Airframe Design and Construction

Column Failing strength

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Column Strength

- For bars with closed-thick sections, they are expected to buckle following *Euler Buckling*.
- For open cross-sections with small thicknesses, the test results follow the curve DEFC.
- Thus, *Euler equation* cannot be used in the range DEF.
- For slenderness ratio lower than 20 (DE), columns falls based on *Crippling stresses*.
- The portion EF of the curve referred to as *transition region* which combines failure due *local buckling and elastic bending instability*.
- Because there is no theory developed to determine the failing strength in this region, a *semi-empirical method was developed*.

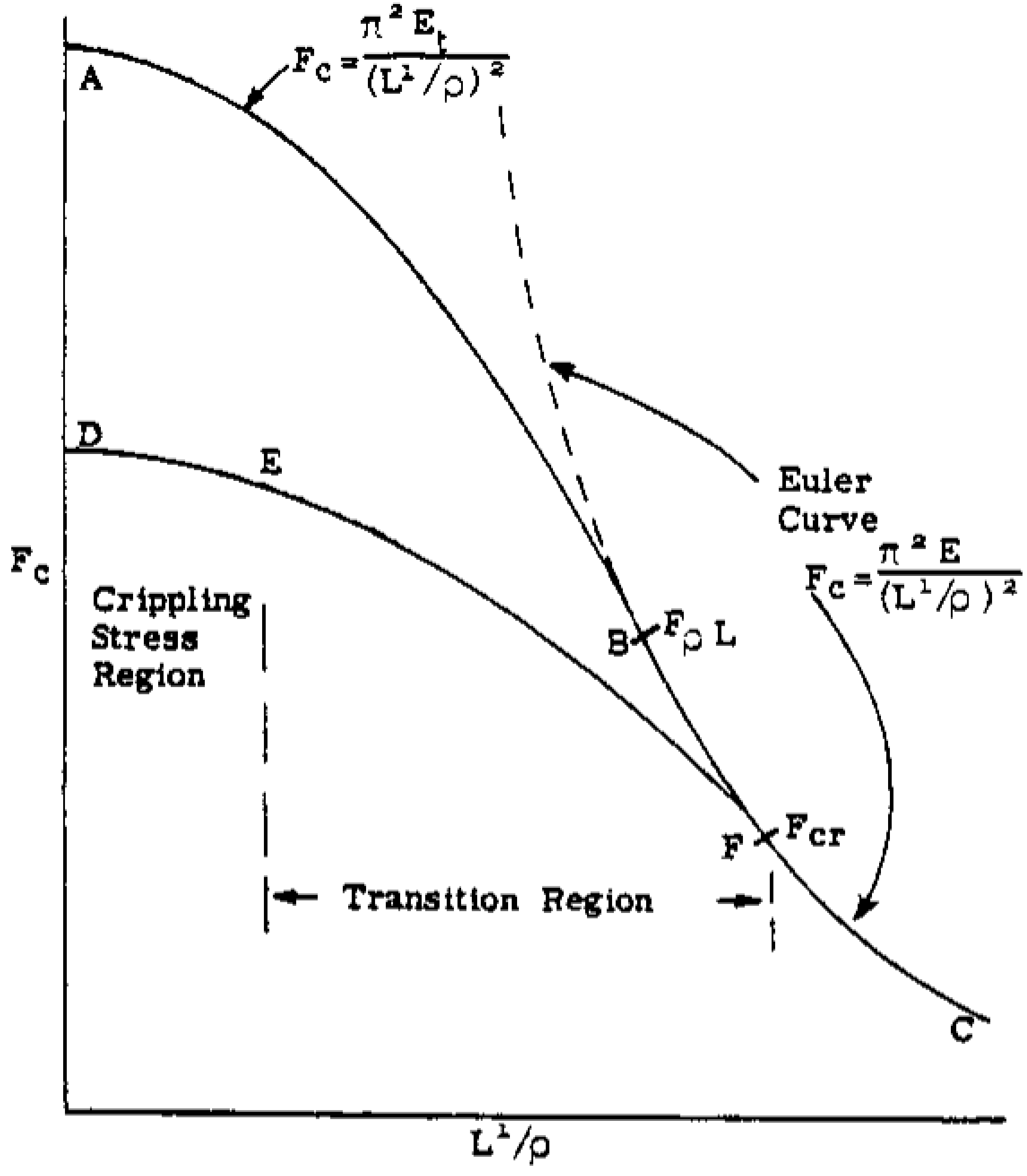


Fig. C7.32

Column Failing Strength in transition region

METHOD 1. JOHNSON-FULER EQUATION.

Possibly the first method used in calculating the column failing stress F_c in the transition range EF in Fig. C7.32 was the well known Johnson-Euler equation which involves the crippling stress. The equation is,

$$F_c = F_{cs} - \frac{F_{cs}^2}{4\pi^2 E}(L'/\rho)^2 \quad \text{-------} \quad (C7.31)$$

where, F_c = column failing stress (psi)
F_{cs} = crippling stress, assumed to occur at $L'/\rho = 0$, where $L' = L/\sqrt{c}$.

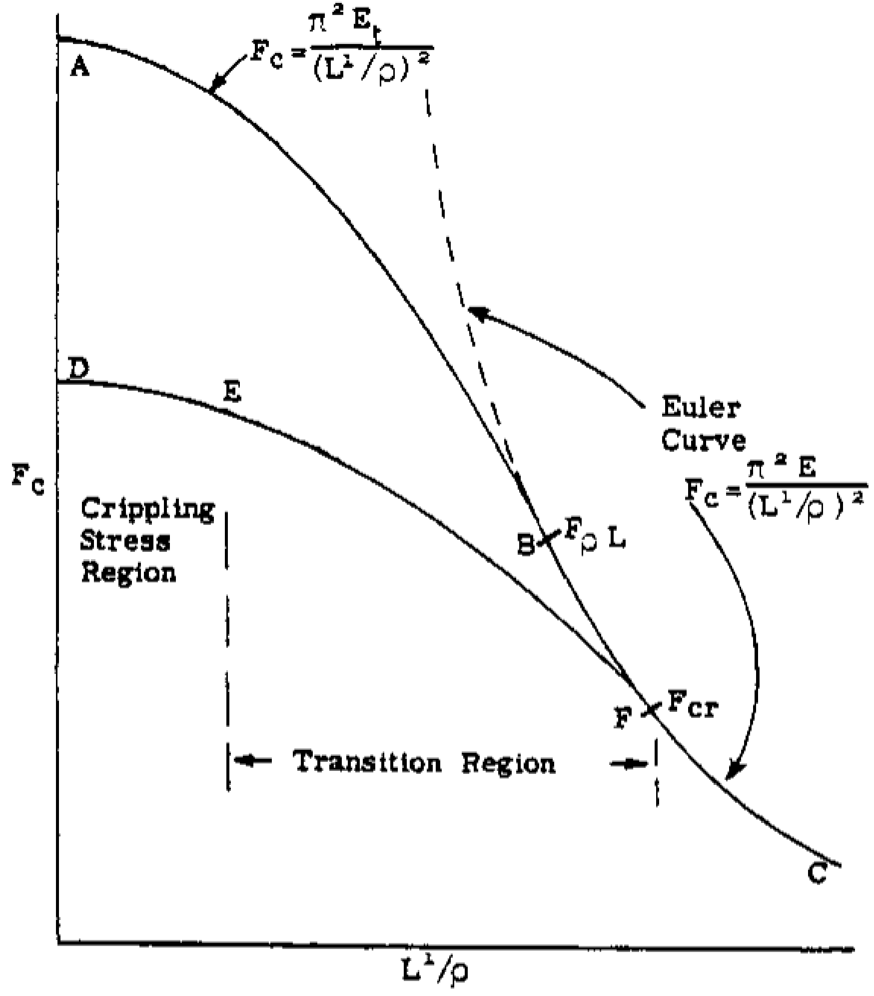


Fig. C7.32

Column Failing Strength in transition region

Method 2: Graphical method

(1) Locate point (O) on the basic column curve by drawing a horizontal line through an F_c value equal to F_{cy}, the yield stress of the particular material being used.

(2) Draw a horizontal line starting at point D. Point D is at an F_c value equal to the (F_{cs}) crippling stress for the column section being considered. Point (E) on this line is determined by projecting vertically downward from point (O).

(3) Locate point (F) at a value of $F_c = .9\ F_{cr}$ where F_{cr} is the buckling stress for the cross-section. Draw a horizontal line through point (F) to intersect the column curve at point (G).

(4) Connect points E and G with a straight line. The line EG then represents the column failing stress F_c for values of L'/ρ between points E and G.

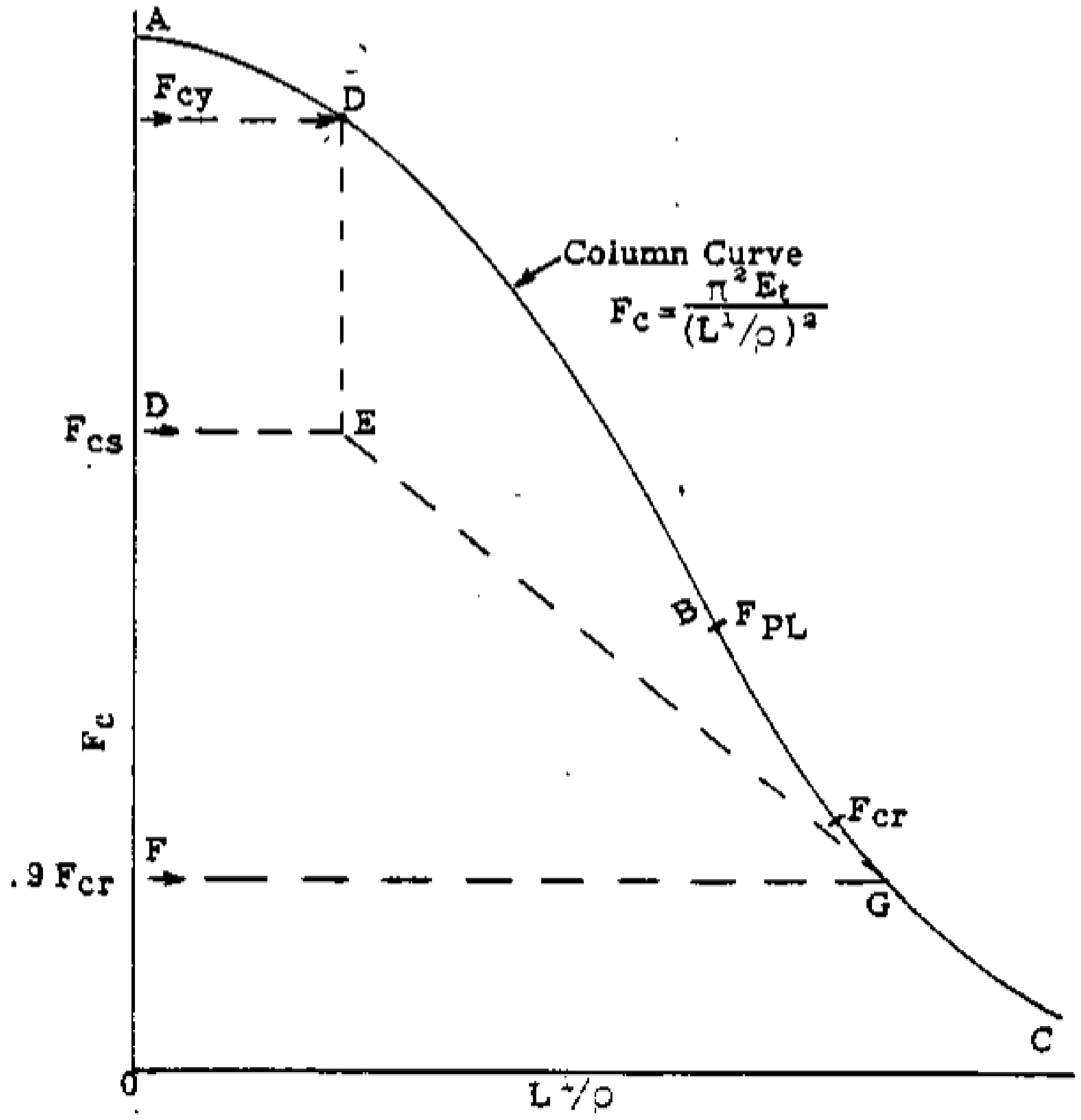


Fig. C7.34

Column Failing Strength in transition region

METHOD 3.

Another method that is widely used also uses a parabolic curve to represent the column strength in the transition range (see Ref. 12)

The parabolic approximation has the following form:-

$$\frac{F_c}{F_{cs}} = 1 - \left(1 - \frac{F_{cr}}{F_{cs}}\right)\left(\frac{F_{cr}}{F_E}\right) \quad \text{- - - - - - - - (C7.32)}$$

where

F_c is the column failing stress.

F_{cs} is the crippling stress.

F_{cr} is buckling stress for the column cross-section.

F_E is the Euler column stress for the particular column being considered as found from equation $F_E = \pi^2 E/(L'/\rho)^2$

The equation applies for $F_c > F_{cr}$. For cases where $F_{cr} > F_{\rho L}$ where $F_{\rho L}$ is the proportional limit stress for the material use $F_{\rho L}$ instead of F_{cr} in equation C7.32.

Column Failing Strength in transition region

$$F_c = F_{cs} - \frac{F_{cs}{}^2}{4\pi^2 E}\frac{L'^2}{\rho} \qquad L' = \frac{L}{\sqrt{C}}$$

In our calculations we will use *method 1*, BUT

- The column failure load does depend on the system (section and effective skin) radius of gyration ρ
- The radius of gyration depends on the effective width.
- Which in turn depends on the column failure stress.

$$\rho = \rho_0 \sqrt{\frac{1 + \frac{wt}{A_0}[1 + \frac{s}{\rho_0}^2]}{[1 + \frac{wt}{A_0}]^2}}$$

$$W = 1.9t\sqrt{\frac{E}{F_c}}$$

where, ρ_0 = radius of gyration of stiffener alone.

ρ = radius of gyration of sheet and stiffener.

S = distance from centerline of sheet to neutral axis of stiffener.

t = sheet thickness. w = sheet effective width.

Thus , the calculation if the column failing strength should be done by means of an iterative solution.

Column Failing Strength in transition region

Column Failing Stress

Iterative Method

1. Get $$W = 1.9t\sqrt{\frac{E}{F_{cs}}} \qquad \rho = \sqrt{\frac{I}{A}}$$

2. Get ρ where $$\rho = \rho_0\sqrt{\frac{1 + \frac{wt}{A_0}[1 + \frac{s}{\rho_0}^2]}{[1 + \frac{wt}{A_0}]^2}}$$

3. Get Fc from Johnson-Euler Equation:

$$F_c = F_{cs} - \frac{{F_{cs}}^2}{4\pi^2 E}\frac{L'^2}{\rho} \qquad L' = \frac{L}{\sqrt{C}}$$

4. Calculate W $$W = 1.9t\sqrt{\frac{E}{F_c}}$$

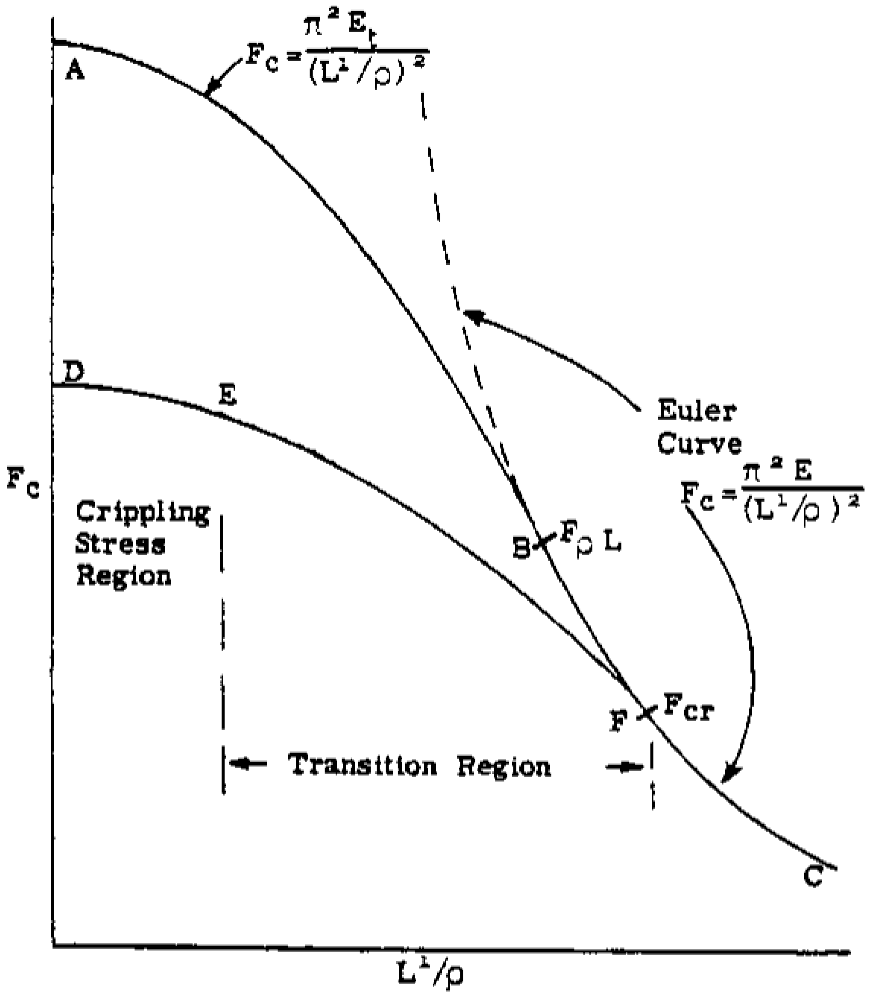
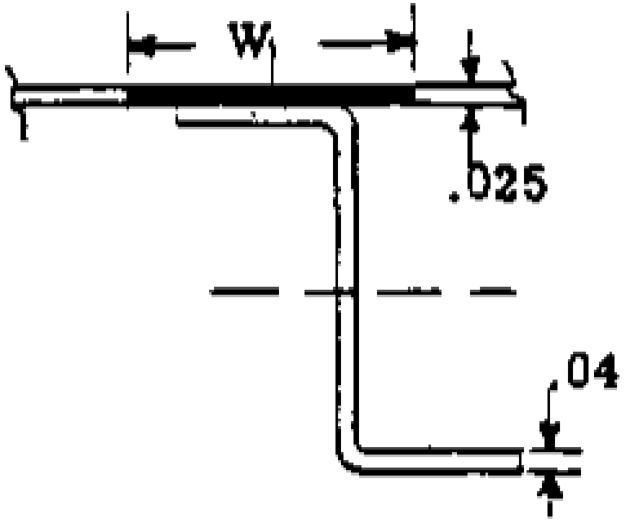


Fig. C7.32

Column Failing Strength in transition region - Examples

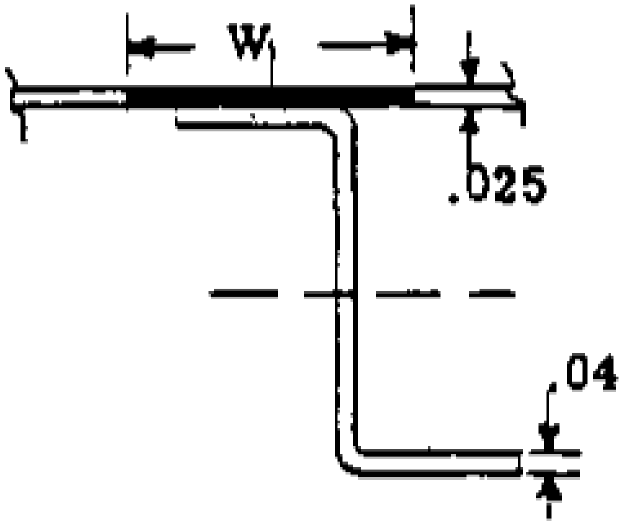
Example Problems.

For an example problem, we will assume that the Z stiffener in Problem 2 is one of several stiffeners riveted to a sheet of .025 thickness and of the same material as the stiffener.



Column Failing Strength in transition region - Examples

The rivet or spot weld spacing is made such as to prevent inter-rivet buckling. Thus the column area will be as shown in Fig. c, namely, the stiffener area plus the area of the sheet for the effective width w. Since the effective sheet width w is a function of the stiffener stress and since the stiffener



stress is a function of the radius of gyration, the design procedure is of the trial and error category.

Column Failing Strength in transition region - Examples

First Trial.

For simplicity in the exam, we will give you the column stand-alone failing strength.

Assume the effective sheet width is based on column strength of Z stiffener acting alone. The average column failing stress by the 3 methods in the Problem 2 solution was (25,860 + 25,000 + 26,300)/3 = 25,700 psi.

The effective width equation to be used is,

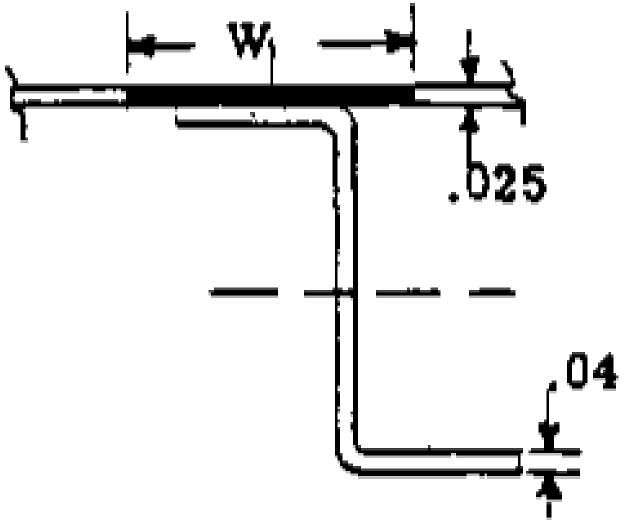
$$w = 1.9\ t\ \sqrt{E/F_{ST}}$$

$$= 1.9 \times .025 \times \sqrt{10{,}500{,}000/25{,}700} = .965 \text{ in.}$$

Effective sheet area = 0.965 x .025 = .0242

Area of stiffener = A_O = .117

Total area = .1412



Column Failing Strength in transition region - Examples

Adding the effective sheet to the stiffener will change the radius of gyration. Mr. R. J. White (Ref. 13) has developed equation C7.33 which gives the variation in the radius of gyration in terms of known variables for any stiffener cross-section. Since the failing stress of a column is directly proportional to the radius of gyration squared, equation C7.33 can be equated to the ratio of the column stresses.

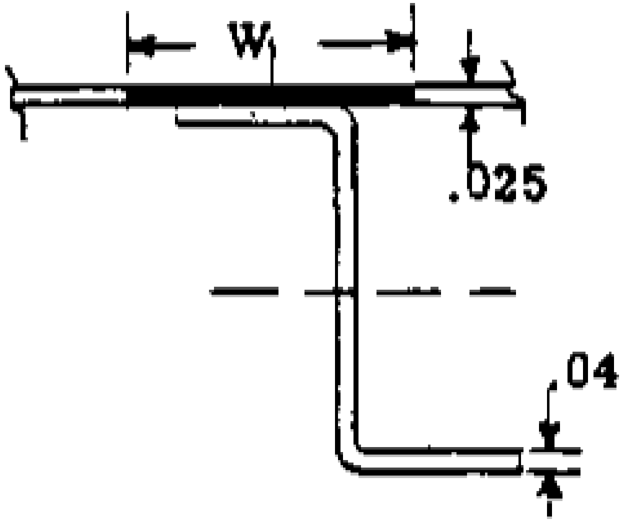
$$\left(\frac{\rho}{\rho_o}\right)^2 = 1 + \left[1 + \left(\frac{S}{\rho_o}\right)^2\right]\frac{wt}{A_o} = \frac{F_c}{F_{ST}} \quad - - - - - \quad (C7.33)$$

where, ρ_o = radius of gyration of stiffener alone.

ρ = radius of gyration of sheet and stiffener.

S = distance from centerline of sheet to neutral axis of stiffener.

t = sheet thickness. w = sheet effective width.



$$\rho = \rho_0 \sqrt{\frac{1 + \frac{wt}{A_0}\left[1 + \frac{s}{\rho_0}^2\right]}{\left[1 + \frac{wt}{A_0}\right]^2}}$$

Column Failing Strength in transition region - Examples

Equation C7.33 has been put in curve form as shown in Fig. C7.36, which will now be used to compute the radius of gyration ρ_o for the stiffener plus effective sheet.

$S = .75 + .0125 = .7625$

ρ_o for stiffener alone = .535

$S/\rho_o = .7625/.535 = 1.425$

$w = .965$

$wt/A_o = .965 \times .025/.117 = \underline{.2063}$

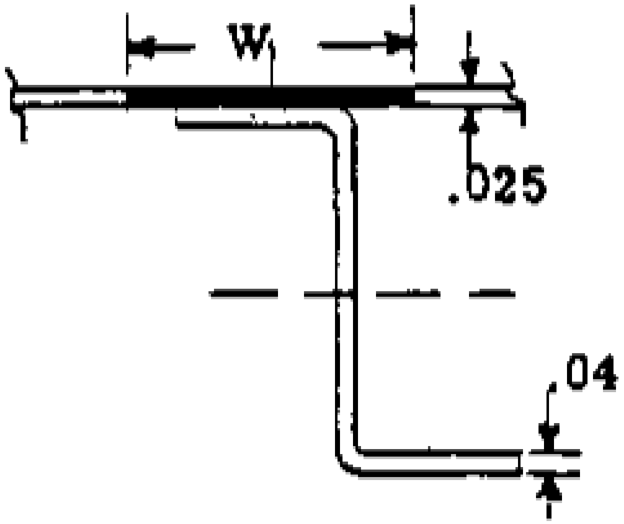
From Fig. C7.36 for the above values of S/ρ_o and wt/A_o, we obtain

$(\rho/\rho_o)^2 = .775$, whence $\rho^2 = .535^2 \times .775$, which gives $\rho = .471$ in.

Then $L'/\rho = 24.5/.471 = 52.$

Use Method 1 for column strength:-

$$F_c = 30{,}600 - \frac{30{,}600^2}{4\,\pi^2 \times 10{,}500{,}000}(52)^2 = \underline{24{,}500}\text{ psi}$$

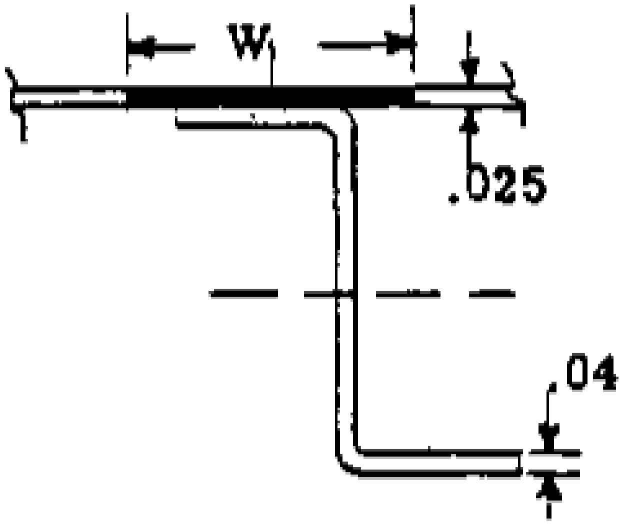


Column Failing Strength in transition region - Examples

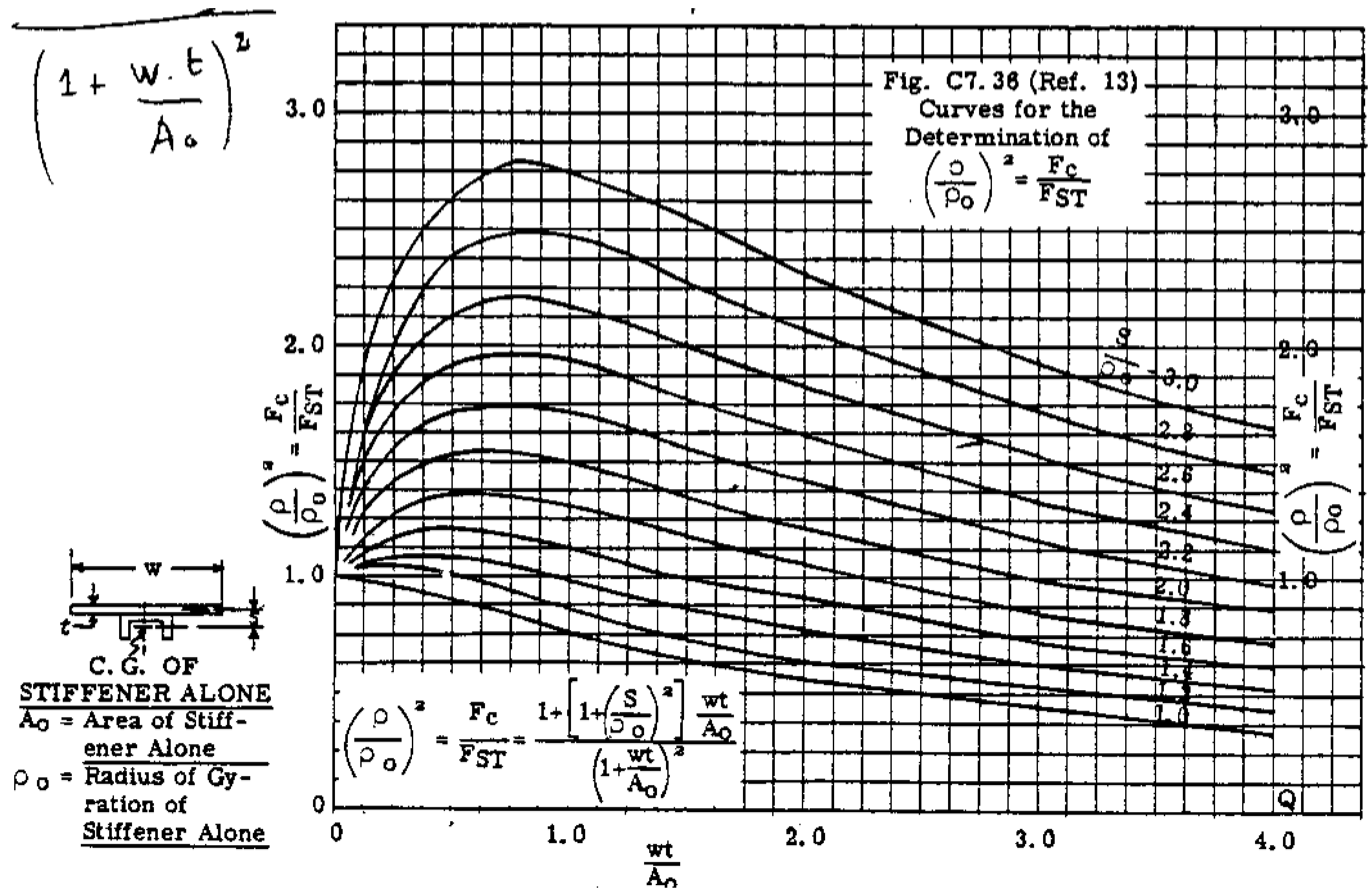
The revised effective width based on this stiffener stress is,

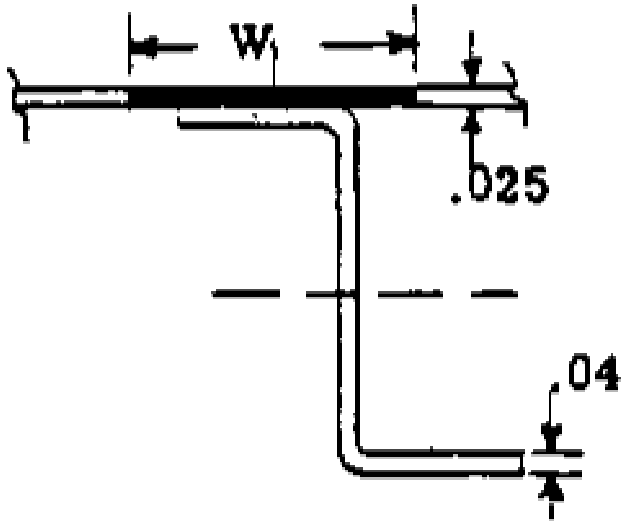
$$w = 1.9 \times .025 \sqrt{10{,}500{,}000/24{,}500} = .98 \text{ in.}$$

This value is only .02 inch more than the value of .96 previously, thus the effect on the radius of gyration ρ will be negligible. The column failing stress for the sheet stiffener combination is therefore 24,500 psi, and the compressive failing load would be 24,500 x .1412 = 3460 lbs.



Column Failing Strength in transition region - Examples





Summary

- *Crippling stress* is the failing stress of the corner stringers in compression side. (*Fcs)*
- *Column failing stress* is the failing stress of middle stringers in compression side. *(Fc)*
- *Ultimate stress* is the failing stress of the stringers in the tension side.