System Dynamics
Block diagram and feedback control

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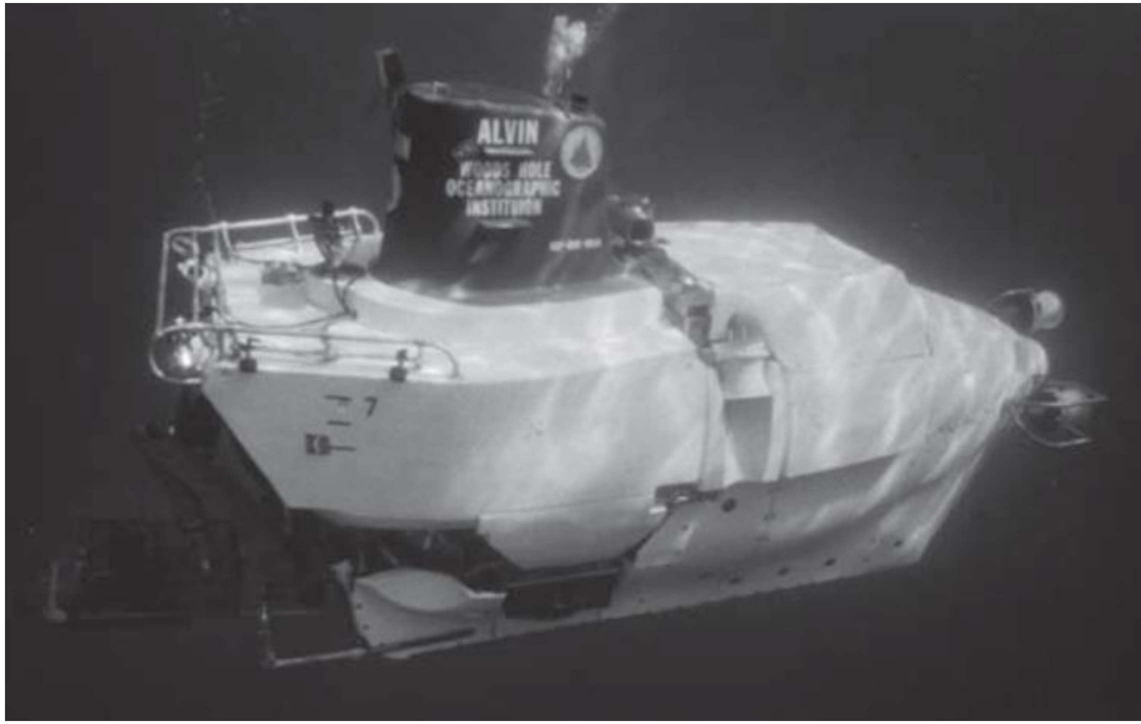
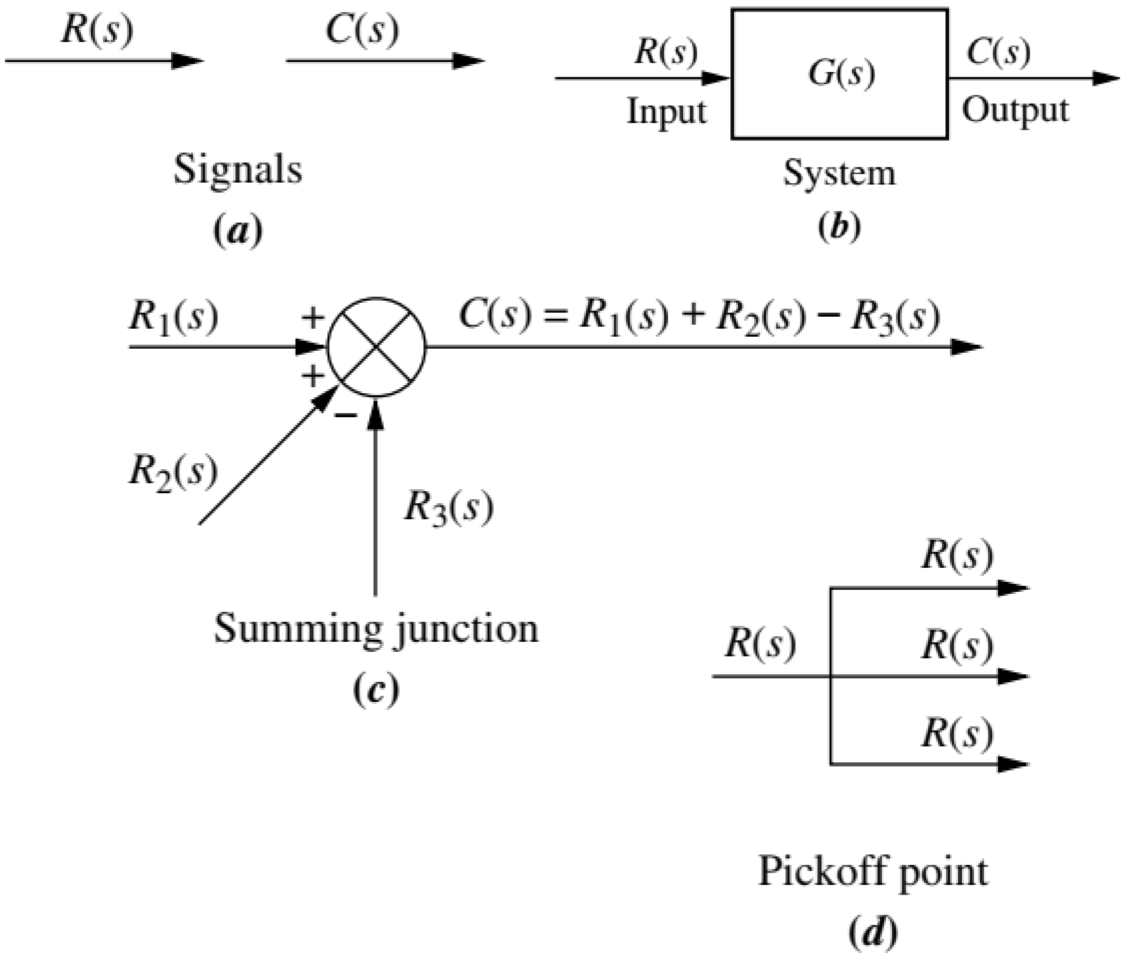
Block diagrams – Closed loop systems

- Multiple subsystems are represented in two ways: as block diagrams and as signal-flow graphs.
- We will concentrate on block diagram, since block diagrams are usually used for frequency-domain analysis and design.
- We will develop techniques to reduce each representation to a single transfer function.
- Block diagram algebra will be used to reduce block diagrams.



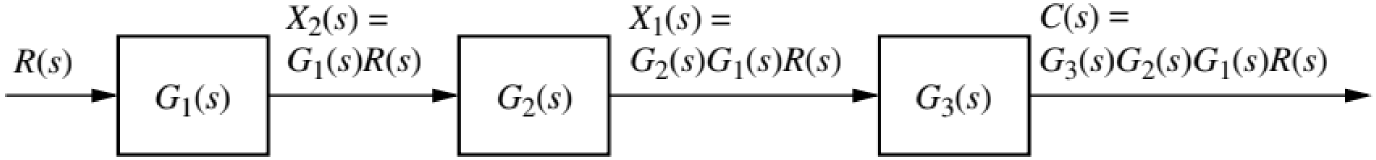
Block diagrams

- A subsystem is represented as a block with an input, an output, and a transfer function.
- Many systems are composed of multiple subsystems.
- When multiple subsystems are interconnected, a few more schematic elements must be added to the block diagram.
- These new elements are summing junctions and pickoff points.

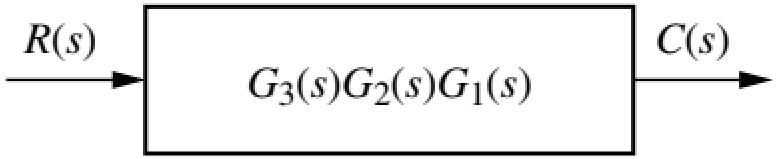


Techniques for reducing complicated systems to one Block diagram – *Cascade form*

For cascaded subsystems, each signal is derived from the product of the input times the transfer function.



(*a*)

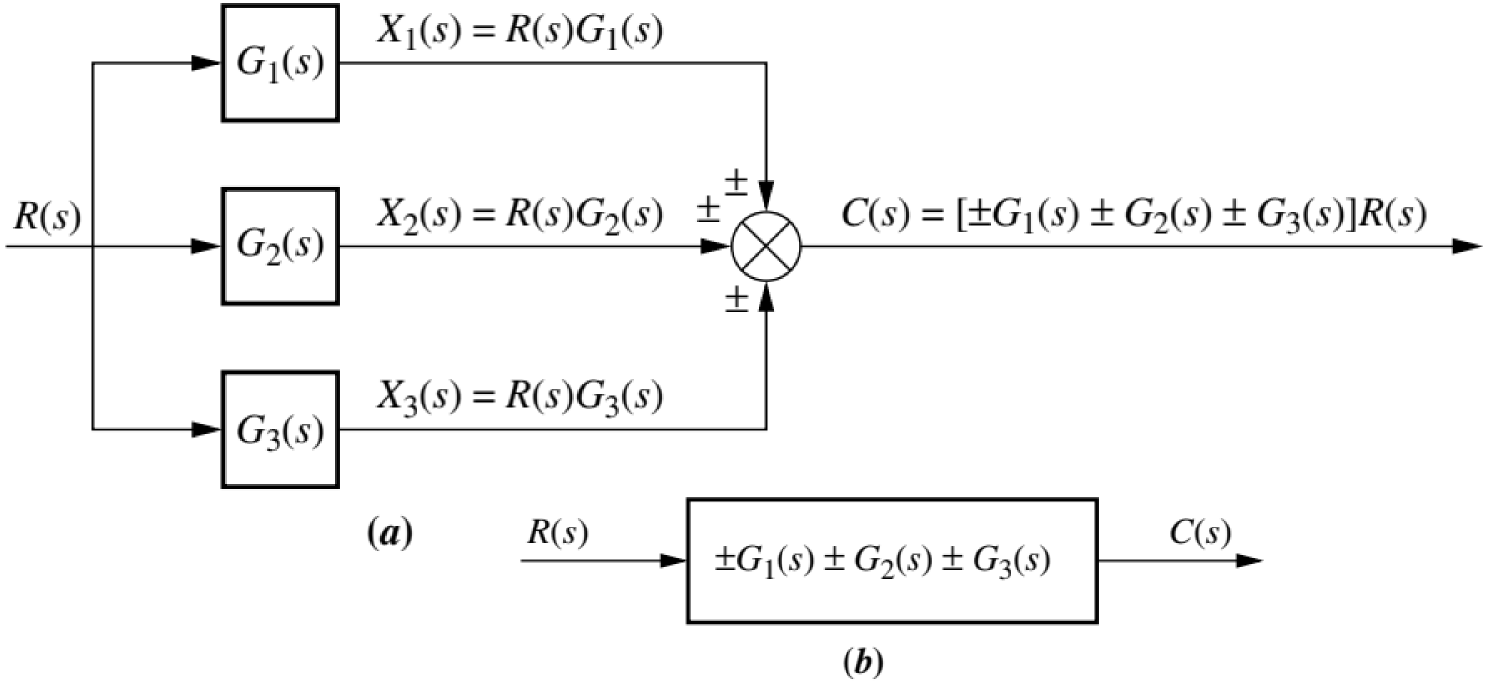


(*b*)

$$G_e(s) = G_3(s)G_2(s)G_1(s)$$

Techniques for reducing complicated systems to one Block diagram – *Parallel form*

Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems.

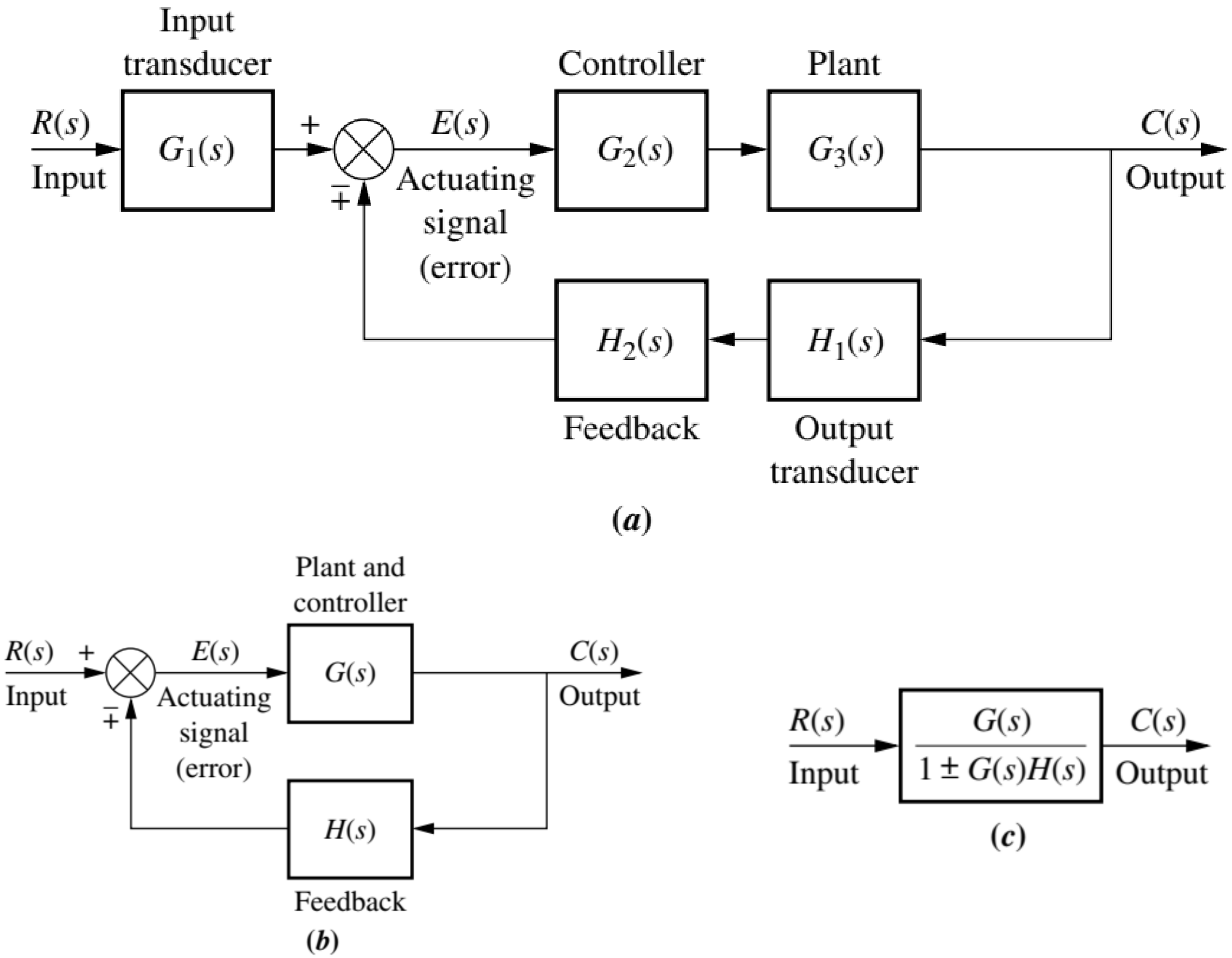


$$G_e(s) = \pm G_1(s) \pm G_2(s) \pm G_3(s)$$

Techniques for reducing complicated systems to one Block diagram – *Feedback form*

- The feedback system forms the basis for our study of control systems engineering.
- It represents a closed loop system.

$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$



Techniques for reducing complicated systems to one Block diagram – *Feedback form*

$$C(s) = E(s)\,G(s) \quad (1)$$

$$\& \quad E(s) = R(s) \mp C(s)H(s) \quad (2)$$

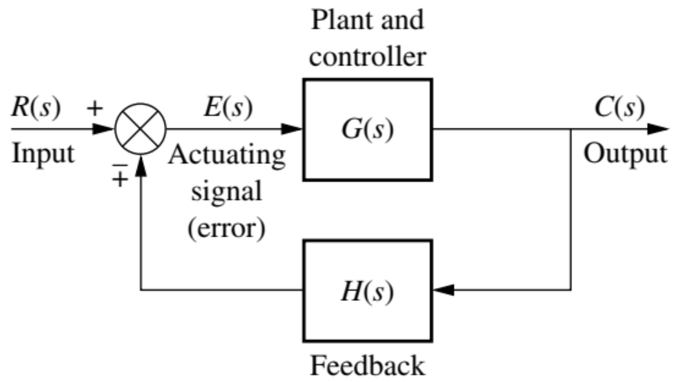
From (1) & (2)

$$\Rightarrow C(s) = [R(s) \mp C(s)H(s)]\,G(s)$$

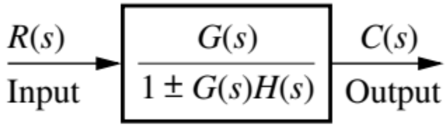
$$\Rightarrow C(s)[1 \pm H(s)G(s)] = R(s)G(s)$$

Finally

$$\frac{C(s)}{R(s)} = G_e(s) = \frac{G(s)}{1 \pm H(s)G(s)}$$



(b)



(c)

$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$

Techniques for reducing complicated systems to one Block diagram – *Moving Blocks*

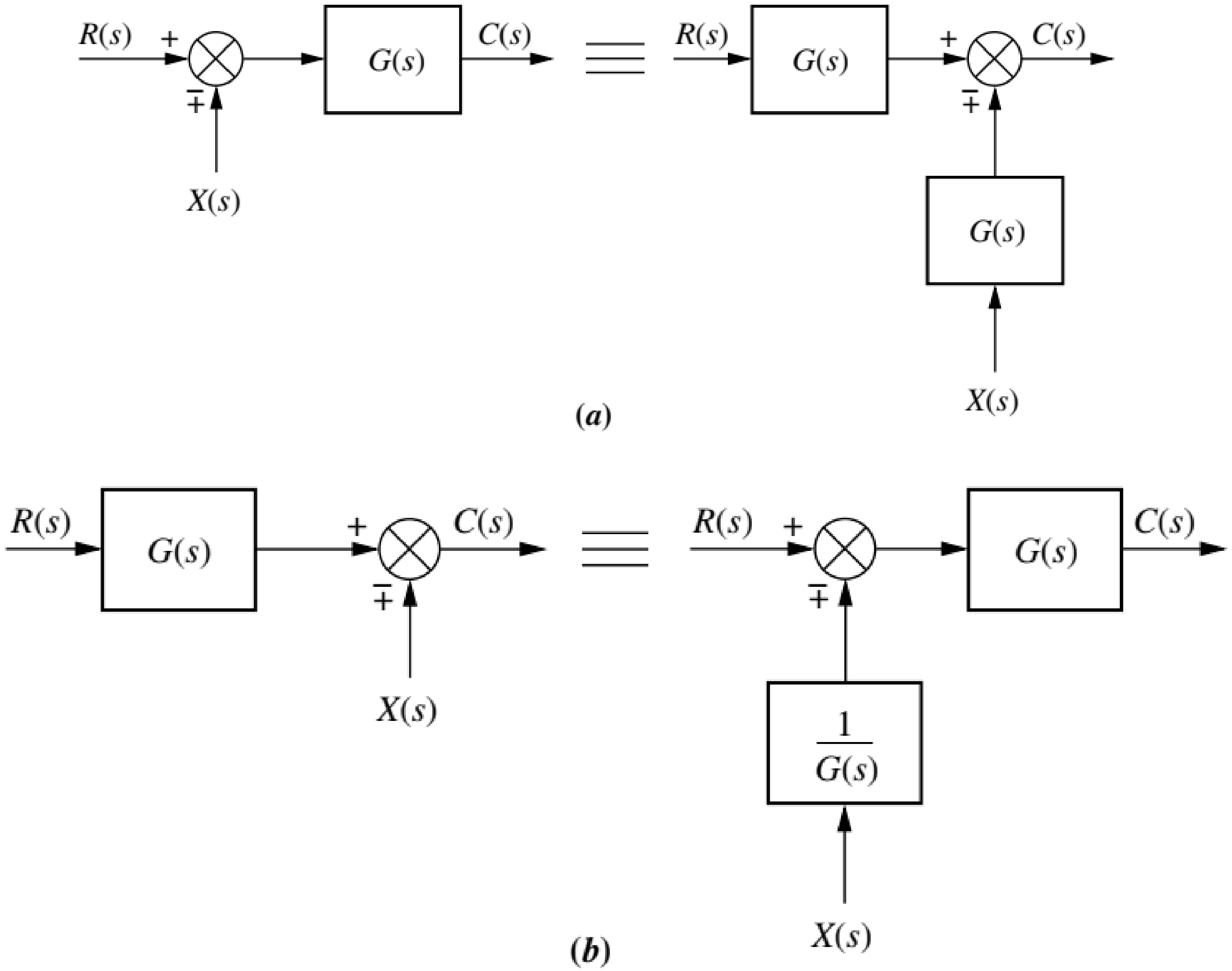
- The familiar forms (cascade, parallel, and feedback) are not always apparent in a block diagram.

- In this section, we discuss basic block moves that can be made in order to establish familiar forms when they almost exist.

- It will explain how to move blocks left and right past summing junctions and pickoff points.

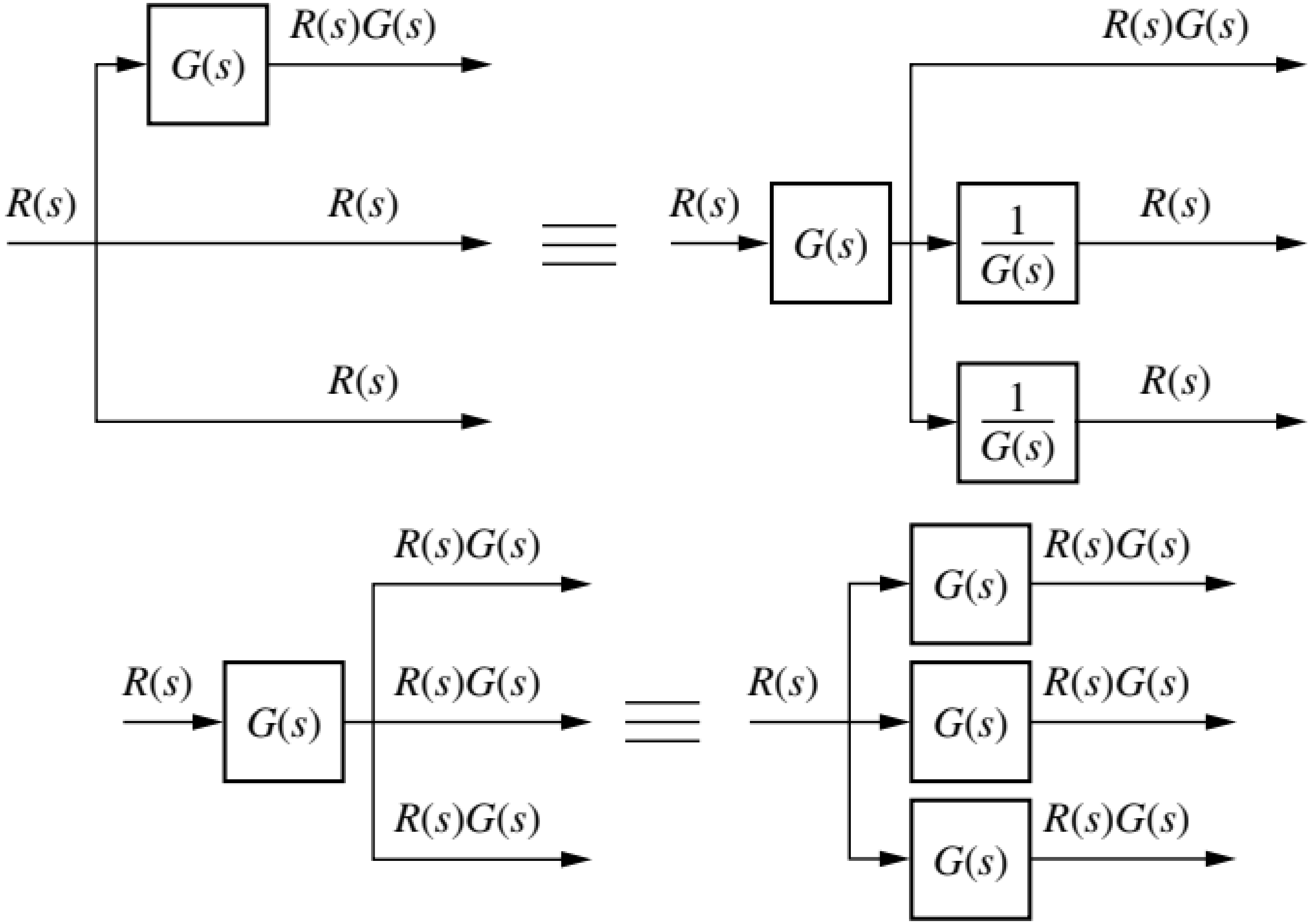
Techniques for reducing complicated systems to one Block diagram – *Moving Blocks*

Equivalent block diagrams formed when transfer functions are moved left or right *past a summing junction.*



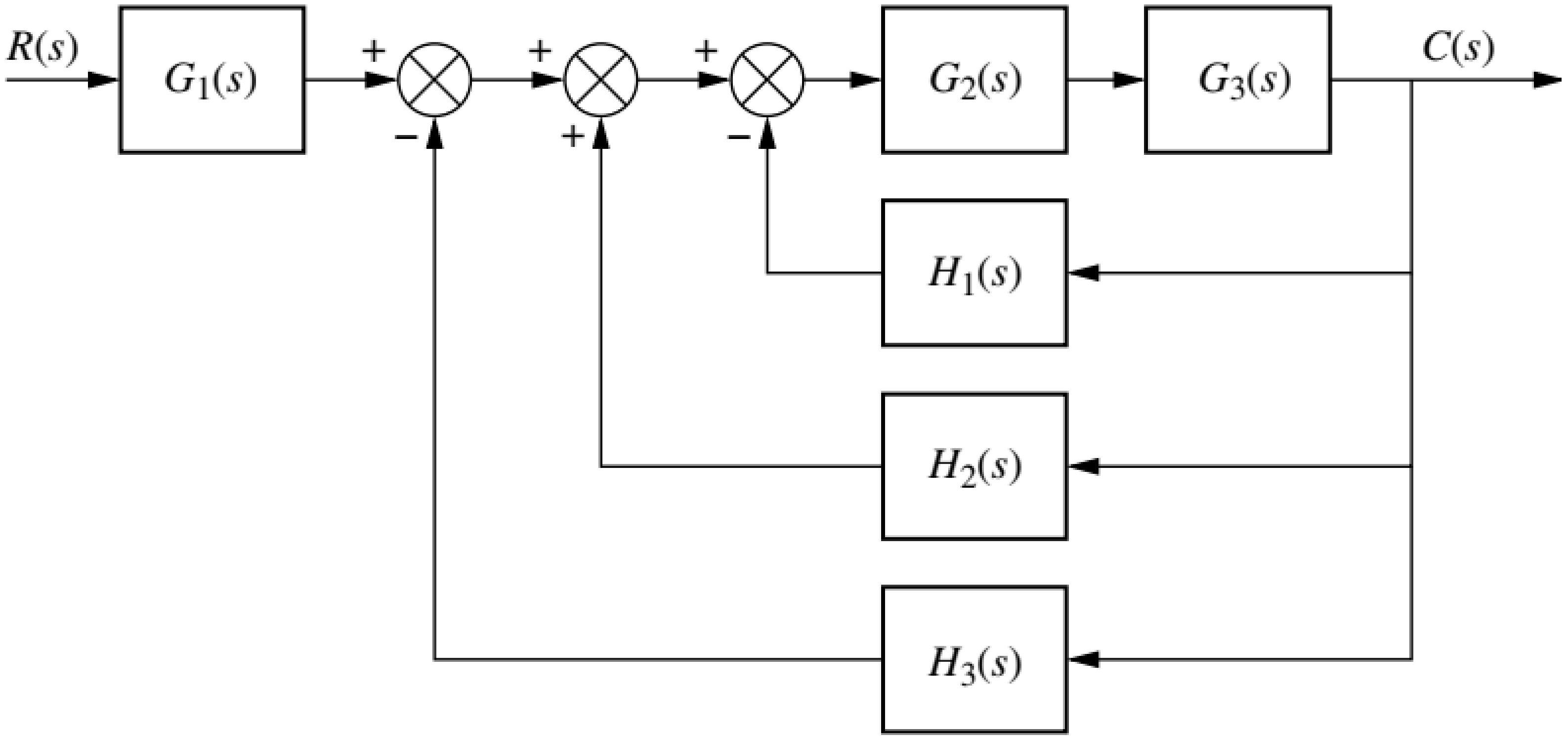
Techniques for reducing complicated systems to one Block diagram – *Moving Blocks*

Equivalent block diagrams formed when transfer functions are moved left or right *past a pickoff point*.



Techniques for reducing complicated systems to one Block diagram – Example

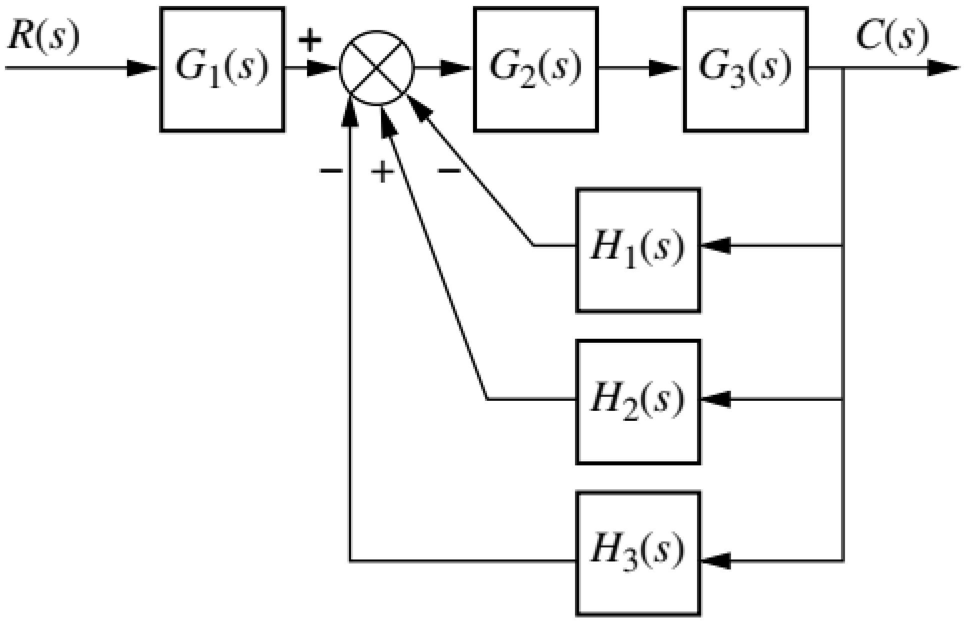
Reduce the block diagram shown in the shown Figure to a single transfer function.

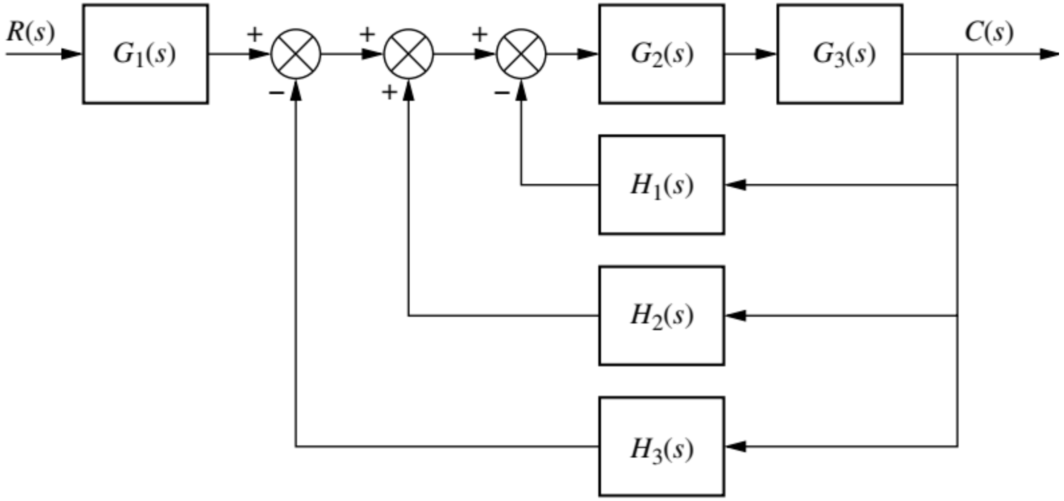


Techniques for reducing complicated systems to one Block diagram – Example

Solution:

Step 1:the three summing junctions can be collapsed into a single summing junction.





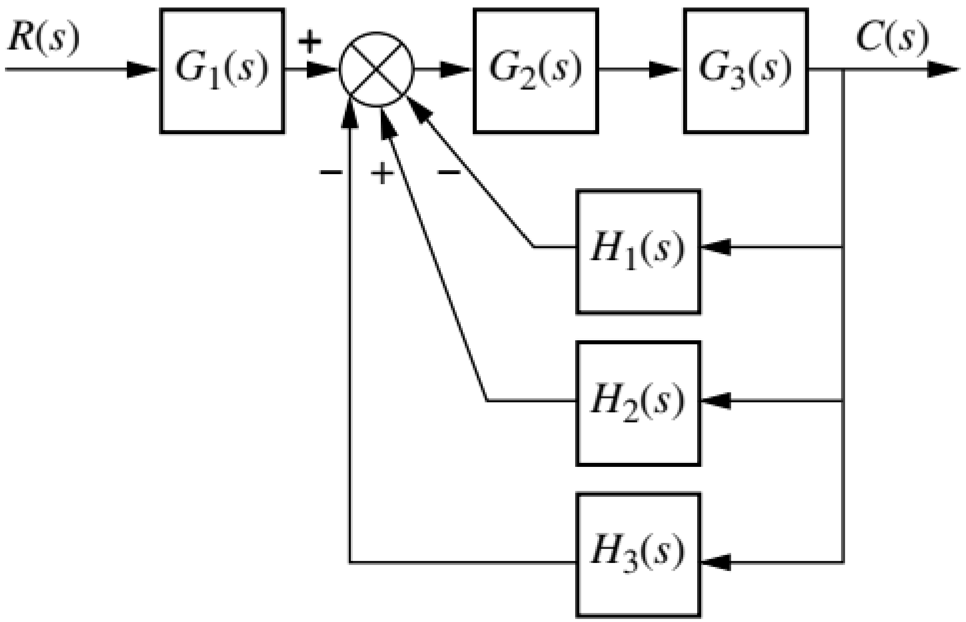
Techniques for reducing complicated systems to one Block diagram – Example

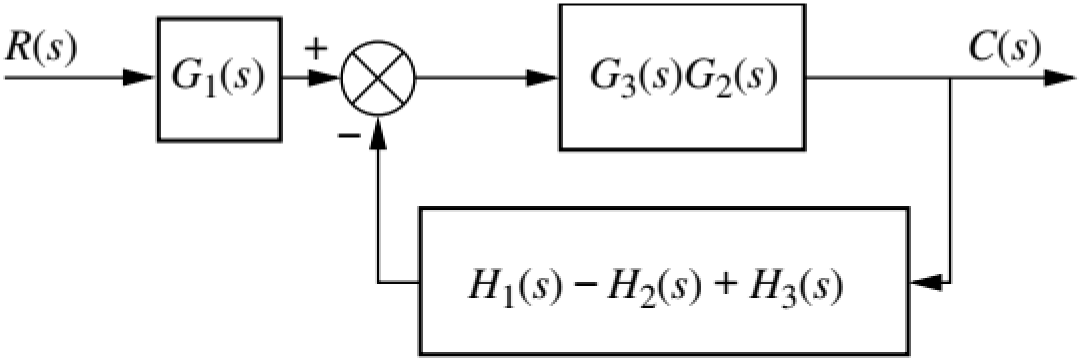
Solution:

Step 2: recognize that the three feedback functions, H1(s), H2(s), and H3(s),are connected in parallel.

They are fed from a common signal source, and their outputs are summed.

Step 3:Also recognize that G2(s) and G3(s) are connected in cascade.

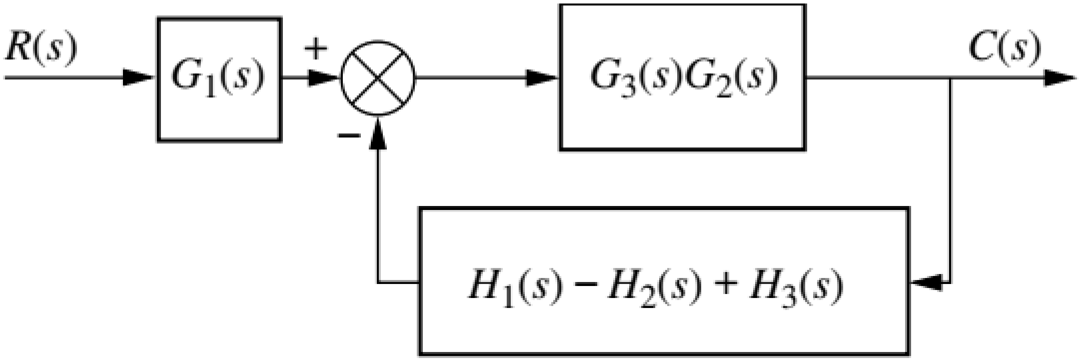


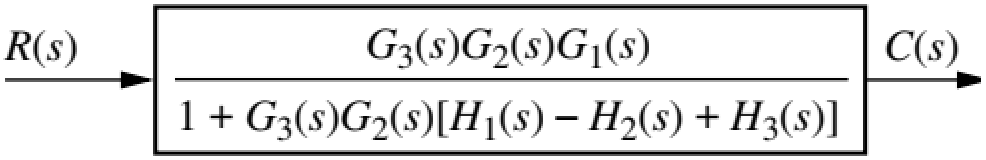


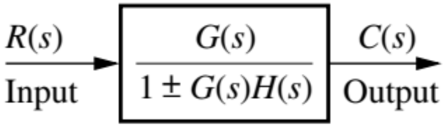
Techniques for reducing complicated systems to one Block diagram – Example

Solution:

Thus, the equivalent transfer function is the product



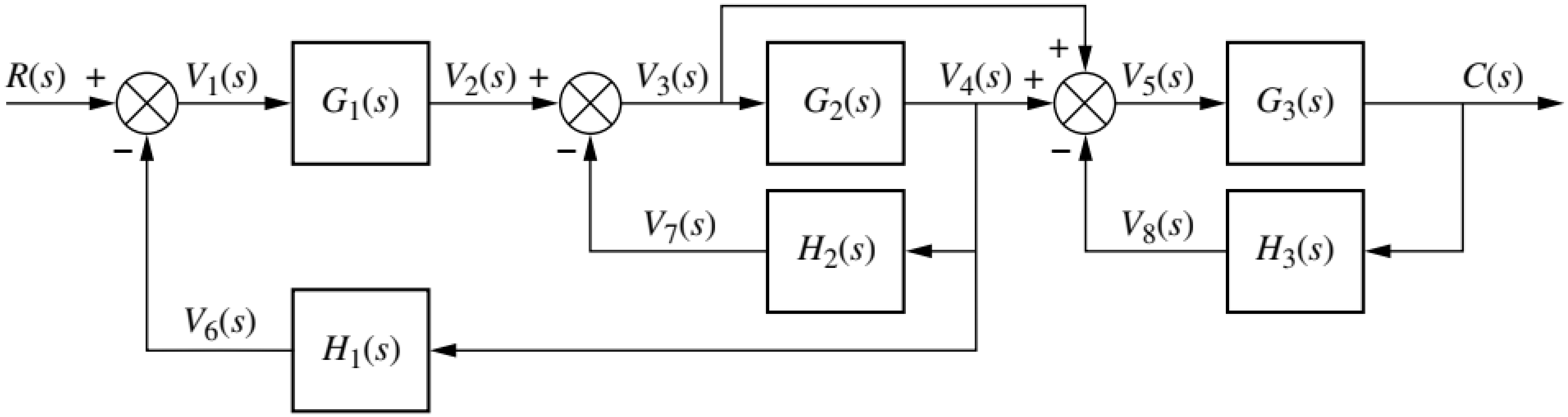




(c)

Techniques for reducing complicated systems to one Block diagram – Example

Reduce the system shown in Figure to a single transfer function

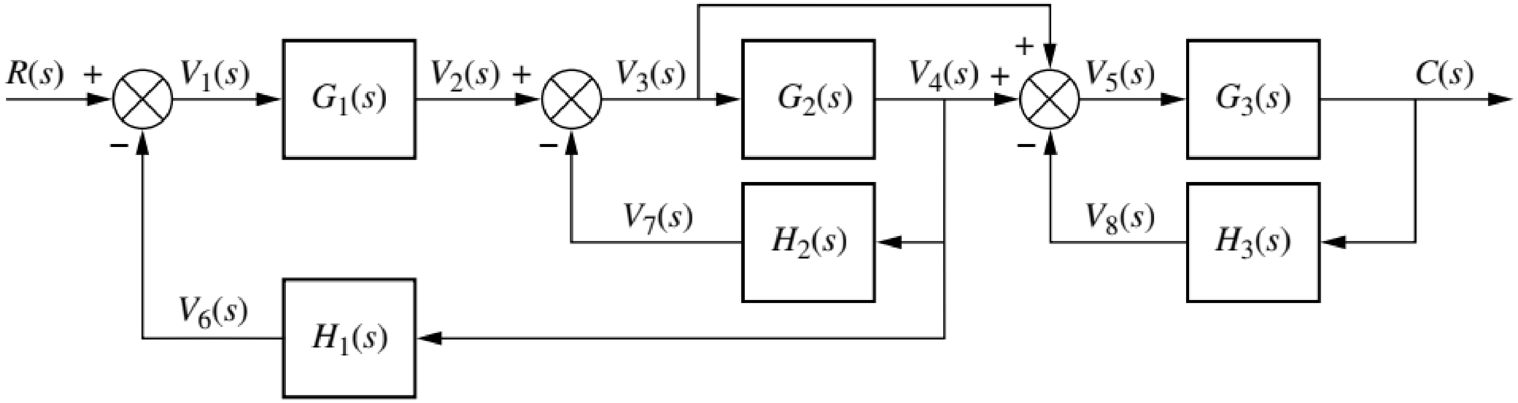


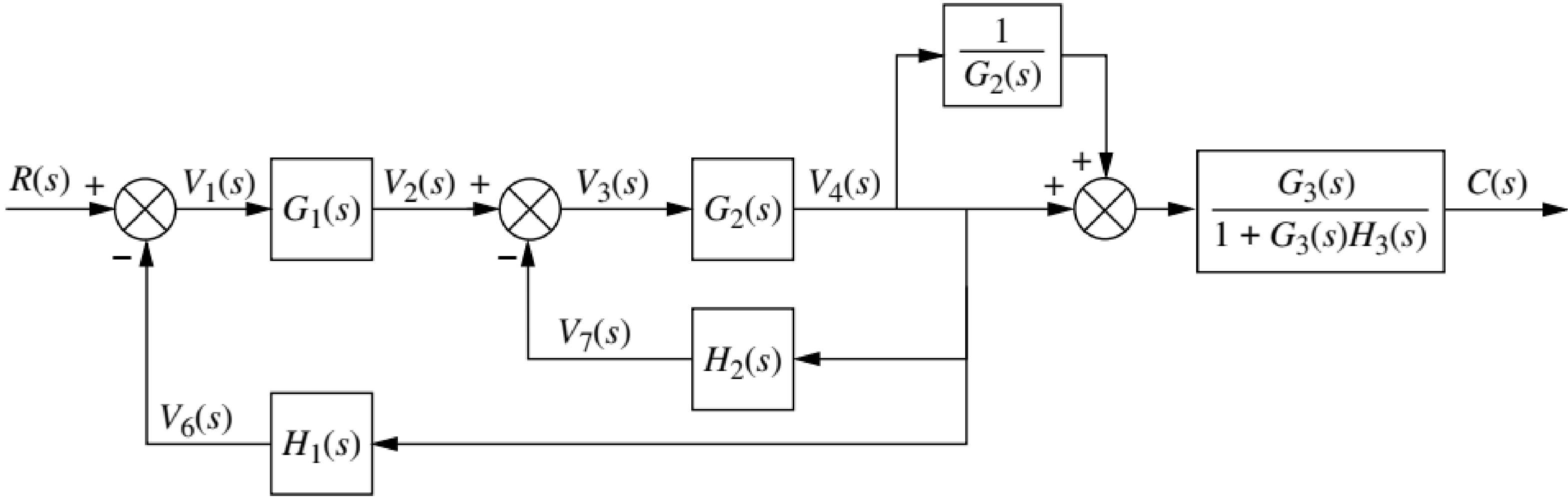
Techniques for reducing complicated systems to one Block diagram – Example

Solution:

Step 1: First, move G2(s) to the left past the pickoff point to create parallel subsystems,

and reduce the feedback system consisting of G3(s) and H3(s).

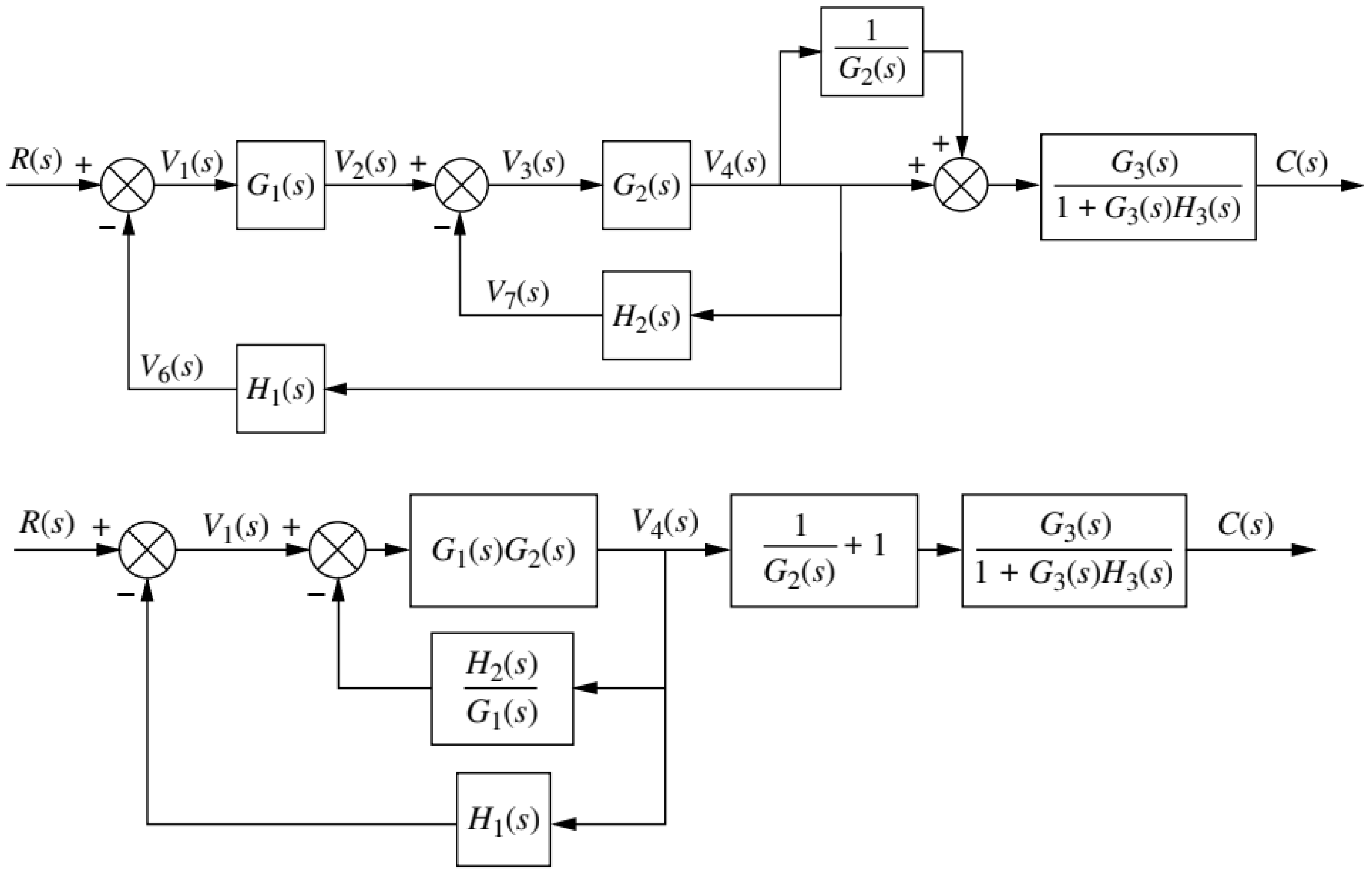




Techniques for reducing complicated systems to one Block diagram – Example

Solution:

Step 2: reduce the parallel pair consisting of 1/G2(s) and unity, and push G1(s) to the right past the summing junction, creating parallel subsystems in the feedback.

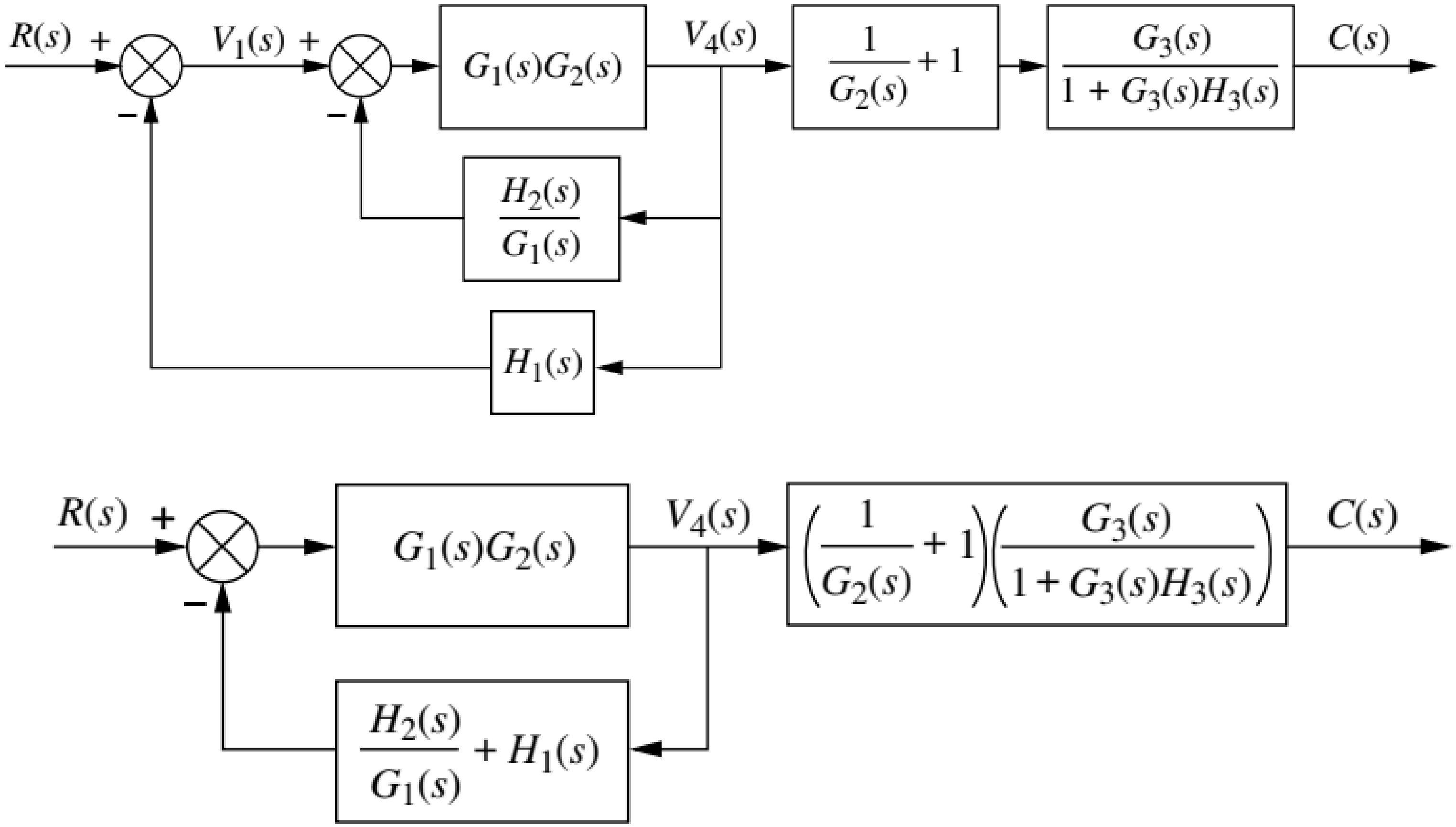


Techniques for reducing complicated systems to one Block diagram – Example

Solution:

Step 3: collapse the summing junctions,

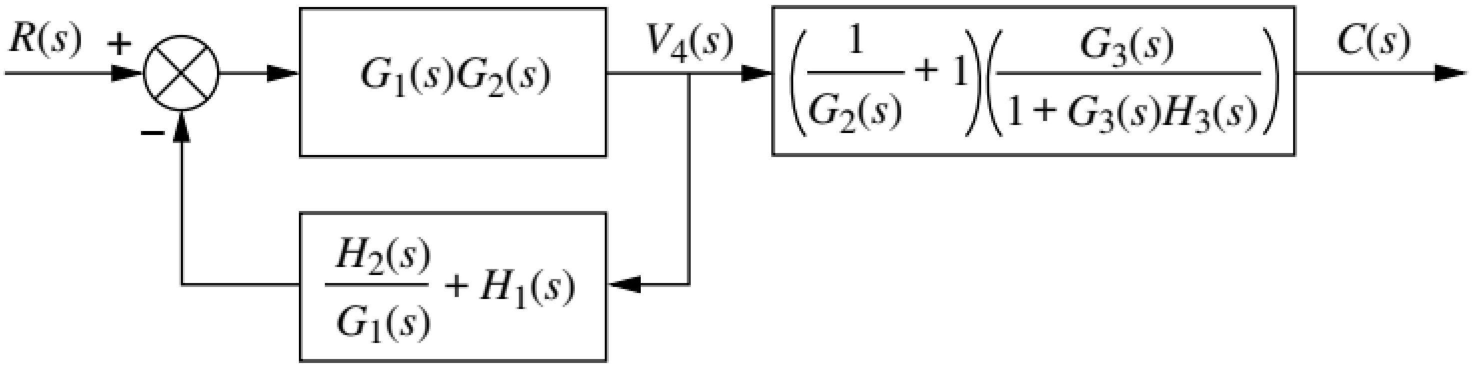
add the two feedback elements together, and combine the last two cascaded blocks.

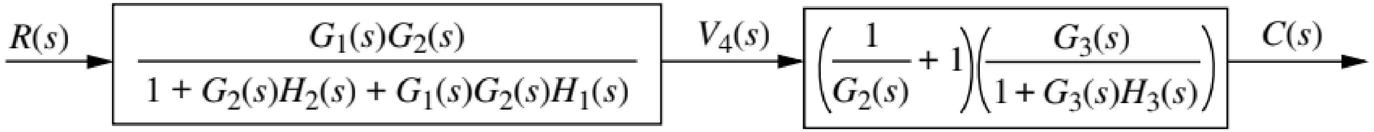


Techniques for reducing complicated systems to one Block diagram – Example

Solution:

Step 4: use the feedback formula.

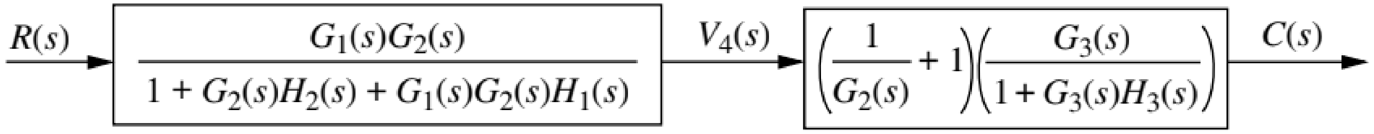


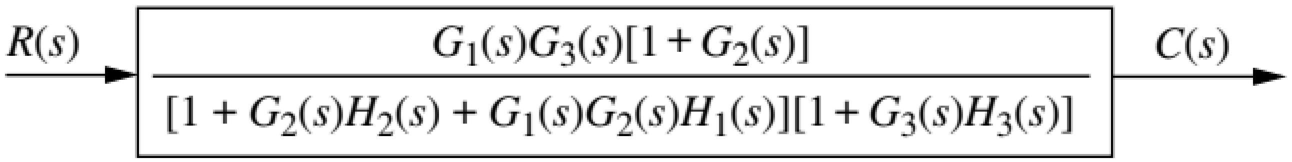


Techniques for reducing complicated systems to one Block diagram – Example

Solution:

Step 5: Finally, multiply the two cascaded blocks and obtain the final result



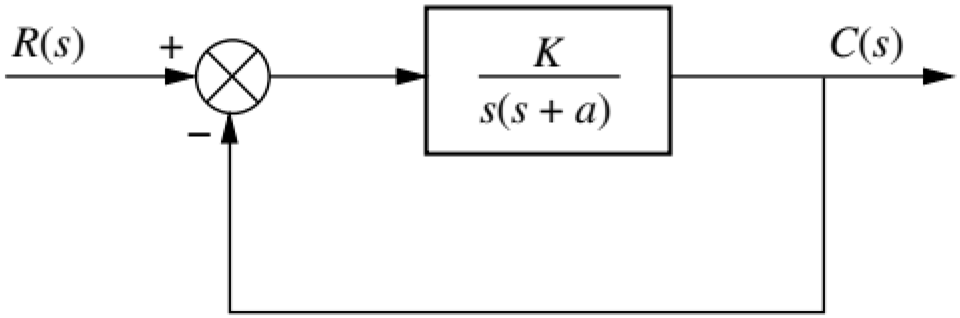


Feedback Control

- Consider the system shown in Figure, which can model a control system such as the antenna azimuth position control system.

- Where *K* models the amplifier gain, that is, the ratio of the output voltage to the input voltage.

- As *K* varies, the poles move through the three ranges of operation of a second-order system: overdamped, critically damped, and underdamped.



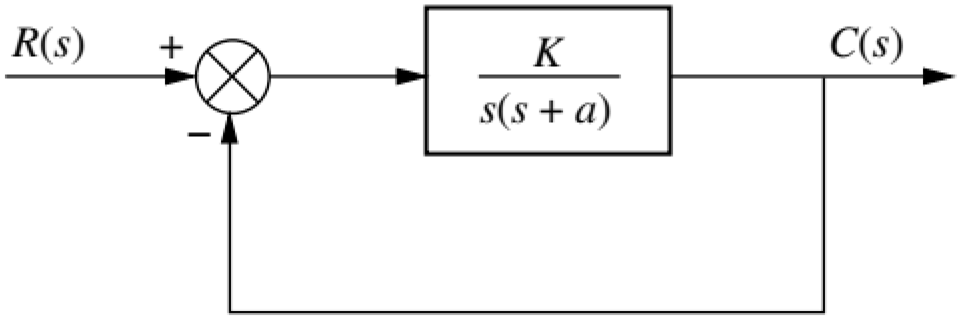
The closed loop T.F.

$$T(s) = \frac{K}{s^2 + as + K}$$

Feedback Control

For K between 0 and $\frac{a^2}{4}$, the poles of the system are real and are located at

$$s_{1,2} = -\frac{a}{2} \pm \frac{\sqrt{a^2 - 4K}}{2}$$

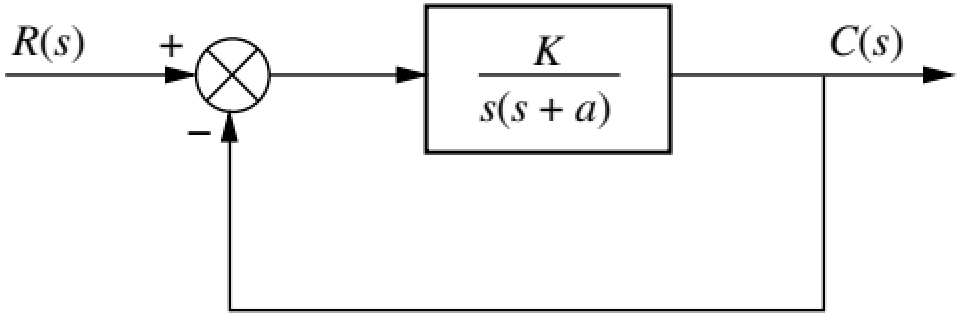


The closed loop T.F.

$$T(s) = \frac{K}{s^2 + as + K}$$

Feedback Control

- As K increases, the poles move along the real axis, and the system remains overdamped until $K = \frac{a^2}{4}$.
- At that gain, or amplification, both poles are real and equal, and the system is critically damped.



The closed loop T.F.

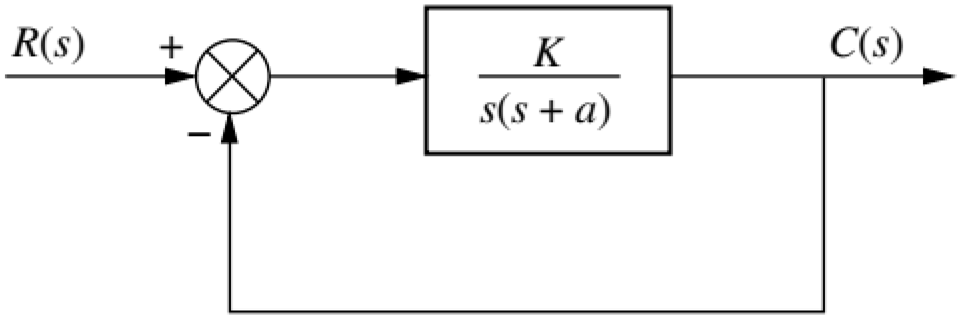
$$T(s) = \frac{K}{s^2 + as + K}$$

Feedback Control

For gains above $\frac{a^2}{4}$, the system is underdamped, with complex poles located at

$$s_{1,2} = -\frac{a}{2} \pm j\frac{\sqrt{4K - a^2}}{2}$$

as K increases, the real part remains constant and the imaginary part increases. Thus, the peak time decreases and the percent overshoot increases, while the settling time remains constant.

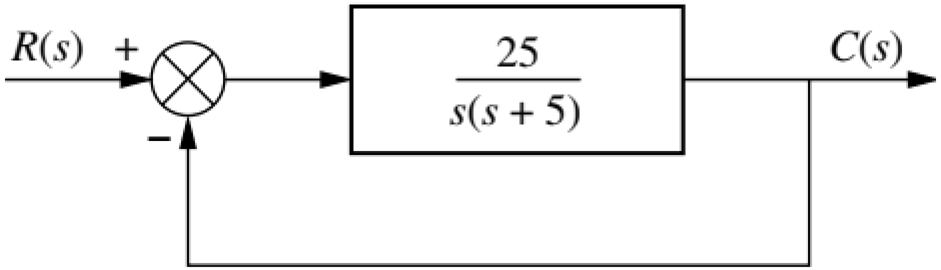


The closed loop T.F.

$$T(s) = \frac{K}{s^2 + as + K}$$

Feedback Control – Example

For the system shown in Figure, find the peak time, percent overshoot, and settling time



Solution:

The closed-loop transfer function takes the form,

$$T(s) = \frac{25}{s^2 + 5s + 25}$$

$$\omega_n = \sqrt{25} = 5$$

$$2\zeta\omega_n = 5$$

$$\zeta = 0.5$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Feedback Control – Example

For the system shown in Figure, find the peak time, percent overshoot, and settling time

Solution:

The closed-loop transfer function takes the form,

$$T(s) = \frac{25}{s^2 + 5s + 25}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.726 \text{ second}$$

$$\%OS = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100 = 16.303$$

$$T_s = \frac{4}{\zeta\omega_n} = 1.6 \text{ seconds}$$

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

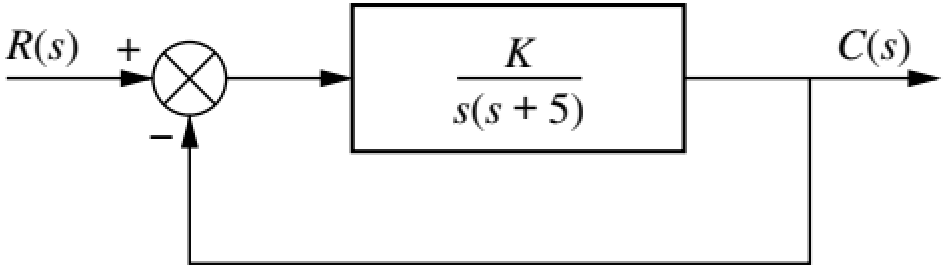
$$\%OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} \times 100$$

$$T_s = \frac{4}{\zeta\omega_n}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Feedback Control – System design Example

Design the value of gain K, for the feedback control system of the Figure, so that the system will respond with a 10% overshoot.



Solution:

The closed-loop transfer function takes the form,

$$T(s) = \frac{K}{s^2 + 5s + K}$$

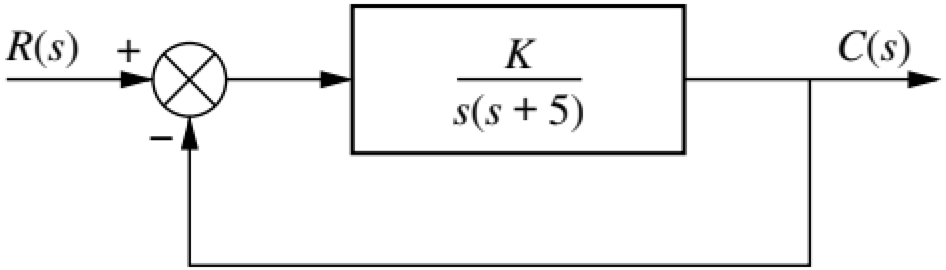
$$2\zeta\omega_n = 5$$

$$\omega_n = \sqrt{K}$$

$$\zeta = \frac{5}{2\sqrt{K}}$$

Feedback Control – System design Example

Design the value of gain *K*, for the feedback control system of the Figure, so that the system will respond with a 10% overshoot.



Solution:

The closed-loop transfer function takes the form,

$$T(s) = \frac{K}{s^2 + 5s + K}$$

A 10% overshoot implies that $\zeta = 0.591$.

$$\zeta = \frac{5}{2\sqrt{K}} \quad \Rightarrow \quad K = 17.9$$