Airframe Design and Construction

Wing Stress Analysis – A19

Instructor: Mohamed Abdou Mahran Kasem, Ph.D.

Aerospace Engineering Department

Cairo University

Typical wing structure arrangement

In general, the wing structural flange arrangement can be classified into two types: (1) *concentrated flange* type where flange material is connected directly to internal webs and (2) *the distributed flange* type where stringers are attached to skin between internal webs.

Distributed Flange Type of Wing Beam.

Fig. d

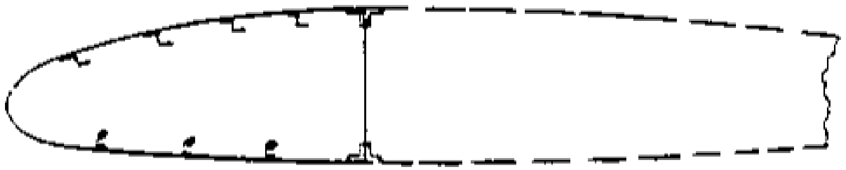


Fig. e

Fig. f

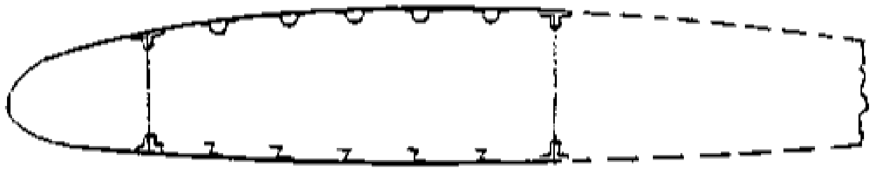


Fig. g

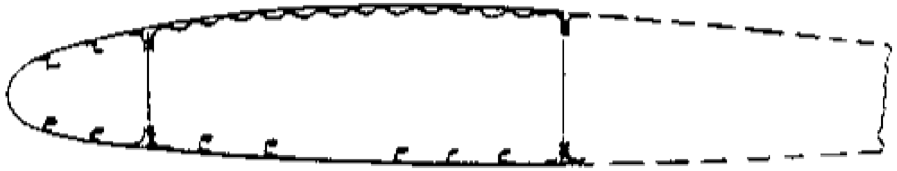


Fig. h

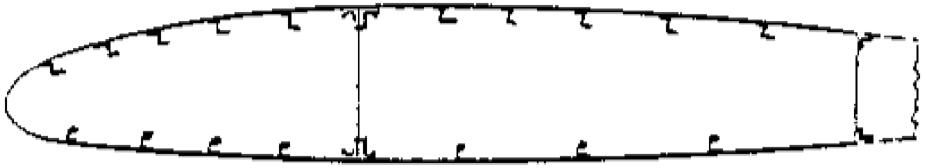
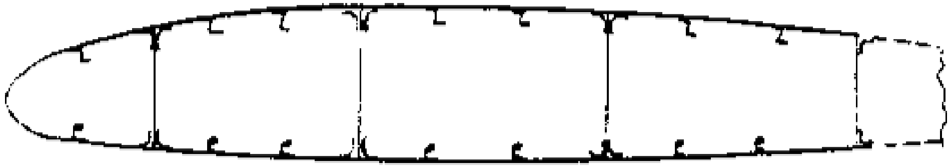


Fig. i



Typical wing structure arrangement

Different arrangements can be used for supersonic aircrafts in where the airfoil is relatively thin.

Fig. j

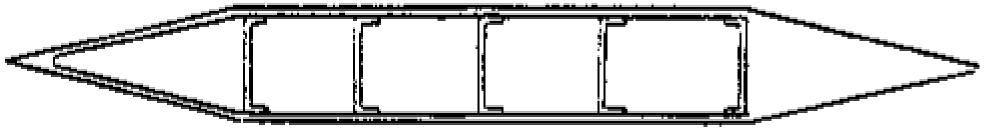


Fig. k

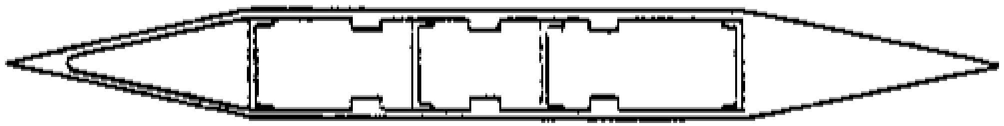
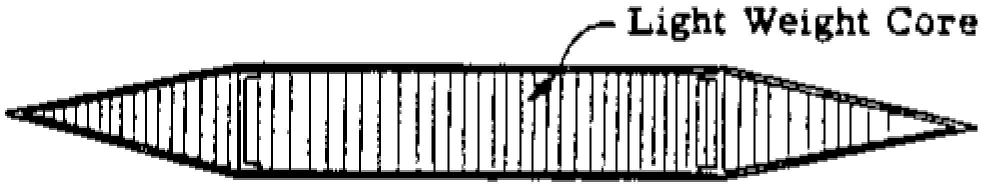


Fig. l



Some design considerations

- *Flanges* should be placed to give the largest moment of inertia in the Z direction, which means they should be placed at the airfoil maximum thickness that is usually between 15 to 50 % of the wings chord.
- *The secondary wing structure* (out of the wing box) should be made as light as possible.
- *Under the applied or limit loads* no part of the structure must stresses beyond the yield stress of the material (in general we can assume the yield stress as the stress which results a permanent strain of 0.002.
- *The term limit loads* refers to the maximum loads an airplane encounter during its lifetime.
- The structure should carry *design loads* without collapse or failure.
- *The design load* equal the airplane loads times a factor of safety which is usually 1.5 in airplane design.

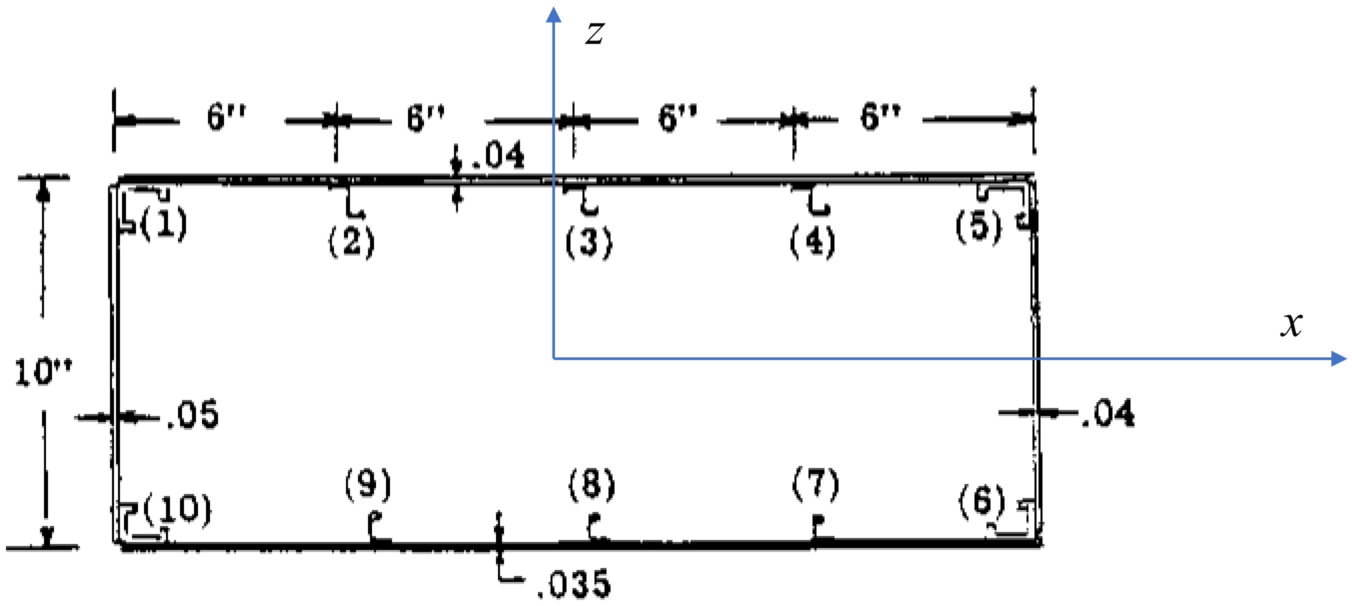
Example problem

The wing section shown in figure is subjected to a design bending moment about the x-axis of 500000 Ib.in. acting in direction to put the upper portion in compression. The material is 2424 aluminum alloy. Z-stringer failing stress is 38000 psi, and corner flange failing stress is 47000 psi.

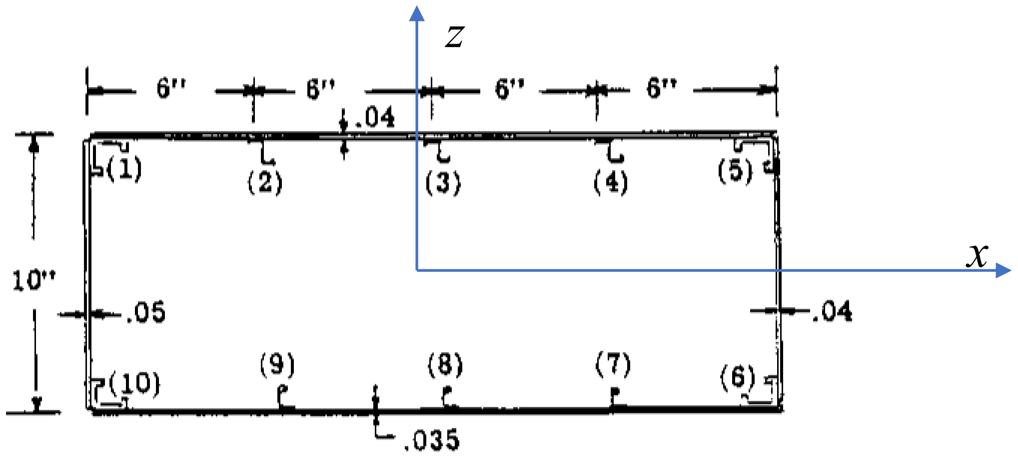
It is required to determine the margin of safety for the wing design load.

Zee stringer area = 0.135 sq.in.
Upper Corner stringers area = 0.27 Sq. in.
Lower Corner stringers area = 0.23 Sq. in.
Angle section area = 0.11 sq.in.

Note that, in this problem we neglect the inter-rivet buckling



Example problem



Solution Strategy:

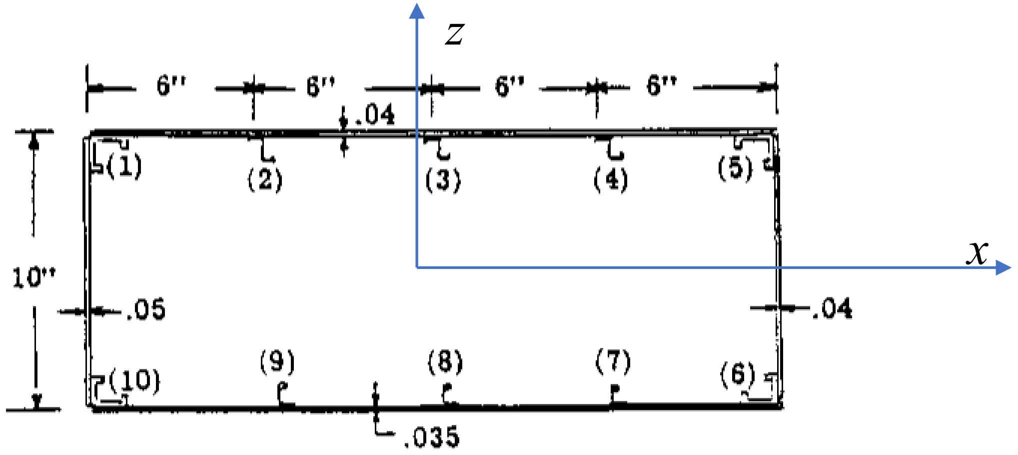
The wing is symmetric about the x-axis and we have only moment about the x-direction

$$\sigma_b = -\frac{M_x z}{I_x}$$

Thus, to determine the wing stress distribution one needs to calculate:

- The section centroid position
- The effective section moment of inertia

Example problem



Solution :

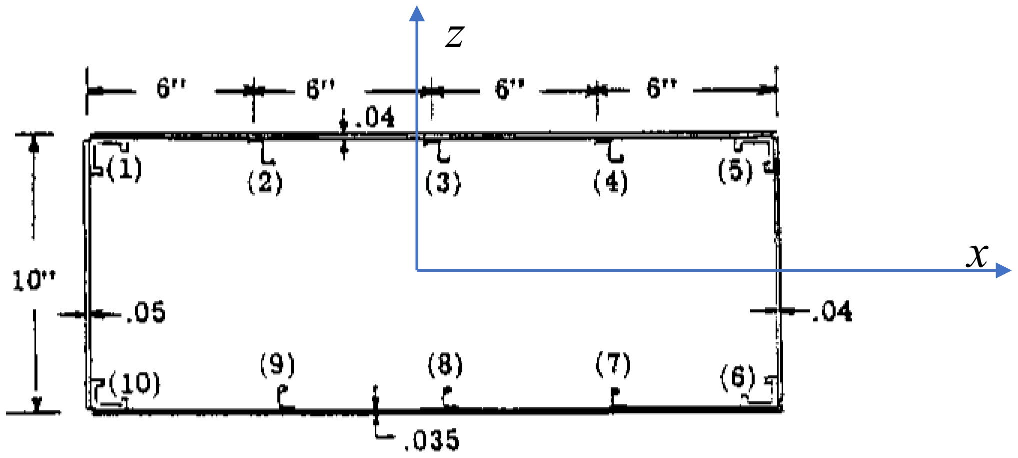
The effective width for zee stringer

$$w = 1.9 * 0.04\sqrt{\frac{10.5 * 10^6}{38000}} = 1.263$$

Thus, the effective skin area is 0.051 in^2, the z-stringer area is 0.135 . Then the total area of the skin and stringer is 0.186 in^2 which is corresponding to stringer 2,3, and 4.

The same procedure is done for the corner stringers.

Example problem



Solution :

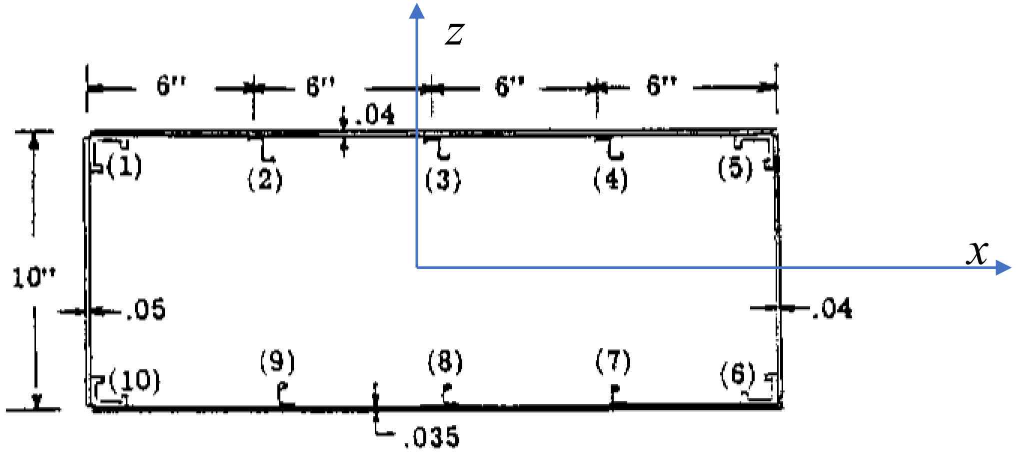
The same procedure is done for the corner stringers – 1.

Skin effective width

$$w = 1.9 * 0.04\sqrt{\frac{10.5 * 10^6}{47000}} = 1.136, \frac{w}{2} = 0.568\ in, w_1 = 0.62 * 0.04\sqrt{\frac{10.5 * 10^6}{47000}} = 0.371$$

$$w_{eff,skin} = 0.568 + 0.371 = 0.939\ in$$

Example problem



Solution :

The same procedure is done for the corner stringers.

Web effective width

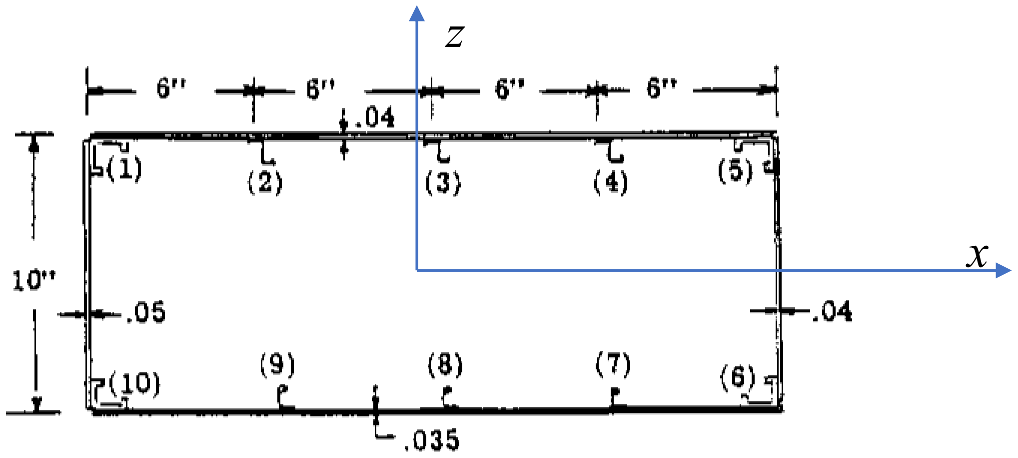
$$w = 1.9 * 0.05\sqrt{\frac{10.5 * 10^6}{47000}} = 1.419, \frac{w}{2} = 0.71\ in, w_1 = 0.62 * 0.04\sqrt{\frac{10.5 * 10^6}{47000}} = 0.463$$

$$w_{eff,web} = 0.71 + 0.463 = 1.173\ in$$

Total sheets effective area

$$A_{eff} = 0.939 * 0.04 + 1.173 * 0.05 = 0.09621\ Sq.in.$$

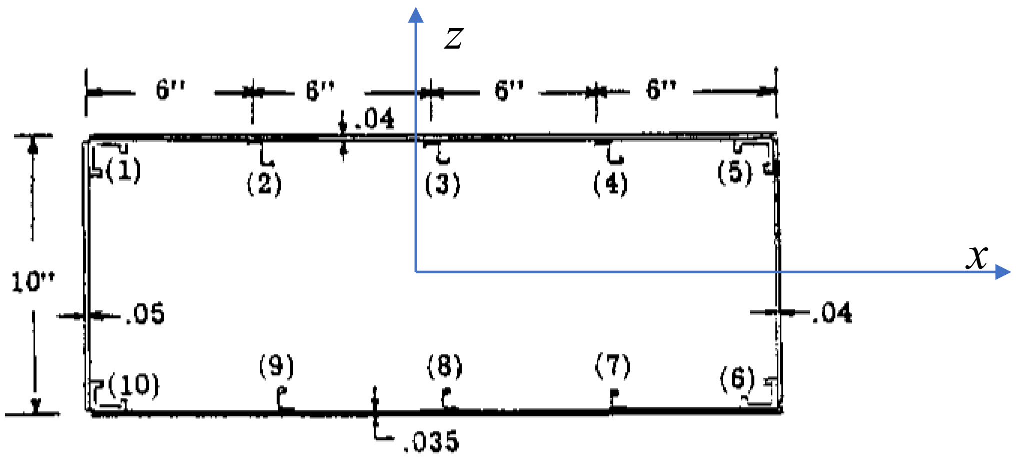
Example problem



Solution :

ID	Sheet	t	W_intermediate	W_corner/2	W1_corner	W_web_tension
1	Upper skin	0.04	1.263	0.568	0.371	-
2	right web	0.04	1.263	0.568	0.371	1.6667
3	left web	0.05	1.579	0.710	0.463	1.6667
4	lower skin	0.035	6.000	3.000	-	-

Example problem



Solution :

N	Ast	Aeff	Atot
1	0.270	0.096	0.366
2	0.135	0.051	0.186
3	0.135	0.051	0.186
4	0.135	0.051	0.186
5	0.270	0.075	0.345
6	0.230	0.172	0.402
7	0.110	0.210	0.320
8	0.110	0.210	0.320
9	0.110	0.210	0.320
10	0.230	0.188	0.418
sum			

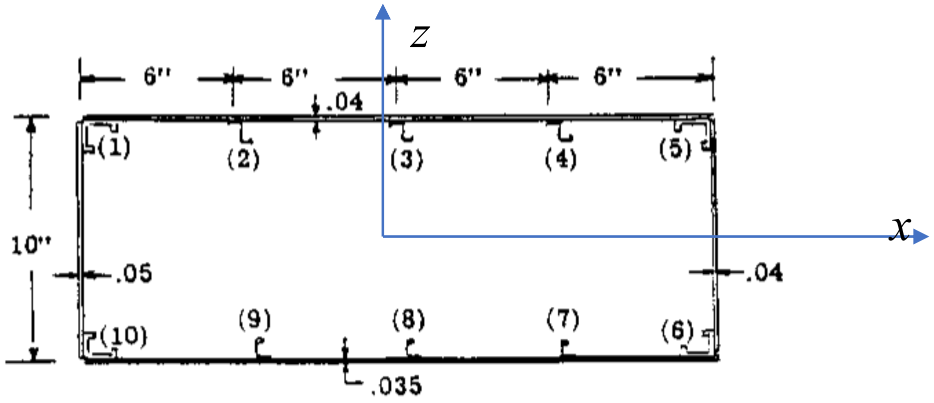
In the tension side the skin is totally effective.

Thus, a skin area of 6*0.035 is added to each angle section.

For the lower corner stringers, the effective area of the shear web is calculated as

$$A_{eff} = \frac{tb}{3}$$

b equals one half the height of the shear web



Example problem

Solution :

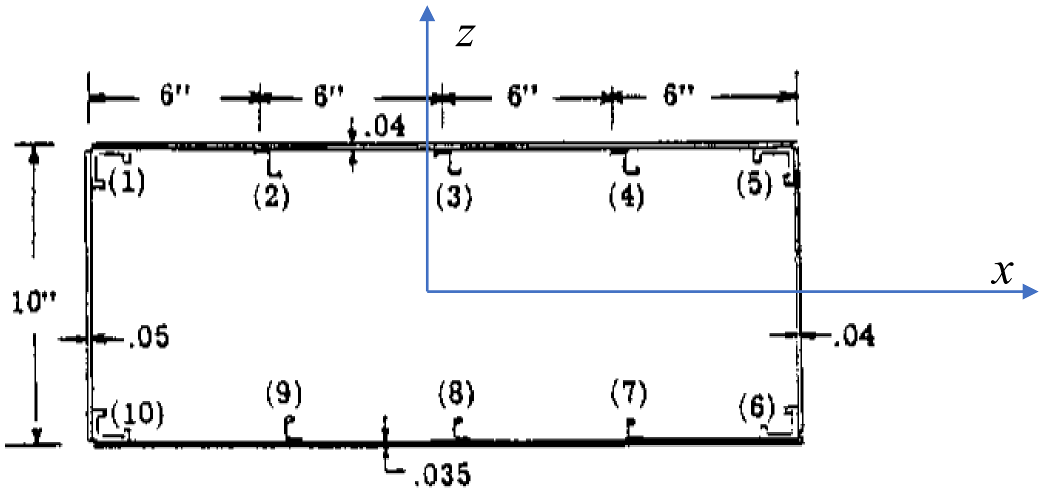
Then, we calculate the effectiveness factor results from the difference between each stringer failing strength.

The next step is to correct for stringer effectiveness in compression. The failing stress for the zee stringer was given as 38000 and for the corner members 47000 psi. The effectiveness factor for the zee stringer thus equals 37000/47000 = .808. This factor is recorded in Column 3 of Table I. For the corner members 1 and 5 and all the tension members the factor is of course unity. The balance of Table I gives the calculation of the effective moment of inertia $\Sigma A_z{}^2$ about the x neutral axis.

Example problem

Solution :

Effective area = stringer effectiveness factor*total area.



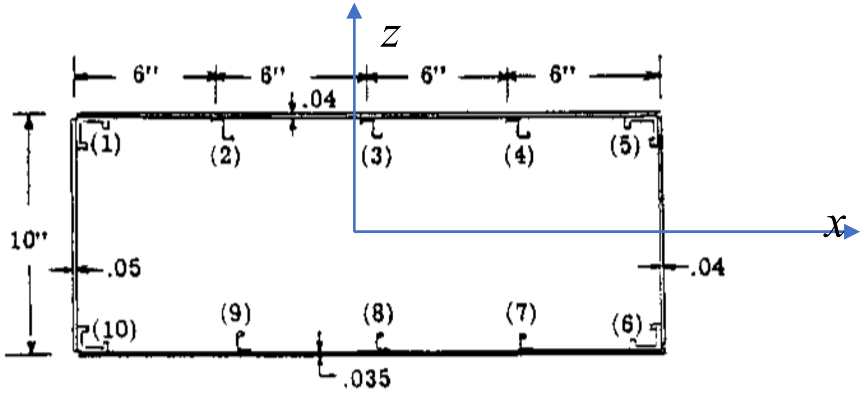
N	Atot	K	A
1	0.366	1.000	0.366
2	0.186	0.809	0.150
3	0.186	0.809	0.150
4	0.186	0.809	0.150
5	0.345	1.000	0.345
6	0.402	1.000	0.402
7	0.320	1.000	0.320
8	0.320	1.000	0.320
9	0.320	1.000	0.320
10	0.418	1.000	0.418
sum			2.941

Example problem

Solution :

After that, the position of each stringer is defined and the centroid position is calculated,

$$\bar{z} = \frac{\sum AZ'}{\sum A} = = -\mathbf{0.99}$$

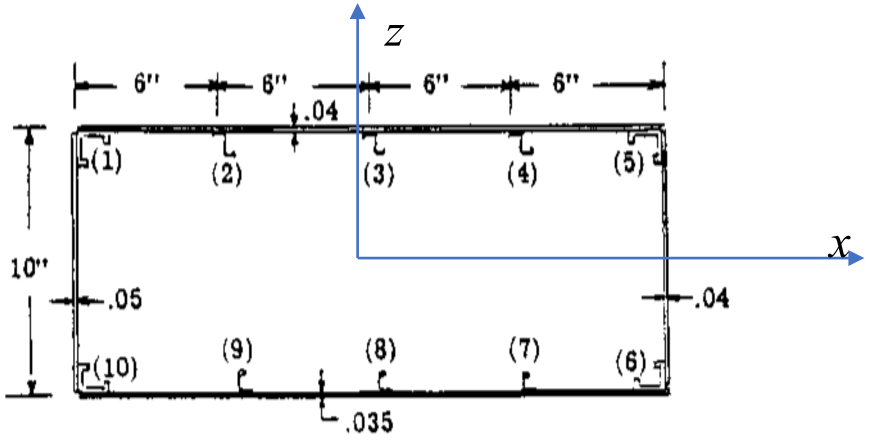


Z'	Az'	Z=Z'-Z_bar
4.500	1.648	5.494
4.600	0.690	5.594
4.600	0.690	5.594
4.600	0.690	5.594
4.600	1.587	5.594
-4.600	-1.848	-3.606
-4.630	-1.482	-3.636
-4.630	-1.482	-3.636
-4.630	-1.482	-3.636
-4.630	-1.937	-3.636
	-2.924	

Example problem

Solution :

Second moment of area and stress calculations

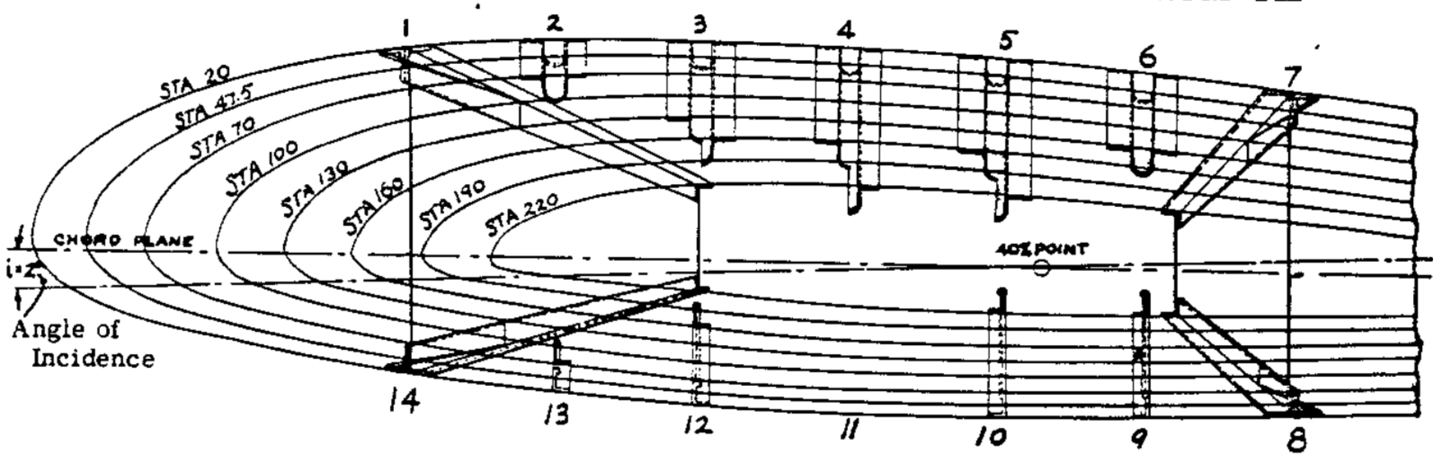


$$\sigma_b = -\frac{M_x z}{I_x}$$

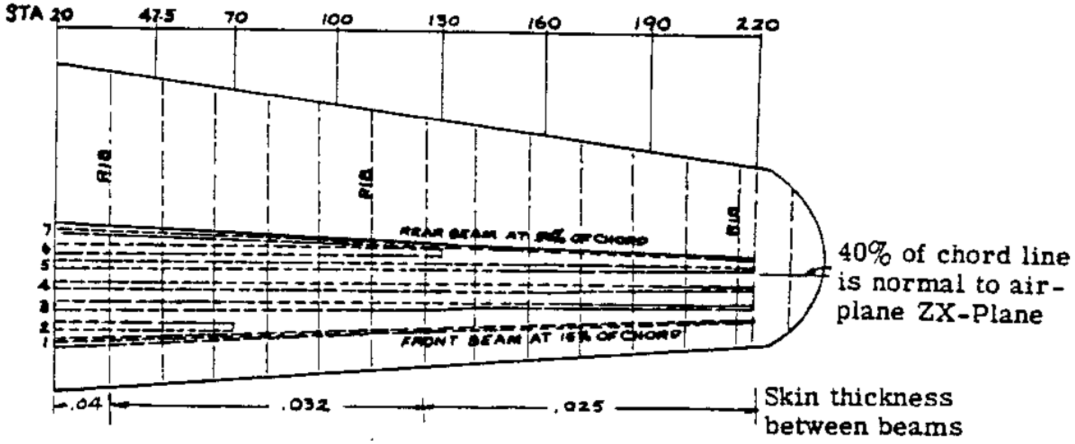
AZ^2	Sigma	S_true	M.S
11.054	-46261.860	-46261.860	0.016
4.694	-47103.891	-38083.997	-0.002
4.694	-47103.891	-38083.997	-0.002
4.694	-47103.891	-38083.997	-0.002
10.799	-47103.891	-47103.891	-0.002
5.223	30363.028	30363.028	
4.230	30615.638	30615.638	
4.230	30615.638	30615.638	
4.230	30615.638	30615.638	
5.530	30615.638	30615.638	
59.380			

Stress analysis of tapered wing

WING BODY SECTION PLAN VIEW LOOKING INBOARD FROM TIP

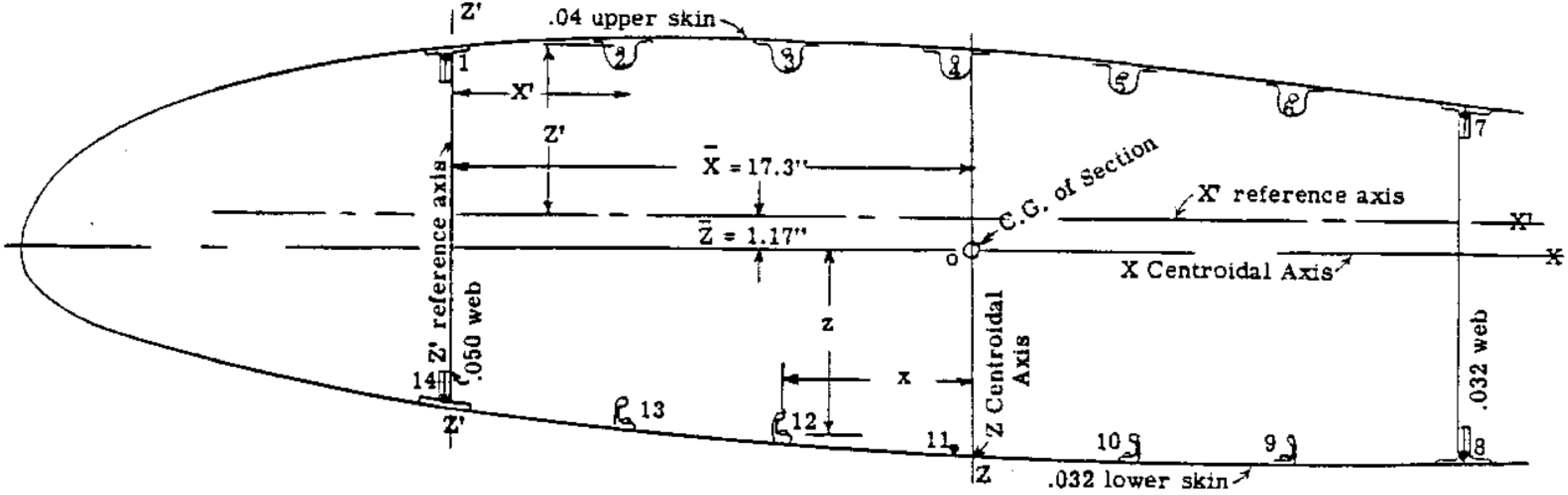


UPPER SURFACE STRINGER AND SKIN ARRANGEMENT



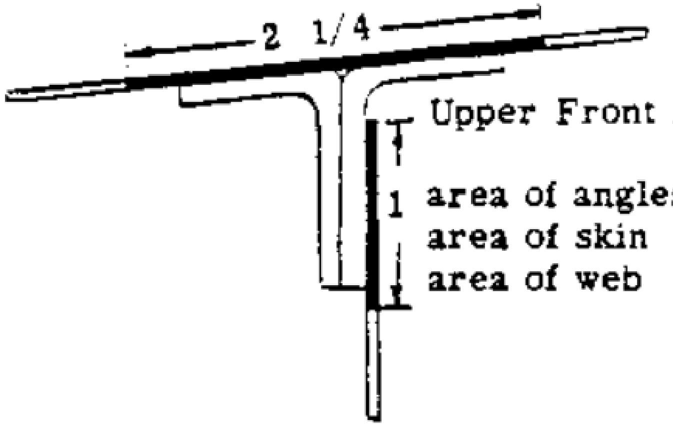
Stress analysis of tapered wing

Wing section at Station 20



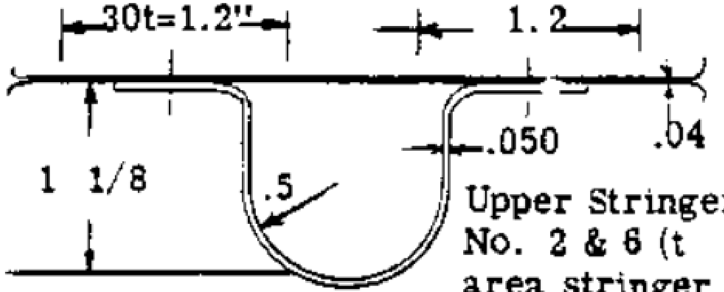
Stress analysis of tapered wing

Upper Stringers of the Wing section at Station 20



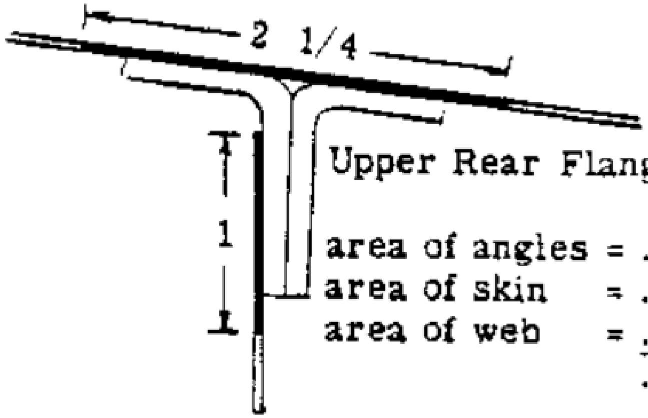
Upper Front Flange

area of angles = .508
area of skin = .090
area of web = .050
.648



Upper Stringer.
No. 2 & 6 (t = .064)
area stringer = .257
area skin = .096
.353

No. 3, 4 & 5 (t = .050)
area stringer = .205
area skin = .096
.301

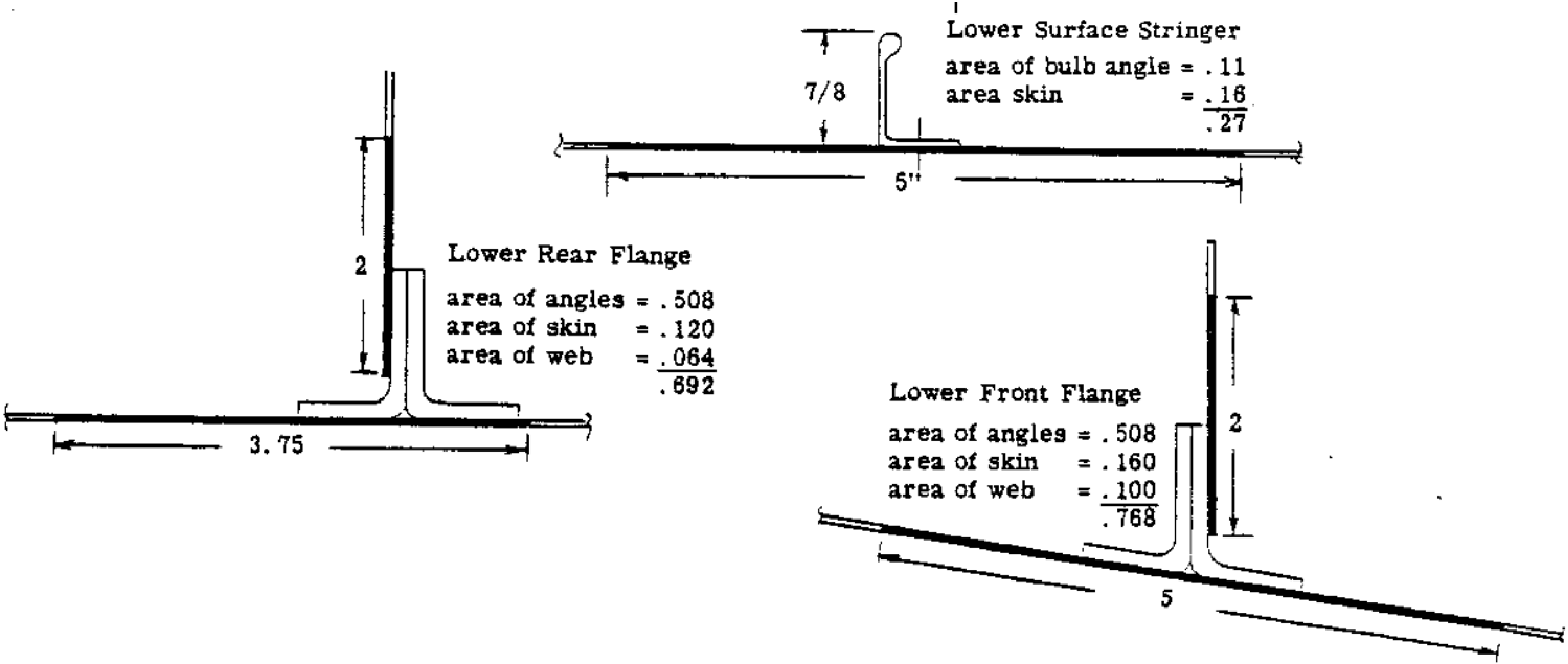


Upper Rear Flange

area of angles = .508
area of skin = .090
area of web = .032
.630

Stress analysis of tapered wing

Lower Stringers of the Wing section at Station 20



Stress analysis of tapered wing

Both the leading and trailing edges are assumed to be ineffective

The design bending moments will be assumed and are as follows: -

<u>Station 20</u> M_x = 1,300,000 in.lb.

M_z = - 285,000 in.lb.

<u>Station 47.5</u> M_x = 1,000,000 in.lb.

M_z = - 215,000 in.lb.

Stress analysis of tapered wing

1	2	3	4	5	6	7	8	9	10	11	12	13
Flange No.	Area A	Z'	AZ'	AZ'^{2}	X'	AX'	AX'^{2}	AX'Z'	$Z = Z' - \bar{Z}$	$X = x' - \bar{X}$	σ_b	$P = \sigma_b A$
1	.648	5.50	3.56	19.61	-0.10	-0.06	0	-0.36	6.68	-17.4	-37660	-24390
2	.353	5.90	2.08	12.30	5.65	1.99	11.30	11.78	7.08	-11.65	-39940	-14070
3	.300	5.85	1.76	10.28	11.20	3.36	37.60	19.67	7.03	- 6.10	-39620	-11890
4	.300	5.55	1.66	9.25	16.85	5.05	85.00	28.10	6.73	- 0.45	-37950	-11370
5	.300	5.05	1.52	7.65	22.40	6.72	150.50	34.00	6.23	5.10	-35115	-10510
6	.353	4.40	1.55	6.83	28.00	9.89	277.00	43.50	5.58	10.70	-31450	-11090
7	.630	3.55	2.24	7.95	35.72	23.15	826.00	80.00	4.73	18.42	-26560	-16700
8	.692	-8.40	-5.81	48.90	35.72	24.70	881.00	-208.00	-7.22	18.42	40750	28230
9	.270	-8.50	-2.30	19.55	27.80	7.50	208.00	- 64.00	-7.32	10.50	41280	11180
10	.270	-8.50	-2.30	19.55	22.60	6.10	138.00	- 52.00	-7.32	5.30	41250	11170
11	.160	-8.30	-1.33	11.05	16.70	2.67	44.60	- 22.20	-7.12	- 0.60	40100	6430
12	.27	-8.00	-2.16	17.30	10.90	2.94	32.00	- 23.50	-6.82	- 6.40	38400	10390
13	.27	-7.50	-2.02	15.20	5.60	1.51	8.45	- 11.35	-6.32	-11.7	35570	9600
14	.768	-6.50	-5.00	32.50	- 0.10	.08	0	0.50	-5.32	-17.40	29950	23020
Σ	5.584		-6.56	238.0		96.44	2700	-163.8				0.00

$$\bar{Z} = \frac{-6.56}{5.584} = -1.176$$

$$\bar{X} = \frac{96.44}{5.584} = 17.3$$

$$I_x = 238 - 5.584 \times 1.176^2 = 230.3 \text{ in.}^4$$

$$I_z = 2700 - 5.584 \times 17.3^2 = 1030 \text{ in.}^4$$

$$I_{xz} = -163.8 - 5.584 \times -1.176 \times 17.3 = -50$$

General Notes:

See Fig. A19.25 for Section at Station 20.
Reference axes X'X' and Z'Z' are assumed as shown.
Properties are calculated with respect to these axes and transferred to the centroidal X and Z axes.

$$\sigma_b = 3.3\ X - 5639\ Z$$

Stress analysis of tapered wing

To solve equation (4) the constants K_1, K_2 and K_3 must be known.

For Station 20 from Table A19.2, I_X = 230.3, I_Z = 1030 and I_{XZ} = − 50, whence −

$$K_1 = I_{XZ}/(I_X I_Z - I_{XZ}^2) = -50/(230.3 \times 10.30 - 50^2) = -50/235500 = -.0002125$$

$$K_2 = I_Z/235500 = 1030/235500 = .004378$$

$$K_3 = I_X/235500 = 230.3/235500 = .00098$$

Stress analysis of tapered wing

Substituting in equation (4)

$$\sigma_b = -\left[.00098\ x - 285000 - (-.0002125\ x\ 1300000)\right] x - \left[.004378\ x\ 1,300,000 - (-.0002125\ x - 285000)\right] z$$

whence, $\sigma_b = 3.3\ x - 5639\ z$ - - - - - - - - - (5)