components. The slopes dx/dy between stations 20 and 47.5 are found by scaling from Fig. A19.24. Fig. A19.29 shows these induced in plane forces as found in Table A19.5.

Shear flow analysis

Shear correction



1	2	3	4	5	6	7	8
Flange No.	P	$\frac{dx}{dy}$	$P_x = p\frac{dx}{dy}$	$M_{c.g.} = P_x Z$	$\frac{dz}{dy}$	$P_z = p\frac{dz}{dy}$	$M_{c.g.} = -(P_z X)$
1	-24390	-.046	1120	7480	.025	-610	-10600
2	-14070	0	0	0	.022	-310	- 3610
3	-11890	0	0	0	.022	-262	- 1600
4	-11370	0	0	0	.022	-250	- 112
5	-10510	0	0	0	.022	-231	1180
6	-11090	0	0	0	.025	-278	2970
7	-16700	.021	- 350	-1660	.025	-417	7770
8	28230	.021	594	-4300	.018	-508	9350
9	11180	0	0	0	.017	-190	2000
10	11170	0	0	0	.015	-167	885
11	6430	0	0	0	.015	- 96	- 57
12	10390	0	0	0	.017	-177	- 1130
13	9600	0	0	0	.018	-173	- 2020
14	23020	-.046	-1060	5640	.018	-416	- 7250
			Σ	7160			- 2224

NOTES:

Column (2); P from Table A19.2

Column (5) and (8): Values of Z and X are found in Columns 10, 11 of Table A19.2

Shear flow analysis

Shear correction

Due to flanges = 7160 - 2224 = 4936 in.lb.
(Ref. Table A19.5)

Due to assumed static shear flow = 256060 in.lb. (Ref. Table A19.4)

Due to external loads = 41800 in.lb.

Then the total unbalanced moment = 4936 + 256060 + 41800 = 302796 in.lb.

Shear flow analysis

Shear correction

8	9
q_1	$q_r = q + q_1$
-328	-121.2
-328	- 82.1
-328	- 39.2
-328	3.3
-328	46.6
-328	86.2
-328	225.2
-328	26.5
-328	- 11.3
-328	- 48.0
-328	- 91.2
-328	-126.8
-328	-167.9
-328	-328

For equilibrium, this must be balanced by a constant shear flow q_1.

hence

$$q_1 = \frac{M}{2A} = -\frac{302796}{2 \times 461.5} = -328 \text{ lb./in.}$$

(Note: 461.5 = total area of cell)

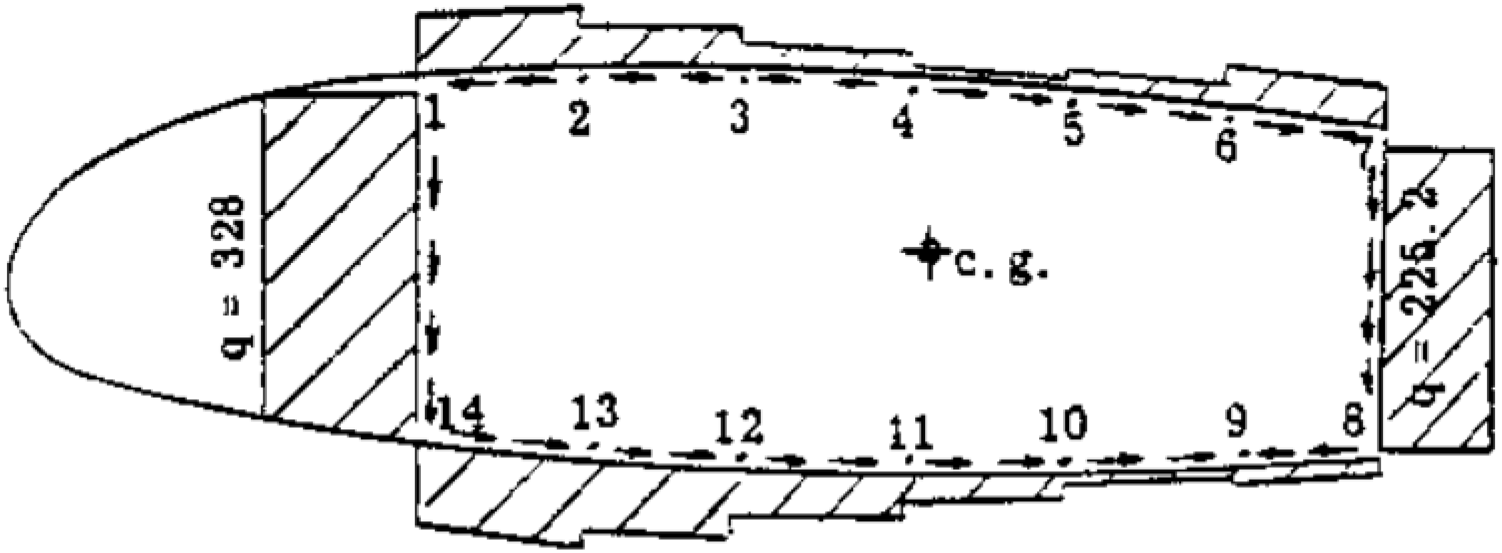
The shear stresses q_1 are listed in Column (8) of Table A19.4.

The final or resultant shear flow q_r at any point therefore equals

$$q_r = q + q_1$$

Shear flow analysis

Shear correction



Shear flow analysis – 2-cell cantilevered wing

No	P_0	P_20	ΔP/d	sum(ΔP/P)	TR	q	L/t	m	T_open	q(L/t)	dx/dy
1	-20951.5	-14503.32503	322.4069	322.4068982	0.916667	295.5397	250	60	17732.38	73884.91	-0.0625
2	-5802.88	-3845.486782	97.86973	420.2766259	0.916667	385.2536	250	60	23115.21	96313.39	-0.02083
3	-5846.85	-3724.466272	106.1193	526.3959255	0.916667	482.5296	250	60	28951.78	120632.4	0.020833
4	-11782.3	-7296.474494	224.29	750.6858874	0.916667	688.1287	240	180	123863.2	165150.9	0.0625
5	10546.91	11134.55335	29.38208	780.0679637	0.916667	715.0623	312.5	60	42903.74	223457	0.0625
6	7016.21	0	-350.81	429.2574846	0.916667	393.486	312.5	60	23609.16	122964.4	0.020833
7	7105.396	0	-355.27	73.98768852	0.916667	67.82205	312.5	60	4069.323	21194.39	-0.02083
8	19714.95	18235.19923	-73.9877	4.26326E-13	0.916667	3.91E-13	240	180	7.03E-11	9.38E-11	-0.0625
Nose				0			589.0486			0	
sum(cell 1)							2167.5		264244.8	823597.3	
sum(cell 2)							829.0486			0	
No	dz/dy	Px	Pz	Px Z'	Pz X'						
1	0.025	1309.466437	-523.787	7175.876076	7856.799						
2	0.025	120.8933612	-145.072	632.2722789	725.3602	sum(L/t)1+(A1/A2)*(L/t)1-8=			3695.387		
3	0.025	-121.8094222	-146.171	-637.063278	-730.857	L/t)1-8+(A1/A2)*sum(L/t)2			5517.887		
4	0.025	-736.3921082	-294.557	-4035.42875	-4263.71						
5	-0.025	659.181989	-263.673	-3612.3173	-3816.66		q1	-283			
6	-0.025	146.1710329	-175.405	-764.474502	-877.026		q2	-72.26			
7	-0.025	-148.0290817	-177.635	774.1920973	888.1745						
8	-0.025	-1232.184562	-492.874	6752.371402	7393.107						
				6285.428021	7175.184						
Ttot	264244.8	2A1	720