Airframe Design and Construction

Wing Ultimate Loads and Bending Stresses

Instructor: Mohamed Abdou Mahran Kasem, Ph.D.

Aerospace Engineering Department

Cairo University

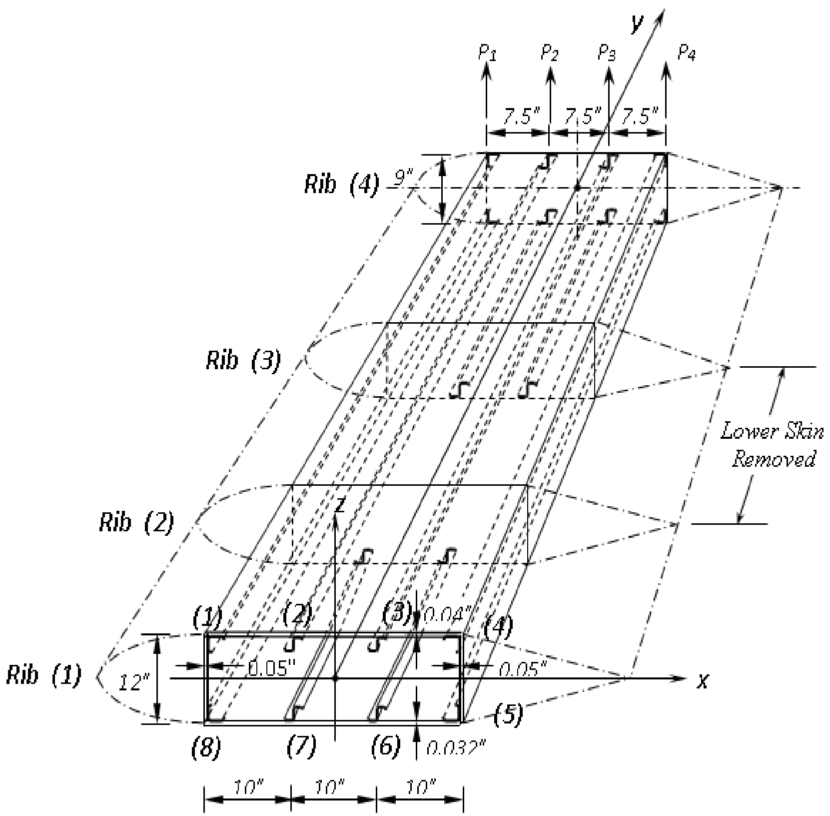
Wing ultimate loads and bending stresses - Example

q1	-283
q2	-72.26

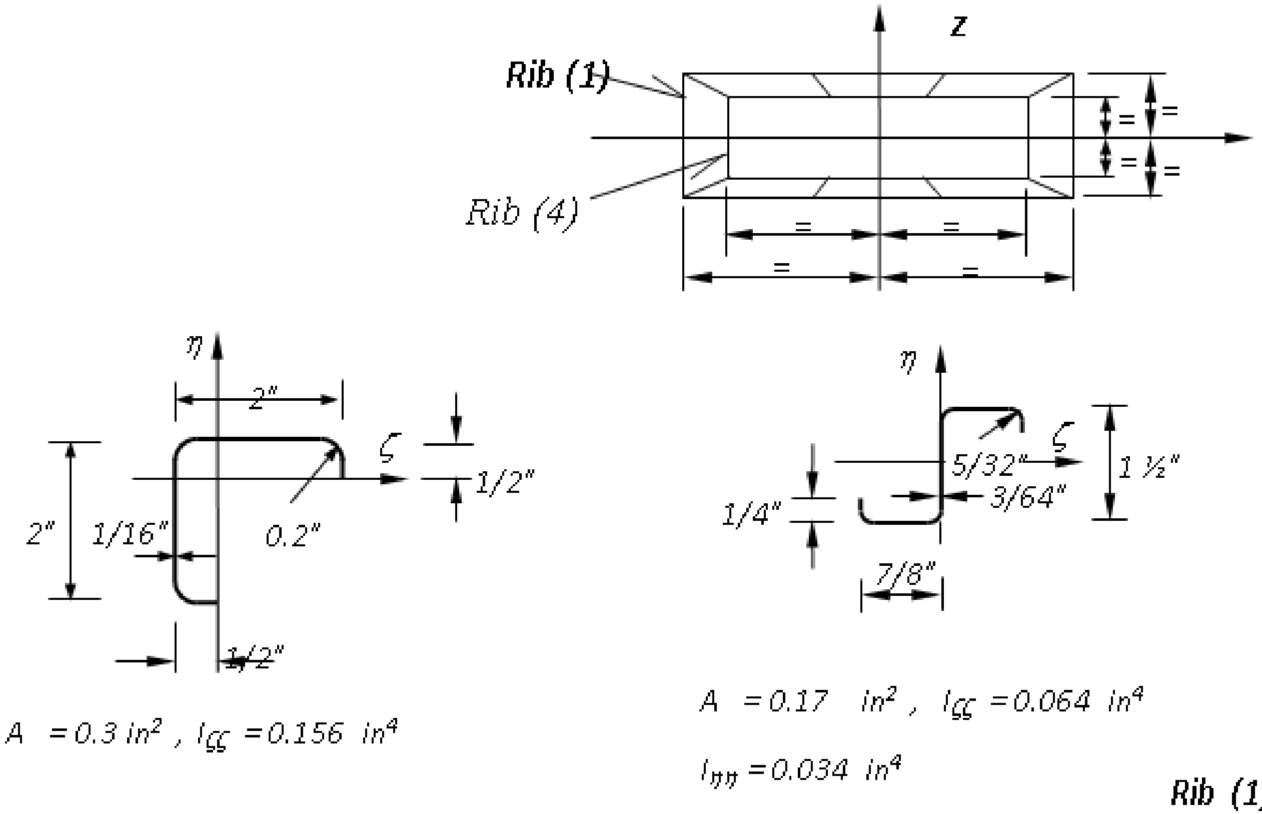
The A[illegible] idealization of tapered wing composed of *3 single*[illegible]dge portion is considered ineffective and the leading-edge portion takes circular shape. <u>*The lower skin of the middle bay is removed.*</u> The corner stringers are formed lipped angles while the other four stringers are formed lipped zee-section as shown. Stringer areas and sheet thickness are constant through the three bays. All parts are made up of *7075-T6 aluminum alloy*. The rib spacing is *20"* and the stringer-rib connections are such that the end fixity coefficient may be taken *1.5*. The skin riveting is *single row countersunk* of diameter *1/8"* while that of the web is *single row Brazier* head of the same diameter. The *rivet pitch* of both skin and web is *1"* and the rivet material is *2117-T3*.

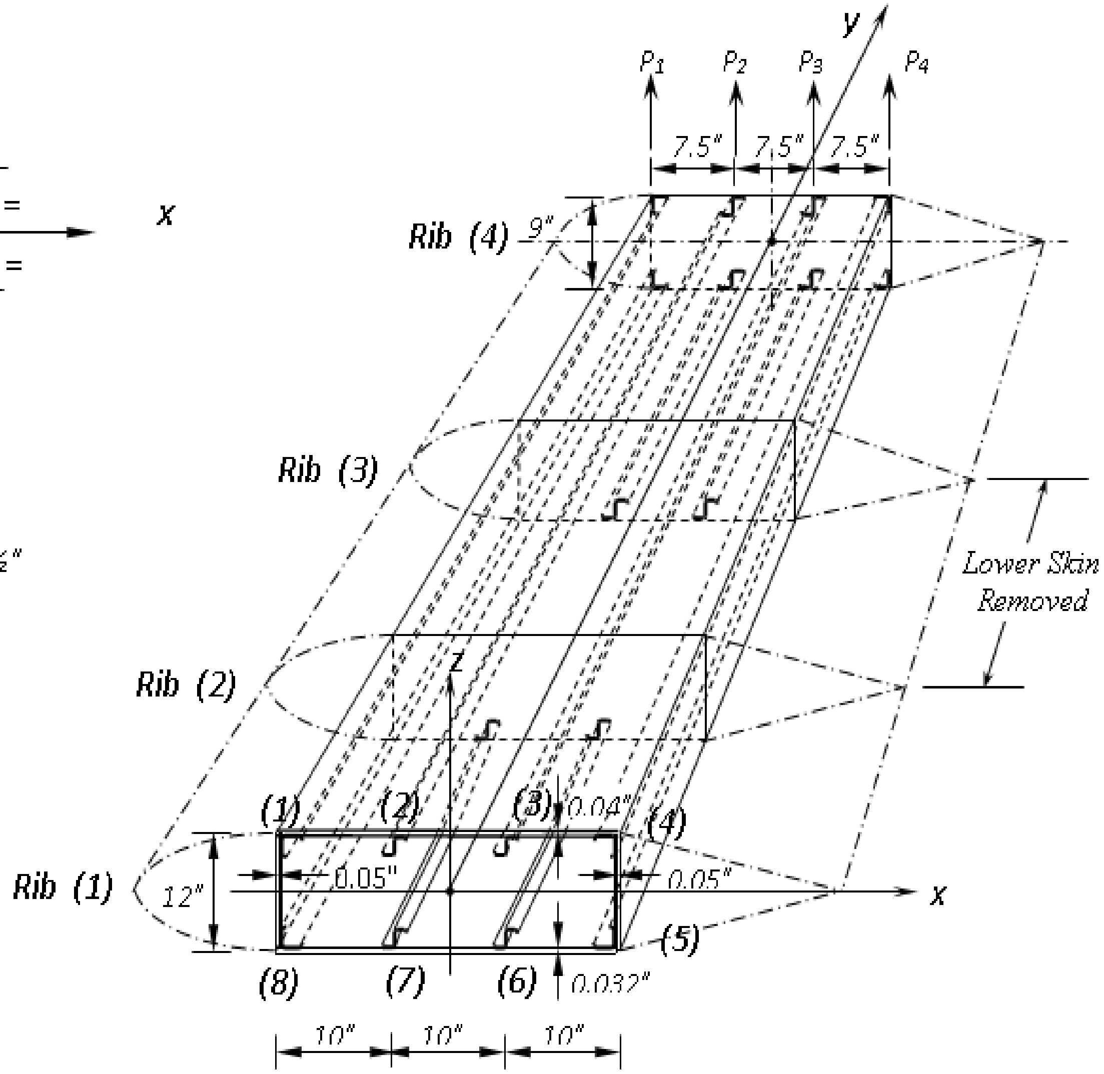
For the case when the design values of the applied forces are $P_1=1.5$ *kips* , $P_2=3.0$ *kips* , $P_3=2.0$ *kips* and $P_4=1.5$ kips and <u>taking the shear lag effect into consideration</u>, *it is required to*

- *Calculate the ultimate loads each stringer can carry*
- *Determine the stresses and margin of safety at both station 0 and station 20.*



Wing ultimate loads and bending stresses - Example





There is two angle sections at the nose with stringer 1 and 8 that are not depicted in the figure.

Wing ultimate loads

For 7075 – T6 material
$F_{cy} = 67000\ psi, F_{tu} = 76000\ psi, E = 10.5e6$

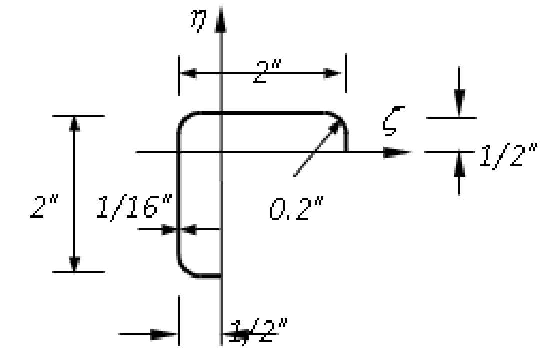
The inter-rivet buckling:

$$\frac{\boldsymbol{P}}{\boldsymbol{t}}_{\boldsymbol{eq}} = \frac{\boldsymbol{P}}{\boldsymbol{t}}_{\boldsymbol{act}} \sqrt{\frac{4}{\boldsymbol{C}}}$$

The rivets fixity coefficient	
For flat head	4
For spot welds	3.5
Brazier	3
counter-sunk	1

	t	p	C-rivet	p-corr	p-corr / t	Material	Fir
Upper-skin	0.04	1.0	1.0	2.00	50.00	7075-T6 Sh.	17,500
Front-web	0.05	1.0	3.0	1.15	23.09	7075-T6 Sh.	56,000
Rear-web	0.05	1.0	3.0	1.15	23.09	7075-T6 Sh.	56,000
Nose skin	0.032	1.0	1.0	2.00	62.50	7075-T6 Sh.	8,500

Wing ultimate loads



$A = 0.3\ in^2$, $I_{\zeta\zeta} = 0.156\ in^4$

Failing Stresses:
Corner section

bL = 0.46875, bF = 1.9375, A = 0.3, t = 0.0625, bL/t = 7.500, bF/t = 31.000

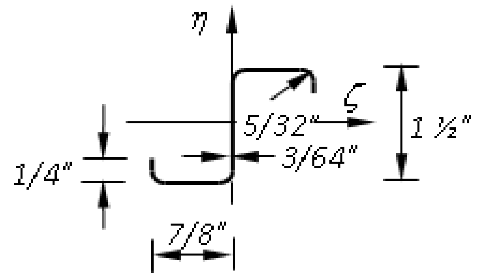
Lip Effectiveness :	376.40625	>=	155	OK		7.500	<=	10.168	OK	
Multi-corne :	g =	8	Fcs/Fcy =	0.702	Max =	0.8	Fcs =	47,017	Xcg	0.5
									w	2

$$0.91\frac{b_L{}^3}{t} - \frac{b_L}{t} \geq 5\frac{b_f}{t} \quad \& \quad \frac{b_L}{t_L} \leq 0.328\frac{b_f}{t_f}$$

In case of lip or bulb use only Gerard method.

$$F_{cs}/F_{cy} = 0.56\left[(gt^2/A)(E/F_{cy})^{1/2}\right]^{0.85}$$

Wing ultimate loads



$A = 0.17\ in^2$, $I_{\zeta\zeta} = 0.064\ in^4$

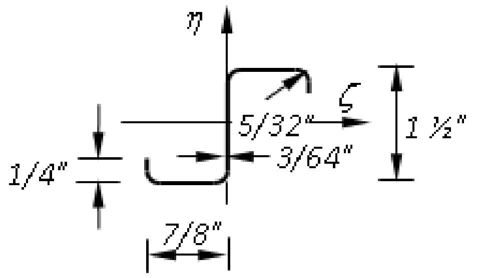
$I_{\eta\eta} = 0.034\ in^4$

<u>Failing Stresses:</u>
<u>Intermediate section</u>

<u>In case of lip or bulb use only Gerard method.</u>

Gerard :		t =	0.046875	bF =	0.828	bL =	0.2265625	bL/t =	4.833333333	bF/t =	17.667
Lip Effectiveness :		97.917	>=	88.333	OK	g =	11				
		4.833	<=	5.795	OK						
				Fcs/Fcy =	0.914	Max =	0.8	Fcs =	53,600	**Cut of velue**	

Wing ultimate loads



$A = 0.17\ in^2$, $I_{\zeta\zeta} = 0.064\ in^4$

$I_{\eta\eta} = 0.034\ in^4$

__Failing Stresses:__
__Intermediate section__

Initial value of F_c is calculated using ρ_0

h =	1.5	Ao =	0.17	I11 =	0.064	ρo =	0.614	L =	20.0
C (str) =	1.50	L' =	16.330	t-skin =	0.040	S =	0.770	S/ρo =	1.255
Fir =	17,500	Fcs =	53,600	Fco =	48,691				
iteration	Fc	w	w-corr	w-cor * t	w-cor*t / Ao	(ρ / ρo)2	ρ	L' / ρ	Fc
1	48,691	1.064	0.382	0.015	0.090	1.037	0.625	26.139	48,865
2	48,865	1.064	0.381	0.015	0.090	1.037	0.625	26.140	48,864

Fc = 48,864

Column Failing Stress

Iterative Method

1. Get $W = 1.9t\sqrt{\frac{E}{F_{cs}}}$ $\rho = \sqrt{\frac{I}{A}}$

2. Get ρ where $\rho = \rho_0\sqrt{\frac{1 + \frac{wt}{A_0}[1 + \frac{s}{\rho_0}^2]}{[1 + \frac{wt}{A_0}]^2}}$

3. Get Fc from Johnson-Euler Equation:

$$F_c = F_{cs} - \frac{F_{cs}^{\ 2}}{4\pi^2 E}\frac{L'^2}{\rho} \qquad L' = \frac{L}{\sqrt{C}}$$

4. Calculate W $W = 1.9t\sqrt{\frac{E}{F_c}}$

Wing ultimate loads

Stringer effective area and ultimate loads:

If we assumed two stringers at corner $\boldsymbol{A_{st} = 0.3 * 2 = 0.6}$

Stringer 1:

Fcs = 47,017

The edge distance assumed to be higher than W_1

Upper Skin									
E	t	Fir	w/2	w/2-corr	w1	w1-corr	Edge Dist.	Total w	(w-corr)*t
1.05E+07	0.040	17,500	0.568	0.211	0.371	0.138	1.000	0.349	0.0140
Front Web									
E	t	Fir	w/2	w/2-corr	w1	w1-corr	Edge Dist.	Total w	(w-corr)*t
1.05E+07	0.050	56,000	0.710	0.710	0.463	0.463	1.000	1.173	0.0587
Nose Skin									
E	t	Fir	w/2	w/2-corr	w1	w1-corr	Edge Dist.	Total w	(w-corr)*t
1.05E+07	0.032	8,500	0.454	0.082	0.296	0.054	1.000	0.136	0.0043

Total Area (1) =	0.677
P-ult (1) =	31,829

Wing ultimate loads

Stringer effective area and ultimate loads:

Stringer 2 and 3:

Fc = 48,864 **Astr = 0.17**

E	t	Fir	w	w-corr	Total w	(w-corr)*t
1.05E+07	0.040	17,500	1.114	0.399	0.399	0.0160

Total Area (2) & (3) =	0.186
P-ult (2) & (3) =	9,087

Wing ultimate loads

Stringer effective area and ultimate loads:

Stringer 4:

Fcs = 47,017 **Astr = 0.3**

Upper Skin									
E	t	Fir	w/2	w/2-corr	w1	w1-corr	Edge Dist.	Total w	(w-corr)*t
1.05E+07	0.040	17,500	0.568	0.211	0.371	0.138	0.875	0.349	0.0140
Rear Web									
E	t	Fir	w/2	w/2-corr	w1	w1-corr	Edge Dist.	Total w	(w-corr)*t
1.05E+07	0.050	56,000	0.710	0.710	0.463	0.463	0.875	1.173	0.0587

Total Area (4) =	0.373
P-ult (4) =	17,520

Wing ultimate loads

The effective area of the Nose is calculated as

$$A_{eff} = \frac{\pi R t}{4}$$

Stringer effective area and ultimate loads:

Ftu = 76,000 **Astr = 0.3**

Stringer 5:

Lower Skin			Rear Web		
t	Eff. Skin width	Eff. Skin area	t	Eff. Web width	Eff. Web area
0.032	5.00	0.16	0.05	2.00	0.1

A5 = 0.56

Stringer 6 and 7:

Ftu = 76,000 **Astr = 0.17**

Lower Skin		
t	Eff. Skin width	Eff. Skin area
0.032	10.00	0.32

Pult = 37,240, and A6=A7= 0.4900

Stringer 8:

Lower Skin			Front Web			Nose Skin		
t	Eff. Skin width	Eff. Skin area	t	Eff. Web width	Eff. Web area	t	Eff. Web width	Eff. Web area
0.032	5.00	0.160	0.05	2.00	0.1000	0.03	4.712	0.151

Pult 8 = 65,360

A8 = 1.0108

Stresses at station (0) – root section

$$\boldsymbol{\sigma_b} = -(\boldsymbol{K}_3\boldsymbol{M_z} - \boldsymbol{K}_1\boldsymbol{M_x})\,\boldsymbol{X} - (\boldsymbol{K}_2\boldsymbol{M_x} - \boldsymbol{K}_1\boldsymbol{M_z})\,\boldsymbol{Z}$$

st. No	Aeff	K1	KSL	A	x'	z'	Ax'	Az'
1	0.677	1	1	0.677	-15	5.48	-10.155	3.710
2	0.186	1.0392856	1	0.193	-5	5.23	-0.966	1.011
3	0.186	1.0392856	1	0.193	5	5.23	0.966	1.011
4	0.373	1	1	0.373	14.475	5.48	5.394	2.042
5	0.560	1	1	0.560	14.475	-5.48	8.106	-3.069
6	0.490	1	0.8	0.392	5	-5.23	1.960	-2.050
7	0.490	1	0.8	0.392	-5	-5.23	-1.960	-2.050
8	1.011	1	1	1.011	-15	-5.48	-15.162	-5.539
sum				3.791			-11.817	-4.935

Xcg	-3.11711
Zcg	-1.30178

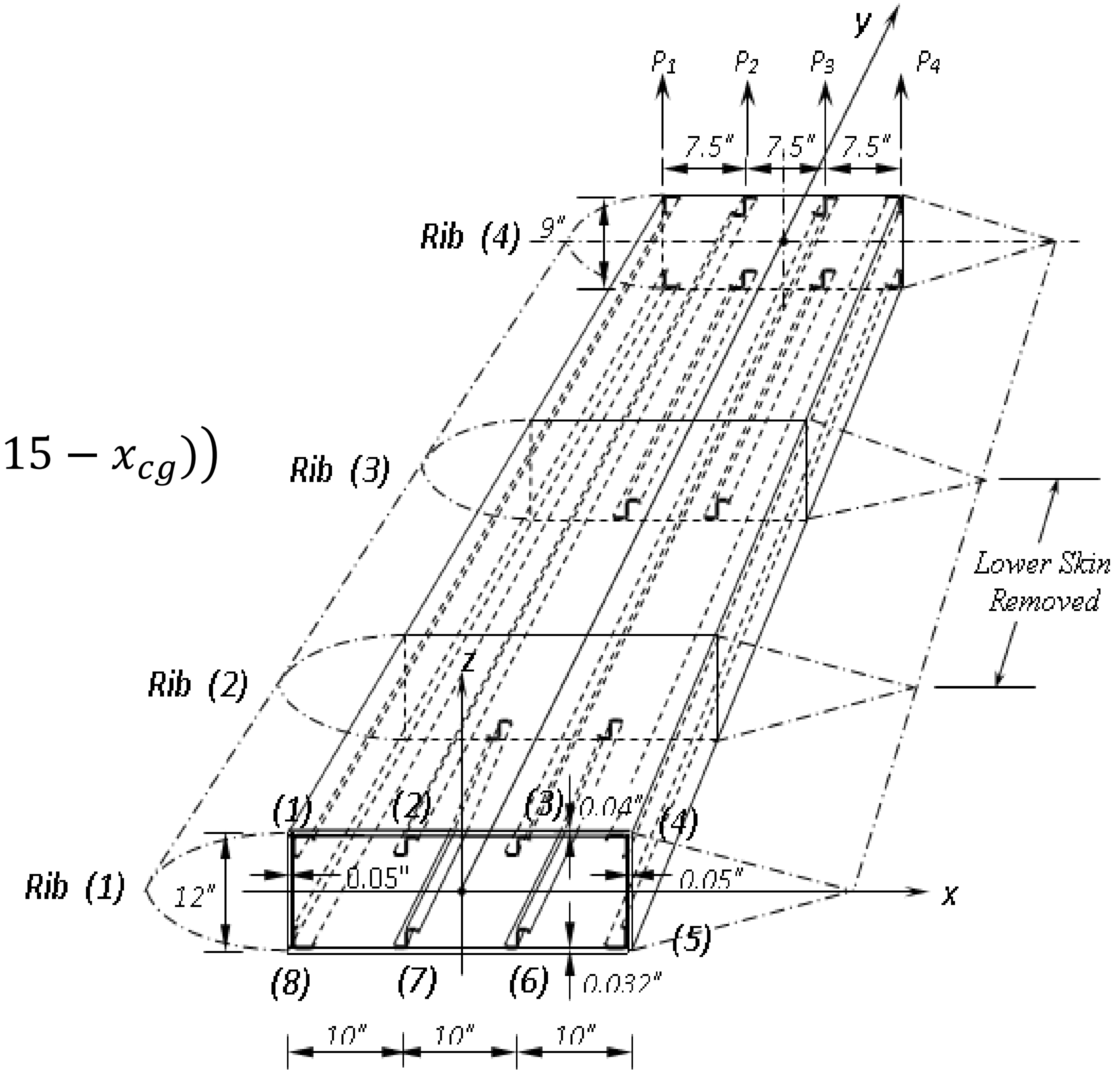
Stresses at station (0) – root section

Moment due to applied loads

$$M_x = (P_1 + P_2 + P_3 + P_4) * 3 * 20 = 480000\ Ib.in$$

$$M_y = \left(P_1 * \left(15 + x_{cg}\right) + P_2 * \left(5 + x_{cg}\right) - P_3 * \left(5 - x_{cg}\right) - P_4 * (15 - x_{cg})\right)$$

$$M_y = -19936.9\ Ib.in$$



Stresses at station (0) – root section

No	X	Z	AX^2	AZ^2	AXZ
1	-11.883	6.782	95.590	31.136	-54.555
2	-1.883	6.532	0.685	8.245	-2.377
3	8.117	6.532	12.734	8.245	10.247
4	17.592	6.782	115.322	17.138	44.457
5	17.592	-4.178	173.310	9.776	-41.162
6	8.117	-3.928	25.828	6.049	-12.499
7	-1.883	-3.928	1.390	6.049	2.899
8	-11.883	-4.178	142.728	17.646	50.185
sum			567.586	104.285	-2.805

K1	-4.7399E-05
K2	0.009590407
K3	0.001762081

Stresses at station (0) – root section

$$\boldsymbol{\sigma_b = -(K_3 M_z - K_1 M_x)\, X - (K_2 M_x - K_1 M_z)\, Z}$$

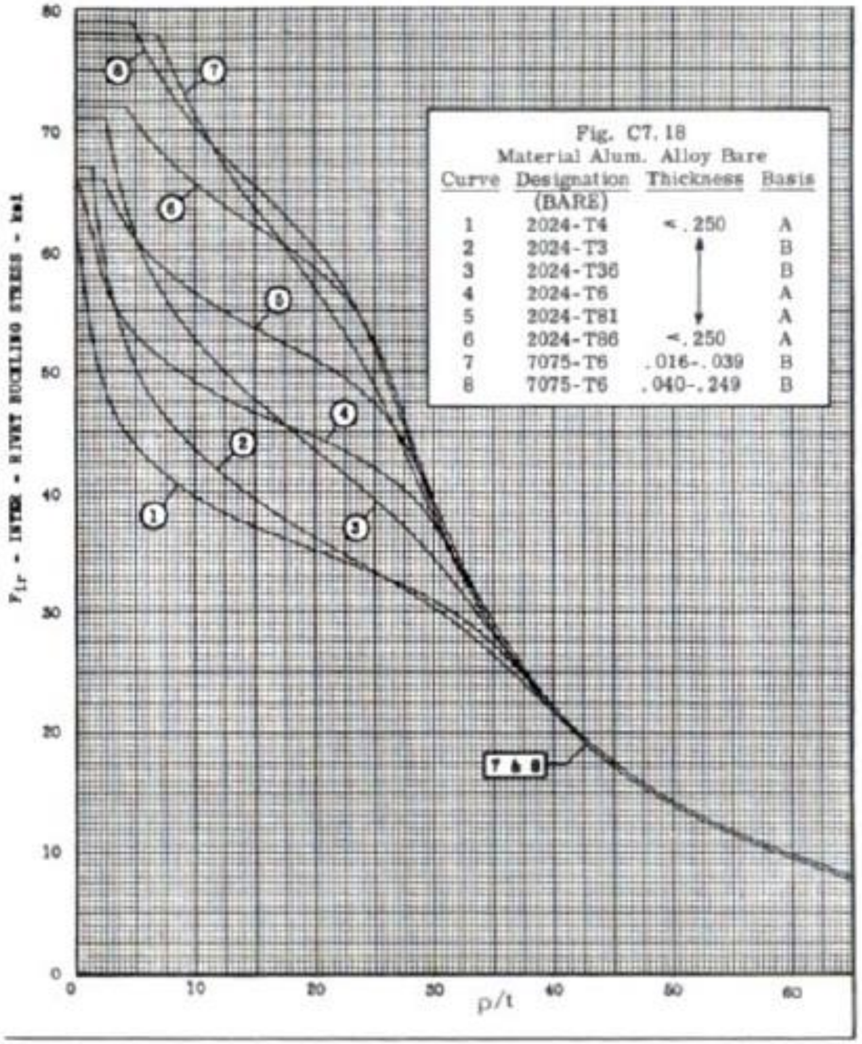
No	Sigma	sigma_True	S_failure	MS	P_0
1	-30948.839	-30948.839	47017.111	0.519	-20951.463
2	-30025.506	-31205.077	48864.208	0.566	-5802.881
3	-30253.023	-31441.531	48864.208	0.554	-5846.852
4	-31619.443	-31619.443	47017.111	0.487	-11782.274
5	18833.771	18833.771	76000.000	3.035	10546.912
6	17898.494	14318.795	76000.000	4.308	7016.210
7	18126.010	14500.808	76000.000	4.241	7105.396
8	19504.375	19504.375	76000.000	2.897	19714.953
sum					0.000

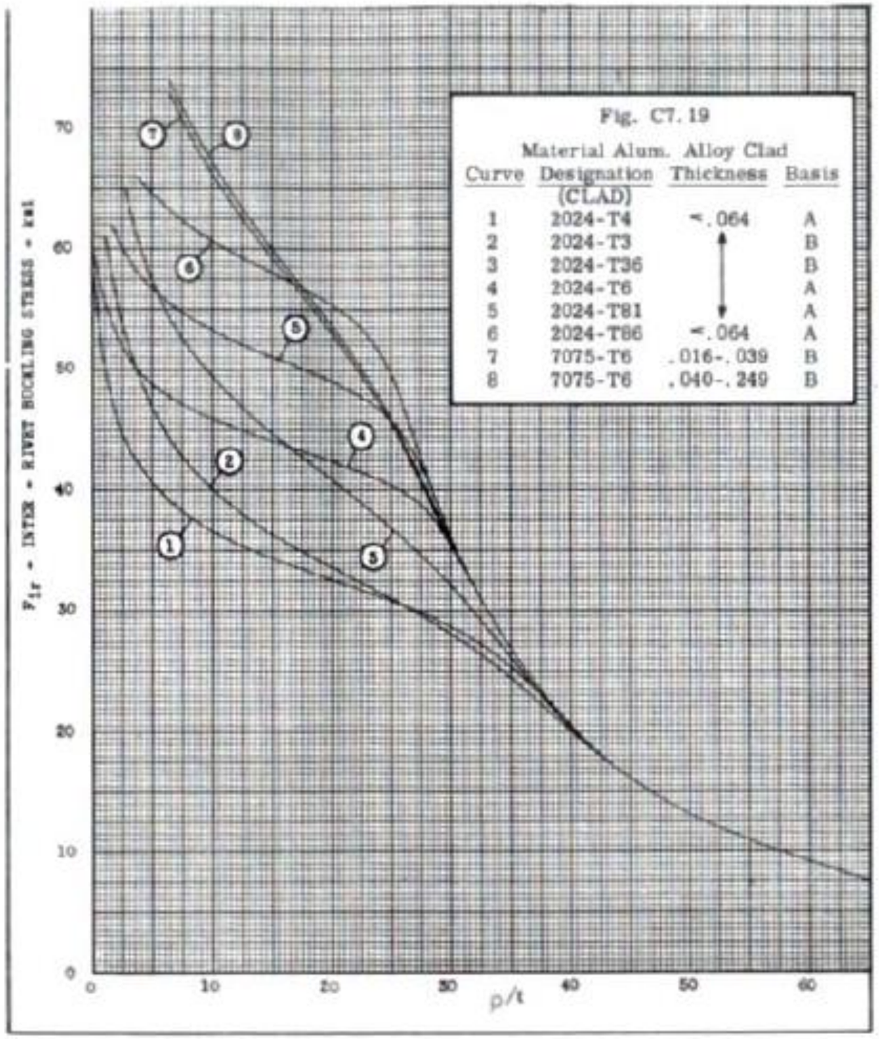
$$MS = \frac{\sigma_{failure}}{\sigma_{true}} - 1$$

Stresses at station (20) – second station

st No	Aeff	K1	Ksl	A	x'	z'	Ax'	Az'	X	Z	AX^2	AZ^2
1	0.676971	1	1	0.676971	-13.75	4.98	-9.30835	3.371315	-10.085	5.166085	68.85346	18.06729
2	0.18596	1.0392856	1	0.193265	-4.58333	4.73	-0.8858	0.9141437	-0.91838	4.916085	0.163004	4.670808
3	0.18596	1.0392856	1	0.193265	4.583333	4.73	0.885798	0.9141437	8.248287	4.916085	13.14864	4.670808
4	0.372627	1	1	0.372627	13.225	4.98	4.927999	1.8556849	16.88995	5.166085	106.2996	9.944843
5	0.538333	1	1	0.538333	13.225	-4.98	7.119458	-2.6809	16.88995	-4.79392	153.5706	12.37177
6	0.463333	1	0	0	4.583333	-4.73	0	0	8.248287	-4.54392	0	0
7	0.463333	1	0	0	-4.58333	-4.73	0	0	-0.91838	-4.54392	0	0
8	0.98913	1	1	0.98913	-13.75	-4.98	-13.6005	-4.925866	-10.085	-4.79392	100.6026	22.73181
Sum				2.963592			-10.8614	-0.551479			442.638	72.45733
No	AXZ	Sigma	Sigma_True	MS	P_20							
1	-35.2703	-23717.44	-23717.4427	0.982385	-16056			Mx	320000	K1	0.000263755	
2	-0.87256	-21837.2	-22695.0927	1.153074	-4220.37			My	-24736.3	K2	0.01383195	
3	7.836759	-21063.52	-21891.016	1.232158	-4070.84			Mz	0	K3	0.002264212	
4	32.51359	-21440.71	-21440.7071	1.19289	-7989.4			I	32001.12			
5	-43.5883	22644.484	22644.48385	2.356226	12190.28							
6	0	20808.557	0		0			Xcg	-3.66495			
7	0	20034.874	0		0			Zcg	-0.18608			
8	47.82132	20367.748	20367.74822	2.731389	20146.35							
Sum	8.440466				0							

Exam Data Sheet





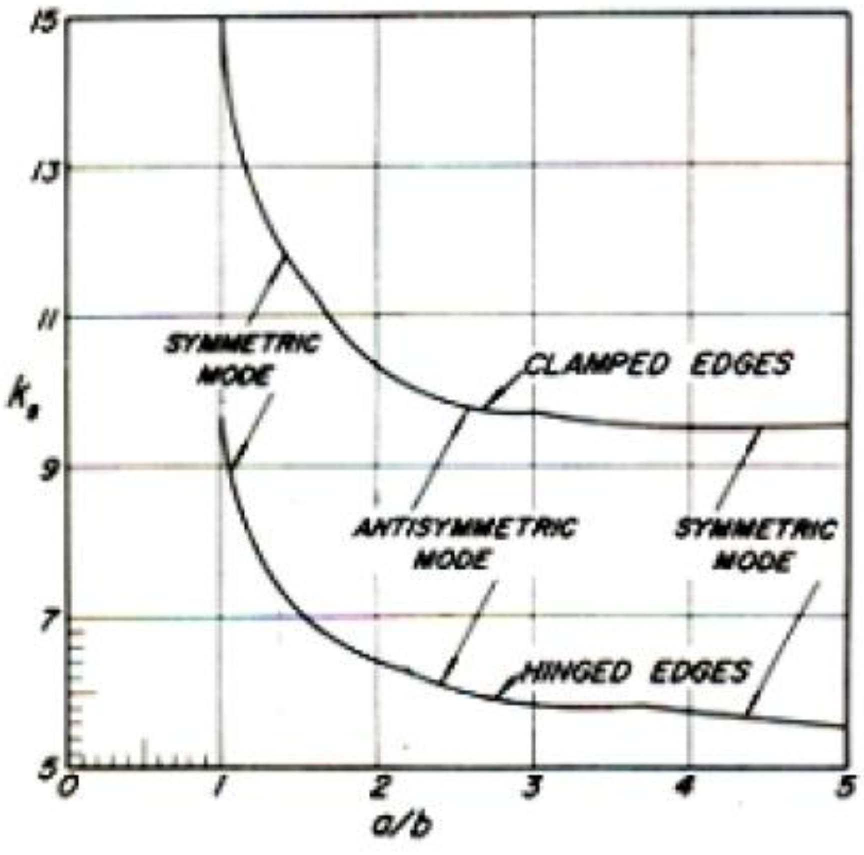
Exam Data Sheet

$$\sigma_b = -(K_3 M_z - K_1 M_x)\, X - (K_2 M_x - K_1 M_z)\, Z$$

$$K_1 = \frac{I_{xz}}{I}, \quad K_2 = \frac{I_{zz}}{I}, \quad K_3 = \frac{I_{xx}}{I},$$

$$I = I_{xx} I_{zz} - {I_{xz}}^2$$

$$\sigma_{cr} = \frac{K_T \pi^2 E}{12(1-\nu^2)} \frac{t^2}{{b_T}^2}$$



$$w = 1.9t\sqrt{\frac{E}{F_{st}}}, \qquad w_1 = 0.62t\sqrt{\frac{E}{F_{st}}}$$

$$\frac{F_{cs}}{\sqrt{F_{cy}E}} = \frac{C_e}{\left(\frac{b'}{t}\right)^{0.75}}$$

$$\frac{F_{cs}}{F_{cy}} = 0.56\left(\frac{gt^2}{A}\sqrt{\frac{E}{F_{cy}}}\right)^{0.85} \quad \frac{F_{cs}}{F_{cy}} = 0.67\left(\frac{gt^2}{A}\sqrt{\frac{E}{F_{cy}}}\right)^{0.4} \quad \frac{F_{cs}}{F_{cy}} = 3.2\left(\frac{t^2}{A}\sqrt[3]{\frac{E}{F_{cy}}}\right)^{0.75}$$

$$0.91\frac{{b_L}^3}{t} - \frac{b_L}{t} \geq 5\frac{b_f}{t} \quad \& \quad \frac{b_L}{t_L} \leq 0.328\frac{b_f}{t_f}$$

$$\rho = \rho_0 \sqrt{\frac{1 + \frac{wt}{A_0}\left[1 + \left(\frac{s}{\rho_0}\right)^2\right]}{\left[1 + \frac{wt}{A_0}\right]^2}} \qquad F_c = F_{cs} - \frac{{F_{cs}}^2}{4\pi^2 E}\left(\frac{L'}{\rho}\right)^2$$

$$\left(\frac{D}{t}\right)^4 - 1.6\left(\frac{D}{t}\right)^3 - 0.374\left(\frac{D}{t}\right)^2 \geq 7.44\,\frac{b_f}{t}$$

Exam Data Sheet

Type of Sections	Max. Fcs
Angles	0.7Fcy
V Groove Plates	Fcy
Multi-Corner sections, Including Tubes	0.8Fcy
Tee and H sections	0.8Fcy
Zee, J and Channels	0.9Fcy