Airframe Design and Construction

Instability of columns and thin sheets

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Column and sheet buckling – A18

- The pressure distribution over an aircraft wing results in a bending moment around the x-axis.

- The bending moment results in tension in the lower side and compression in the upper side.

- The upper side structural components (stringers and sheets) are susceptible to buckling.

- In this lecture, we deals with the buckling of both columns and sheets

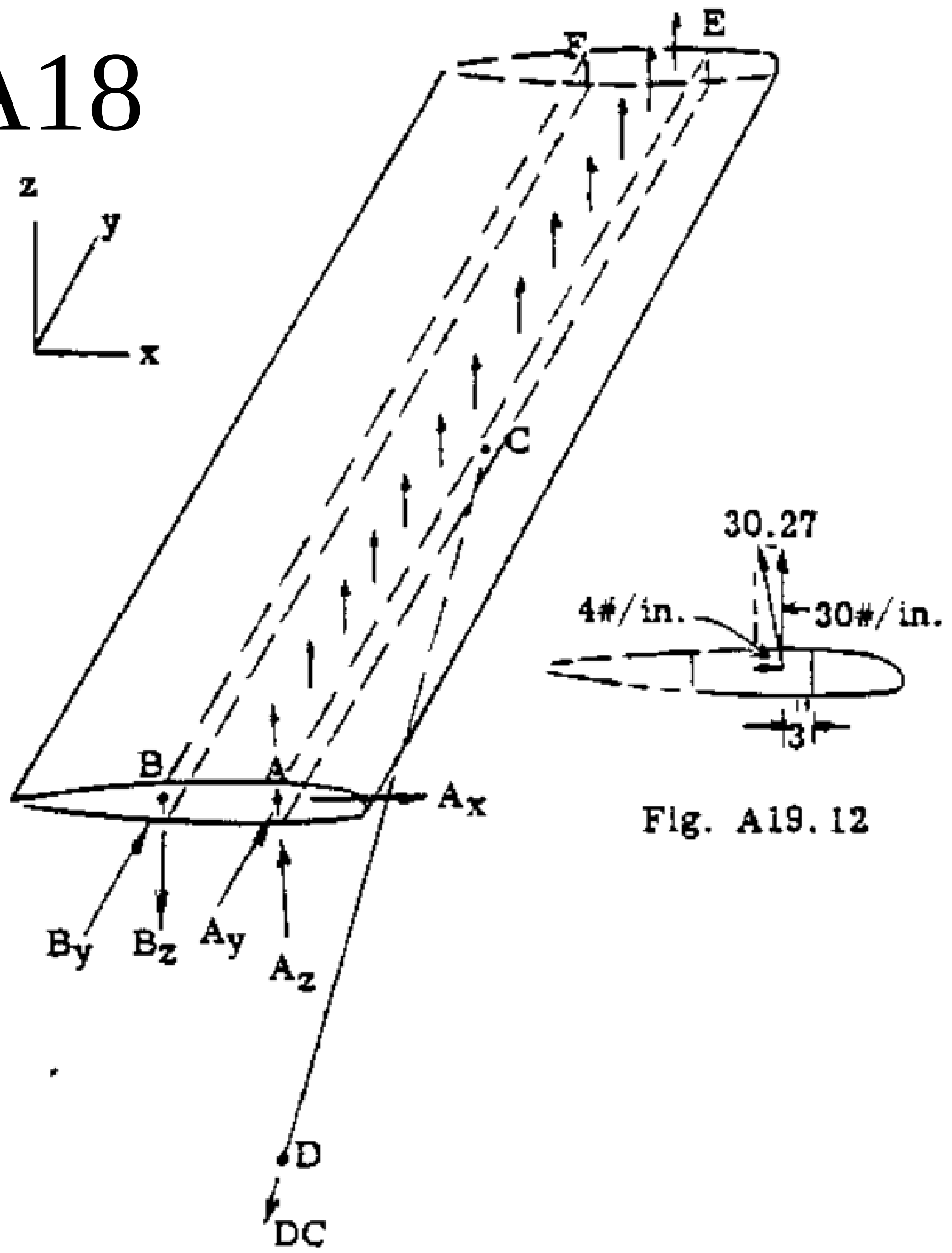


Fig. A19.12

Column Buckling

The government equation for column buckling has the form

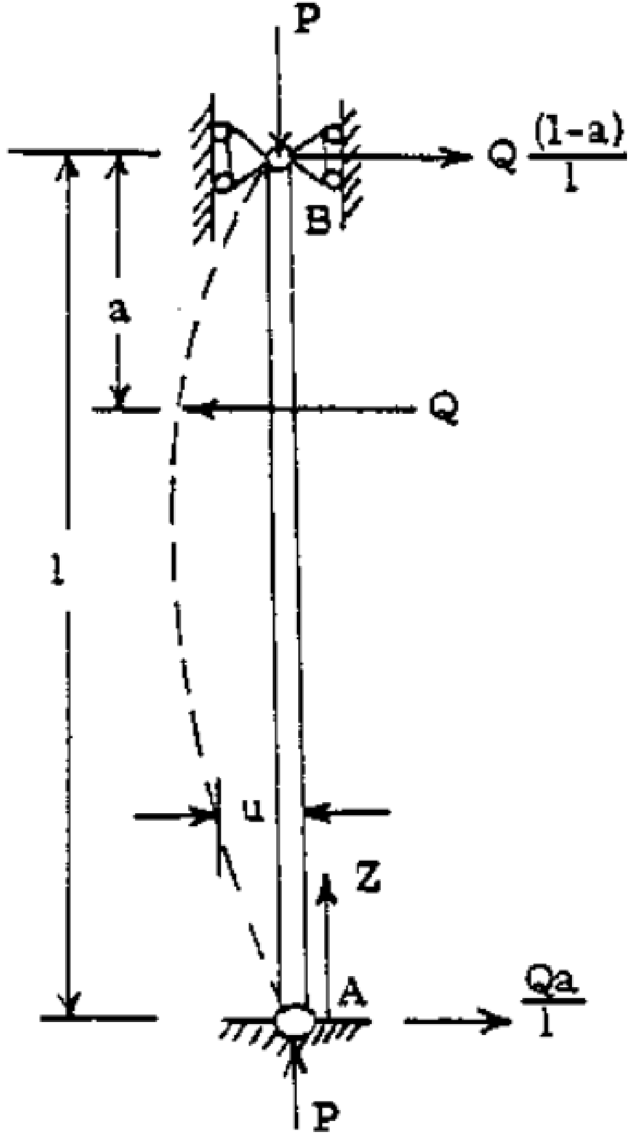
$$\frac{d^4u}{dz^4} + k^2\frac{d^2u}{dz^2} = 0$$

where

$$k^2 = \frac{P}{EI}$$

It is a fourth order differential equation that has the general solution

$$u(z) = c_1\, sin(kz) + c_2\, cos(kz) + c_3\, \mathrm{z} + a_4$$



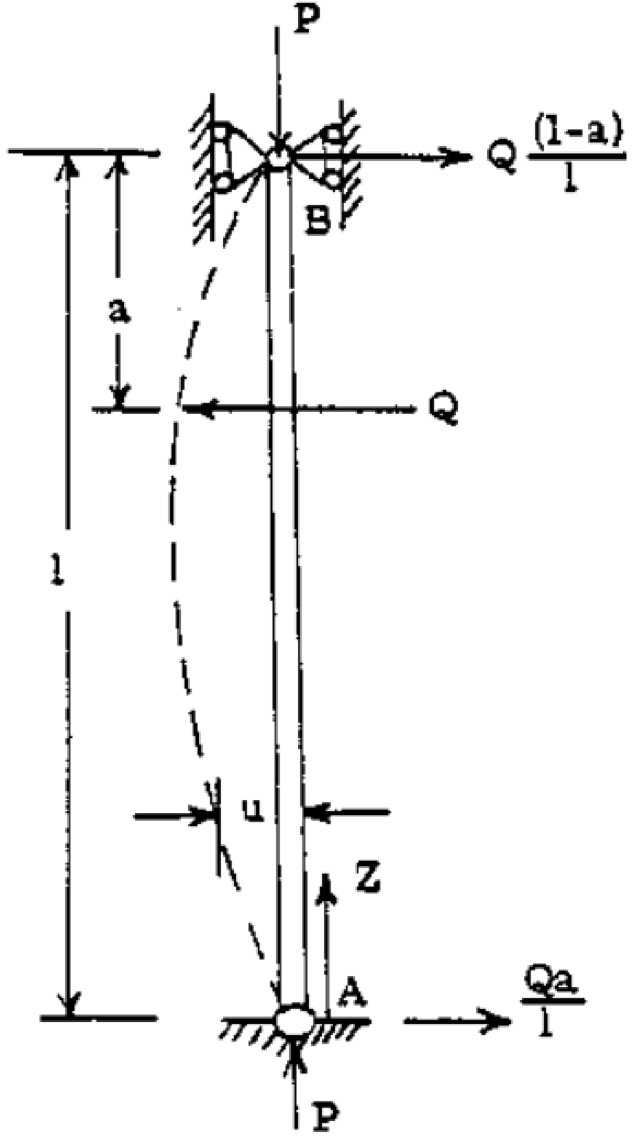
Column Buckling

The coefficients C_1, C_2, C_3 and C_4 depend on the conditions of the end supports. The various end conditions are:-

Free end:- $\dfrac{d^2u}{dz^2} = 0$, $\dfrac{d^3u}{dz^3} = 0$

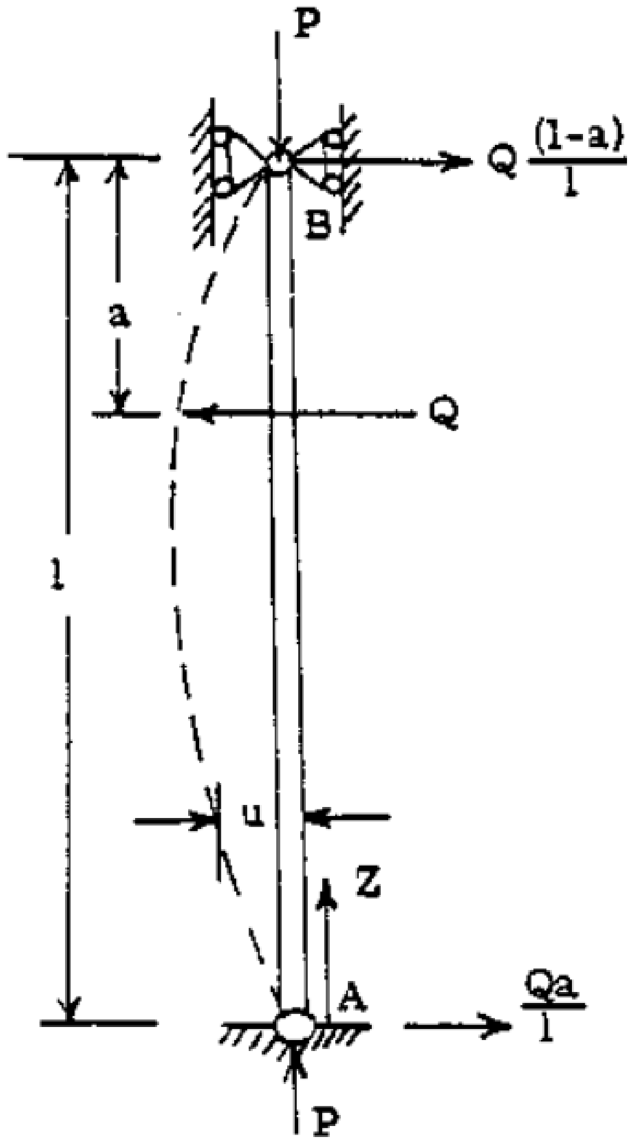
Pin end: - $u = 0$, $\dfrac{d^2u}{dz^2} = 0$

Fixed end:- $u = 0$, $\dfrac{du}{dz} = 0$



Column Buckling

Thus we have 4 end conditions. These give systems of four linear homogenous equations. A trivial solution of these is the zero solution. For the buckling state, however, C_1, C_2, C_3, C_4 are not all zero. The condition of non-zero solution of the above system is that the determinant of the coefficients of C_1, C_2, C_3 and C_4 is equal to zero. From this equation, we calculate the buckling load.



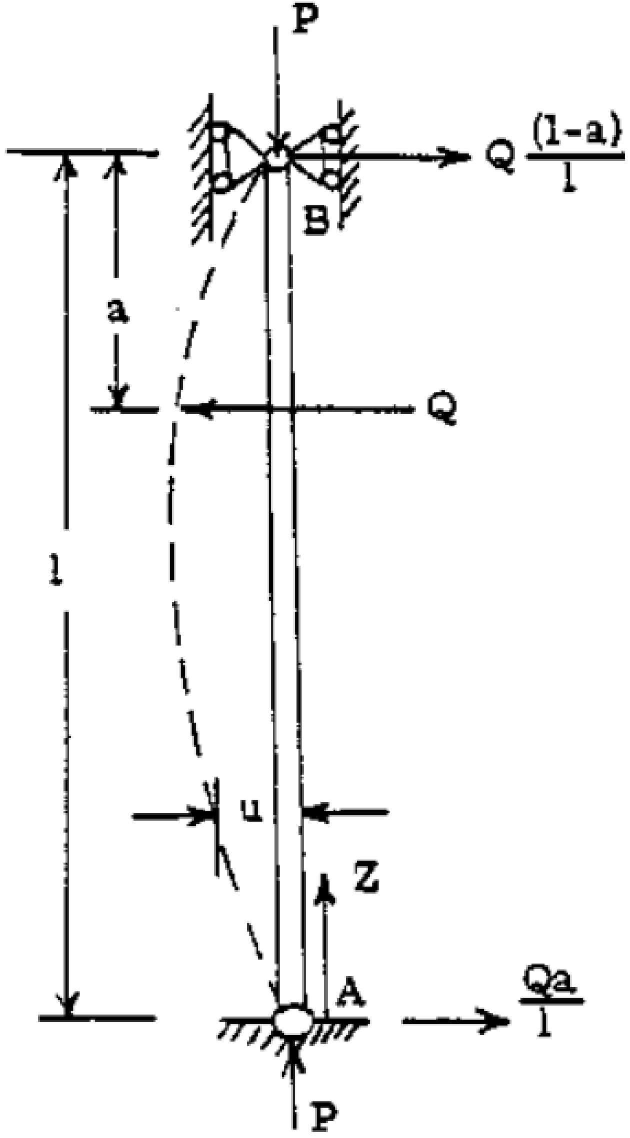
Column Buckling

For example, in a simply supported beam, $(u = 0, \ \frac{d^2u}{dz^2} = 0$, at both ends), the end conditions furnish give

$$C_2 + C_4 = 0 \quad , \quad C_2 = 0$$

$$C_1 \sin kl + C_2 \cos kl + C_3 l + C_4 = 0$$

For buckling we must have:-



Column Buckling

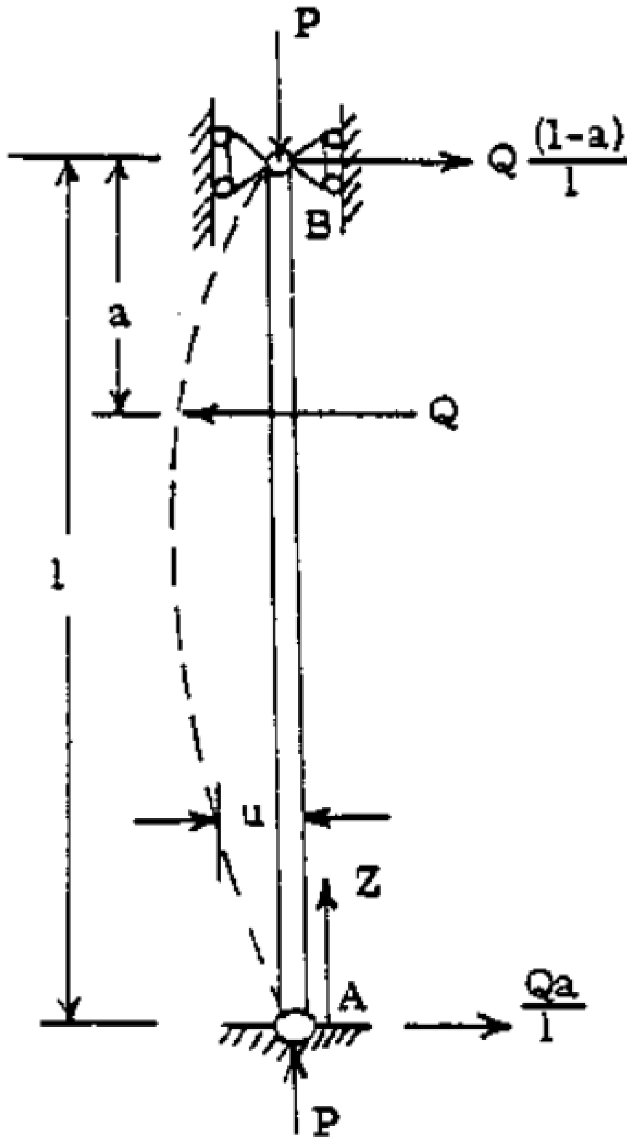
$$\begin{vmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ \sin kl & \cos kl & 1 & 1 \\ -\sin kl & -\cos kl & 0 & 0 \end{vmatrix} = 0$$

or $\sin kl = 0$ or $kl = n\pi$ ($n = 1, 2, 3 - - -$)

whence $P_n = \dfrac{n^2\pi^2 EI}{l^2}$ - - - - - - - - - - - - (16a)

Thus for P_n equal to the right hand side of equation (16a), we have a possible form of equilibrium of the bent form. The smallest value of P_n occurs at $n = 1$, and this is the buckling load:

$$P_{cr} = \frac{\pi^2 EI}{l^2} \quad - - - - - - - - - - - - - - \quad (16b)$$



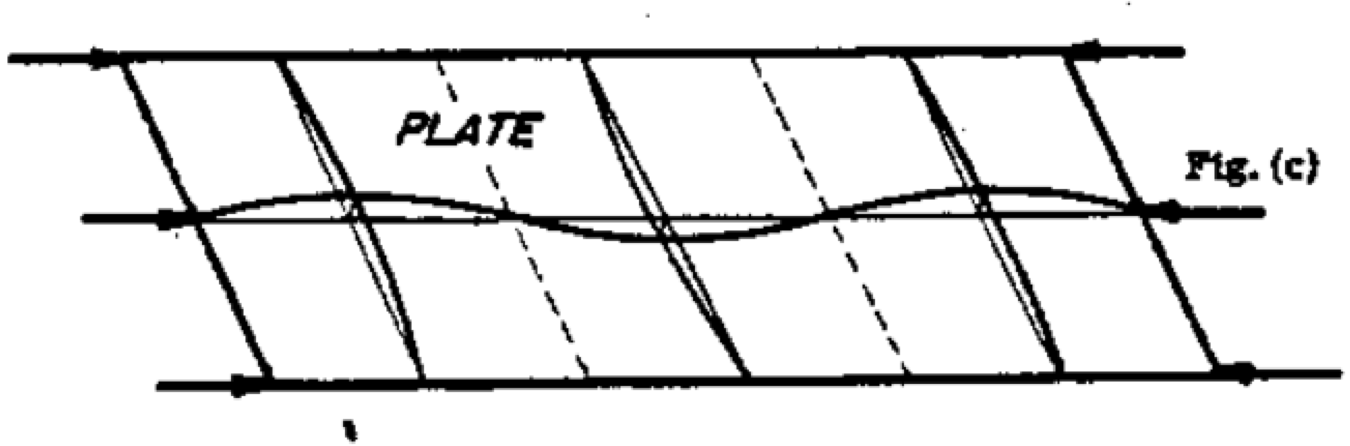
Buckling of straight sheets – C5&C6

Buckling under compression

The general buckling equation under compression load takes the form,

$$\sigma_c = \frac{K_c \pi^2 E}{12(1-v^2)}\left(\frac{t}{b}\right)^2$$

K_c: Buckling coefficient under compression
E: Young's Modulus
v: Poisson ratio
t: plate thickness
b: panel width



Buckling Compression strength of straight sheets

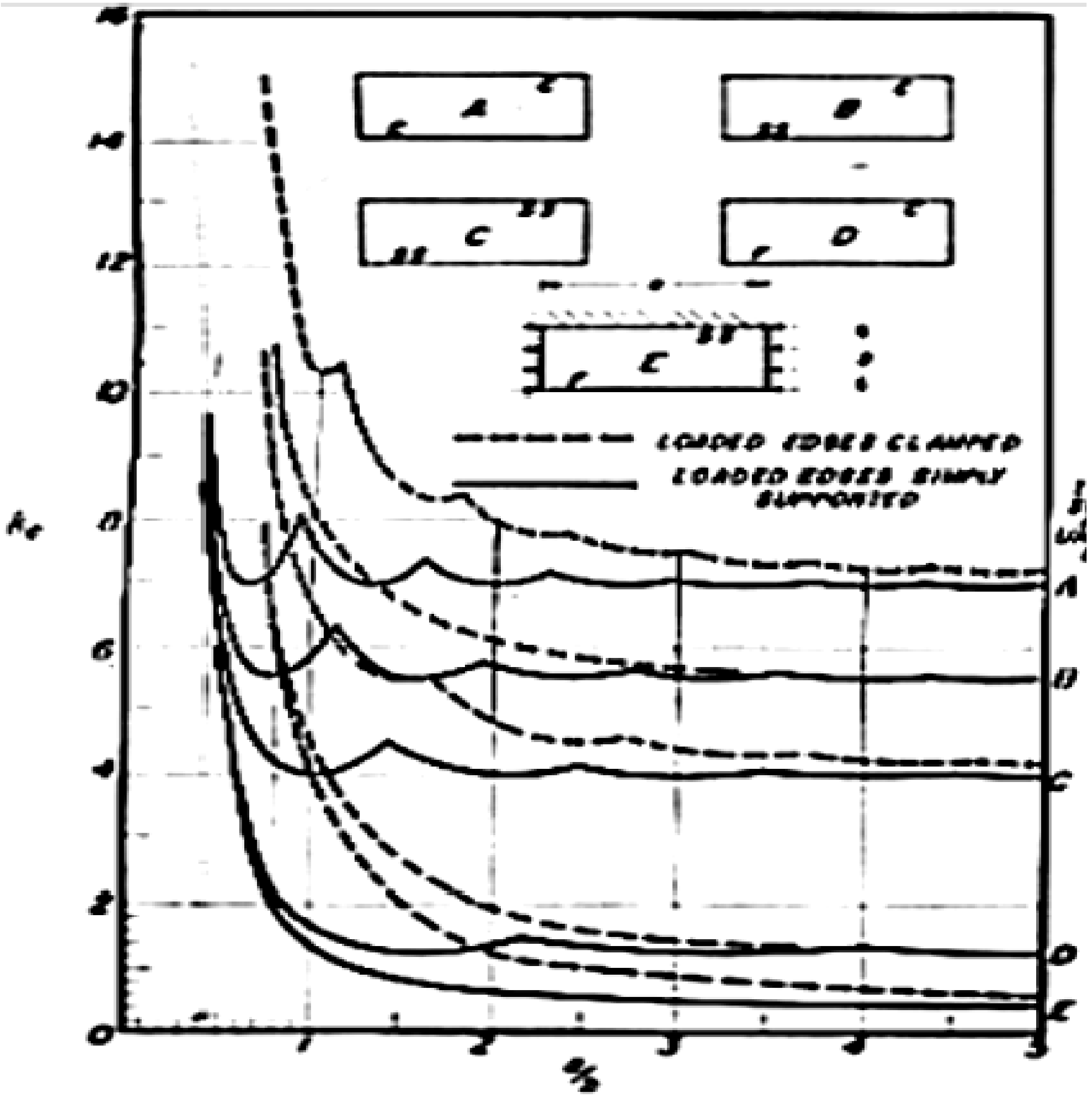
For straight sheets, the buckling coefficient K_c is calculated in terms of the plate aspect ratio

$$\alpha = \frac{a}{b}$$

Or K_c can be calculated from the equation

$$K_c = \left(\frac{m}{\alpha} + \frac{\alpha}{m}\right)^2$$

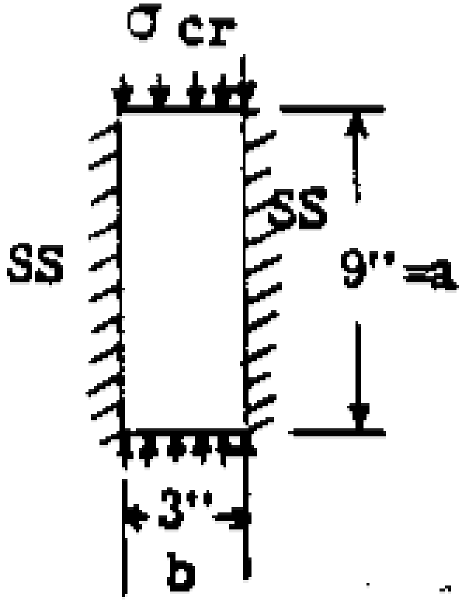
$$b = min(b_{avg}, frame\ spacing)$$



Buckling under Compression – Example

- The sketch shows a 3 x 9 inch sheet panel. The sides are simply supported. The material is aluminum alloy 2024-T3. The thickness is .094". E = 10,700,000.

ν_e = 0.3. Find the buckling stress σ_{cr}.

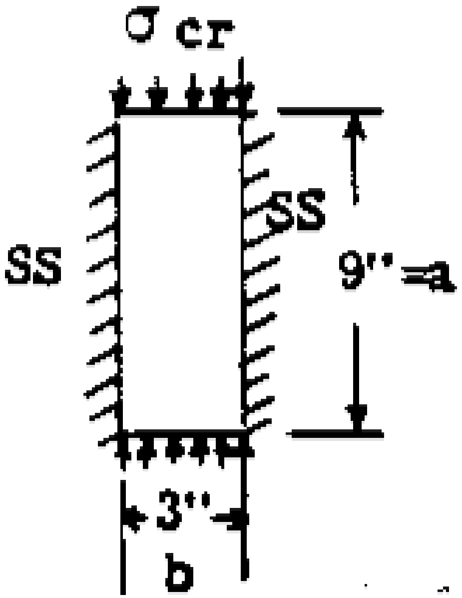


$$\sigma_c = \frac{K_c \pi^2 E}{12(1-v^2)} \left(\frac{t}{b}\right)^2$$

Buckling under Compression – Example

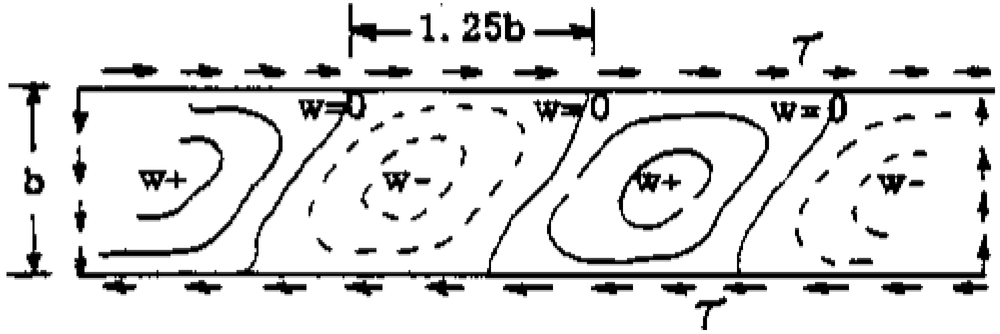
For a/b = 9/3 = 3, we find k_c from curve (c) of Fig. C5.2 equals 4.0.

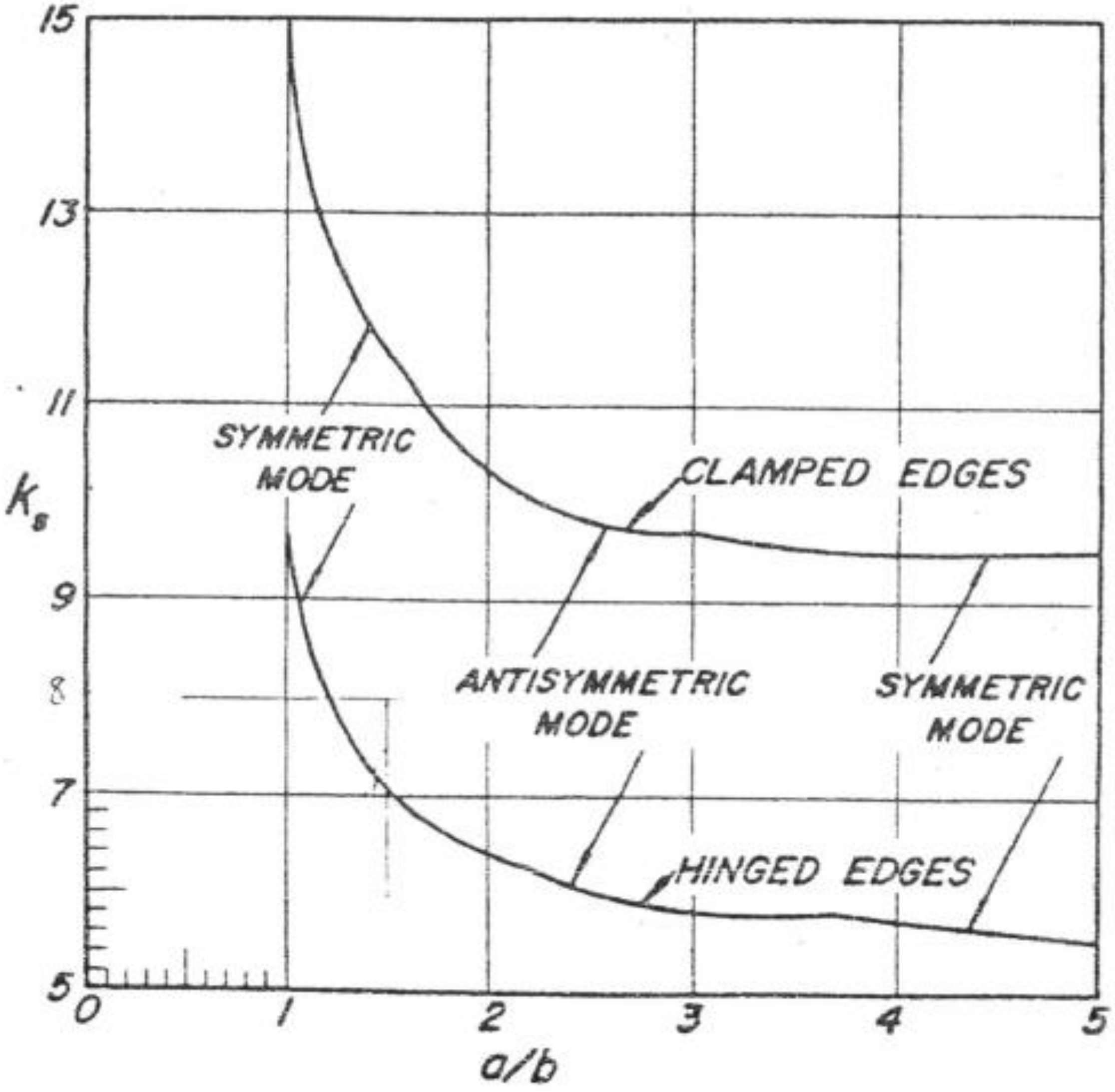
$$\sigma_c = \frac{4\pi^2 E}{12(1-v^2)}\left(\frac{0.049}{3}\right)^2 = 37\,978.2\ psi$$



Shear buckling of straight sheets

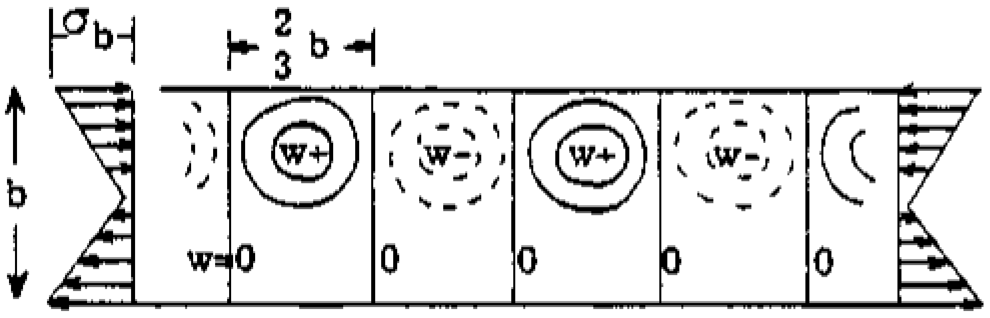
$$\sigma_s = \frac{K_s \pi^2 E}{12(1 - v^2)} \left(\frac{t}{b}\right)^2$$

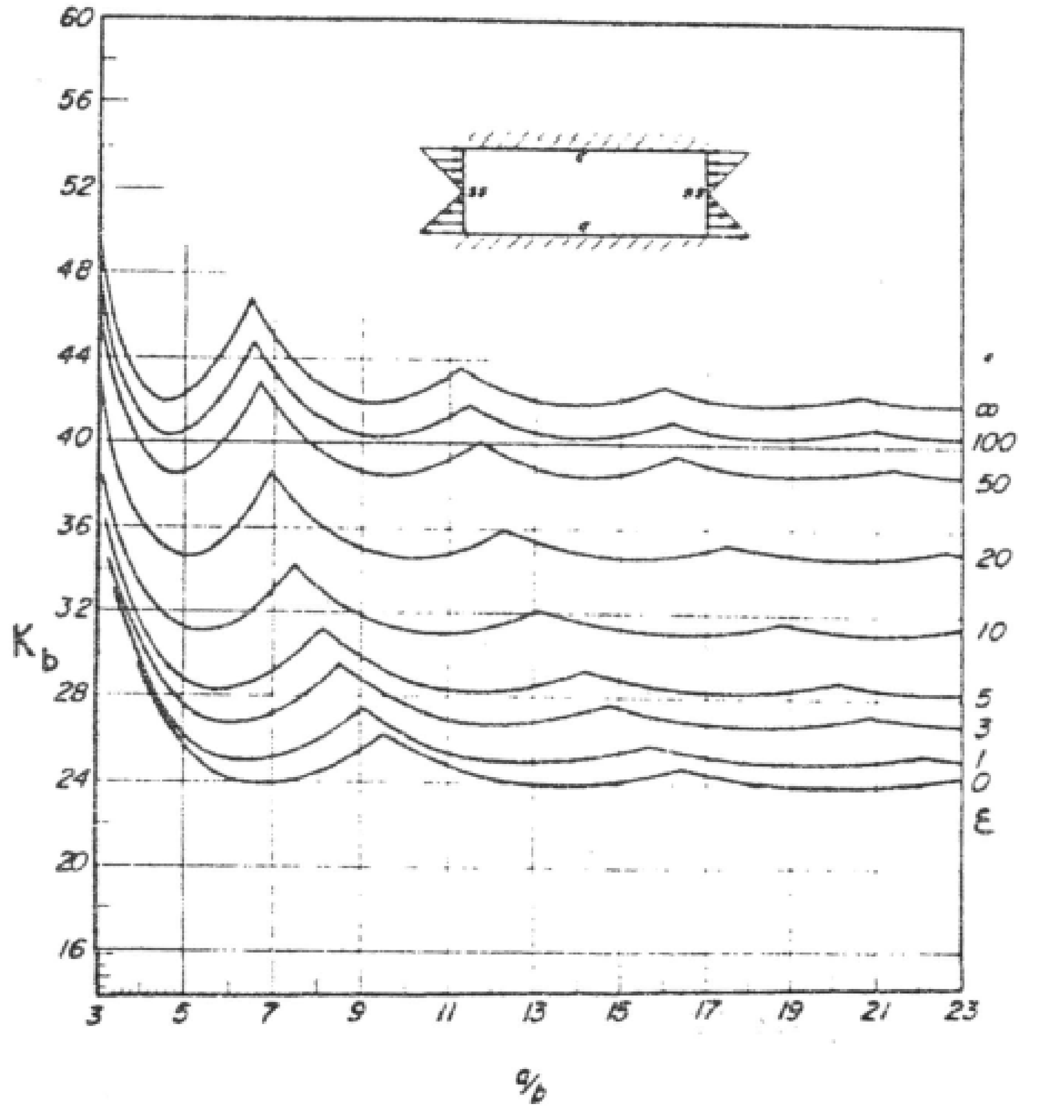




Bending buckling of straight sheets

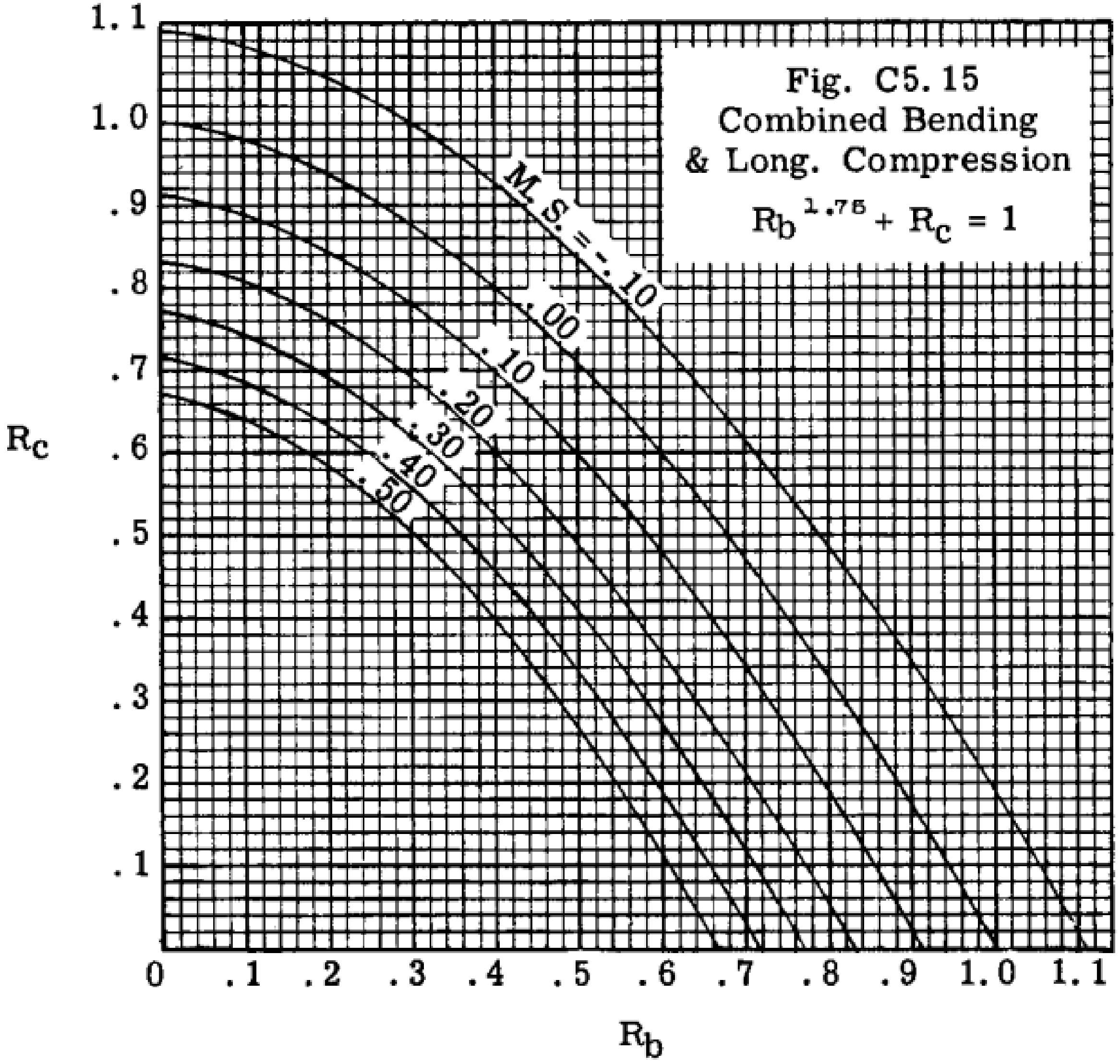
$$\sigma_b = \frac{K_b \pi^2 E}{12(1-v^2)}\left(\frac{t}{b}\right)^2$$





Buckling under combined bending and compression

$$R_b^{1.76} + R_c = 1.0$$

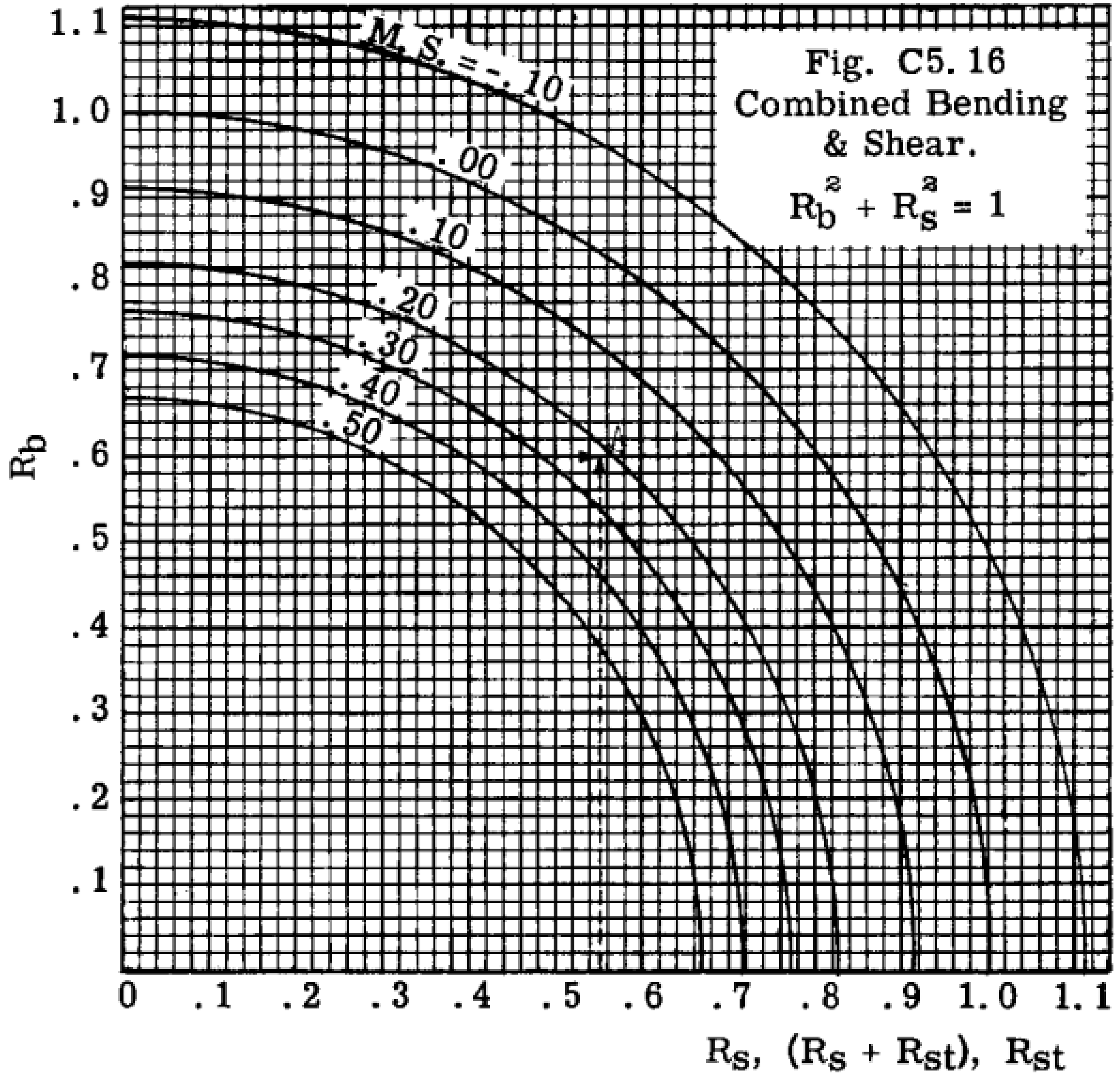


Buckling under combined bending and shear

$$R_b^2 + R_s^2 = 1 \quad \text{------------------} \quad (C5.9)$$

The expression for margin of safety is,

$$M.S. = \frac{1}{\sqrt{R_b^2 + R_s^2}} - 1 \quad \text{----------} \quad (C5.10)$$



Buckling under combined shear and longitudinal direct stress

$$R_L + R_S^2 = 1.0 \quad \text{- - - - - - - - - - - - -} \quad (C5.11)$$

$$M.S. = \frac{2}{\left(R_L + \sqrt{R_L^2 + 4R_S^2}\right)} - 1 \quad \text{- - - - -} \quad (C5.12)$$

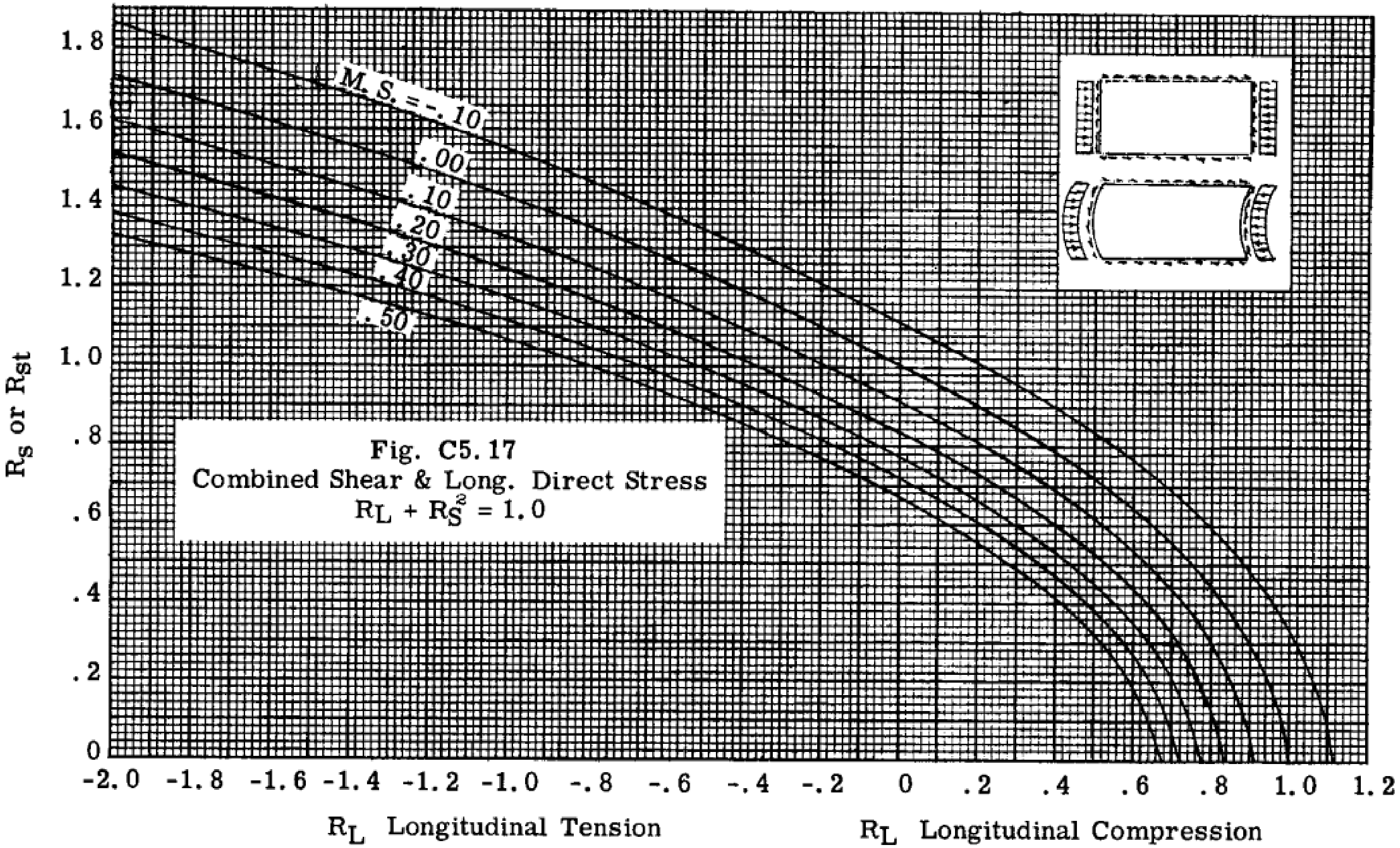


Fig. C5.17
Combined Shear & Long. Direct Stress
$R_L + R_S^2 = 1.0$

Buckling under combined compression, bending, and shear

Buckling check in case the load is given

a. Calculate τ_{cr} for individual buckling under shear.
b. Calculate $R_s = \frac{\tau}{\tau_{cr}}$.
c. Calculate σ_{cr} for individual buckling under bending.
d. Calculate $R_b = \frac{\sigma}{\sigma_{cr}}$
e. Calculate σ_{cr} for individual buckling under uniaxial compression.
f. Calculate $R_c = \frac{\sigma}{\sigma_{cr}}$
g. From Fig. C5-18 determine the buckling value of R_c corresponding to the calculated values of R_s and R_b.
h. If the value of R_c calculated in step 6 is equal to or greater than the value of R_c calculated in step 7 then the plate will buckle.

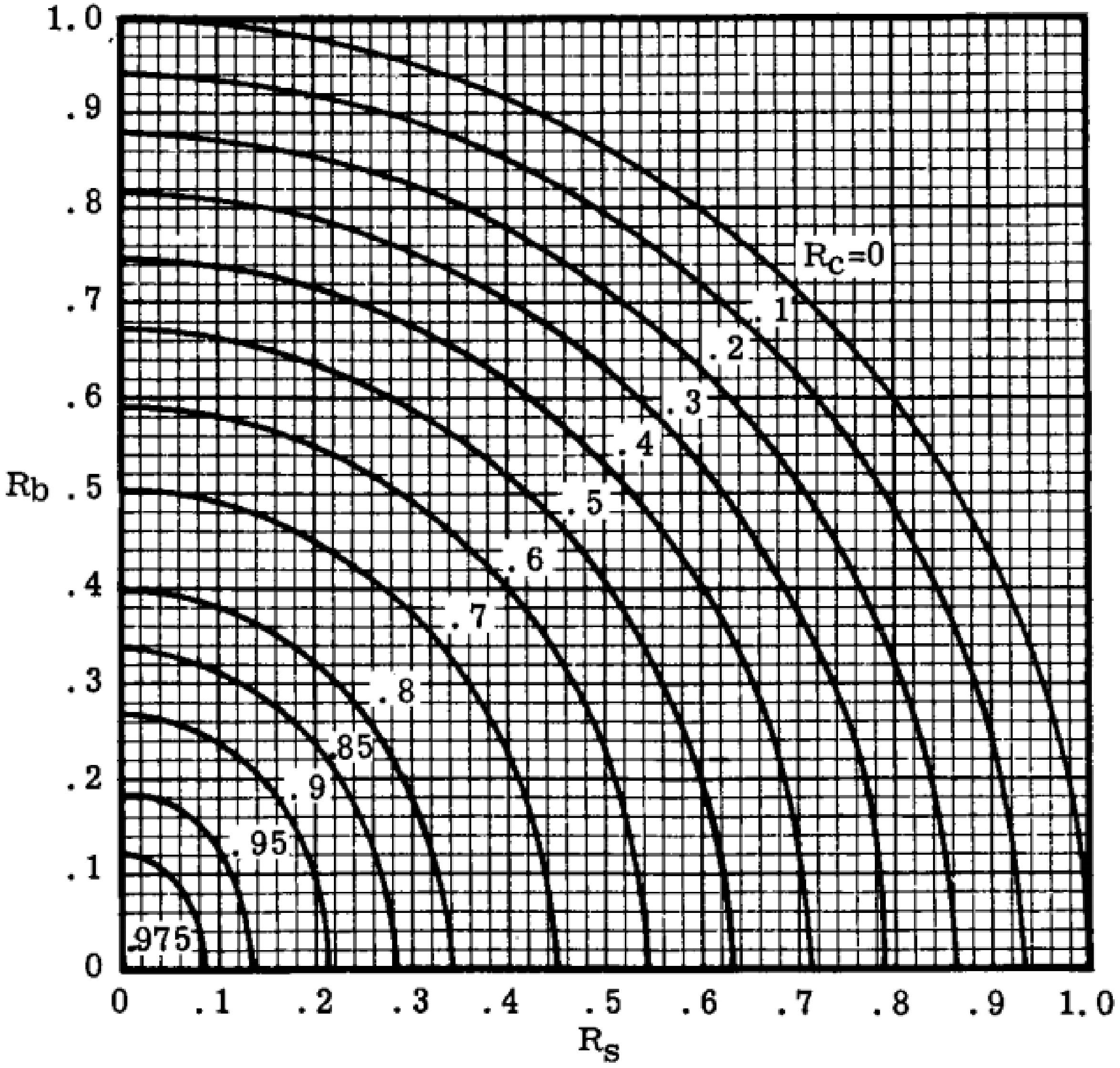


Fig. C5.18 (Ref. 5)

Buckling under combined compression, bending, and shear

To determine the buckling load for given (for example): $\boldsymbol{\sigma_x = \sigma}$ and $\boldsymbol{\tau = \sigma}$.

a. Calculate τ_{cr} for individual buckling under shear.
b. Calculate σ_{cr} for individual buckling under uniaxial compression.
c. Assume σ_0 value.
d. Calculate $R_{s0} = \frac{\sigma_0}{\tau_{cr}}$.
e. Calculate $R_{c0} = \frac{\sigma_0}{\sigma_{cr}}$
f. Calculate $R_{b0} = \frac{\sigma_0}{\sigma_{cr}}$
g. From Fig. C5-18 determine the buckling value of R_{c1} corresponding to the calculated values of R_{s0} and R_{b0}.
h. Using R_{c1}, calculate a new stress value σ_1
i. Calculate the error between the new and the old stress values, $\varepsilon = \frac{\sigma_1 - \sigma_0}{\sigma_1}$
j. If ε is less than 0.05, then the buckling stress $\sigma_{buckling} = \sigma_1$, otherwise GoTo step (d) and repeat.

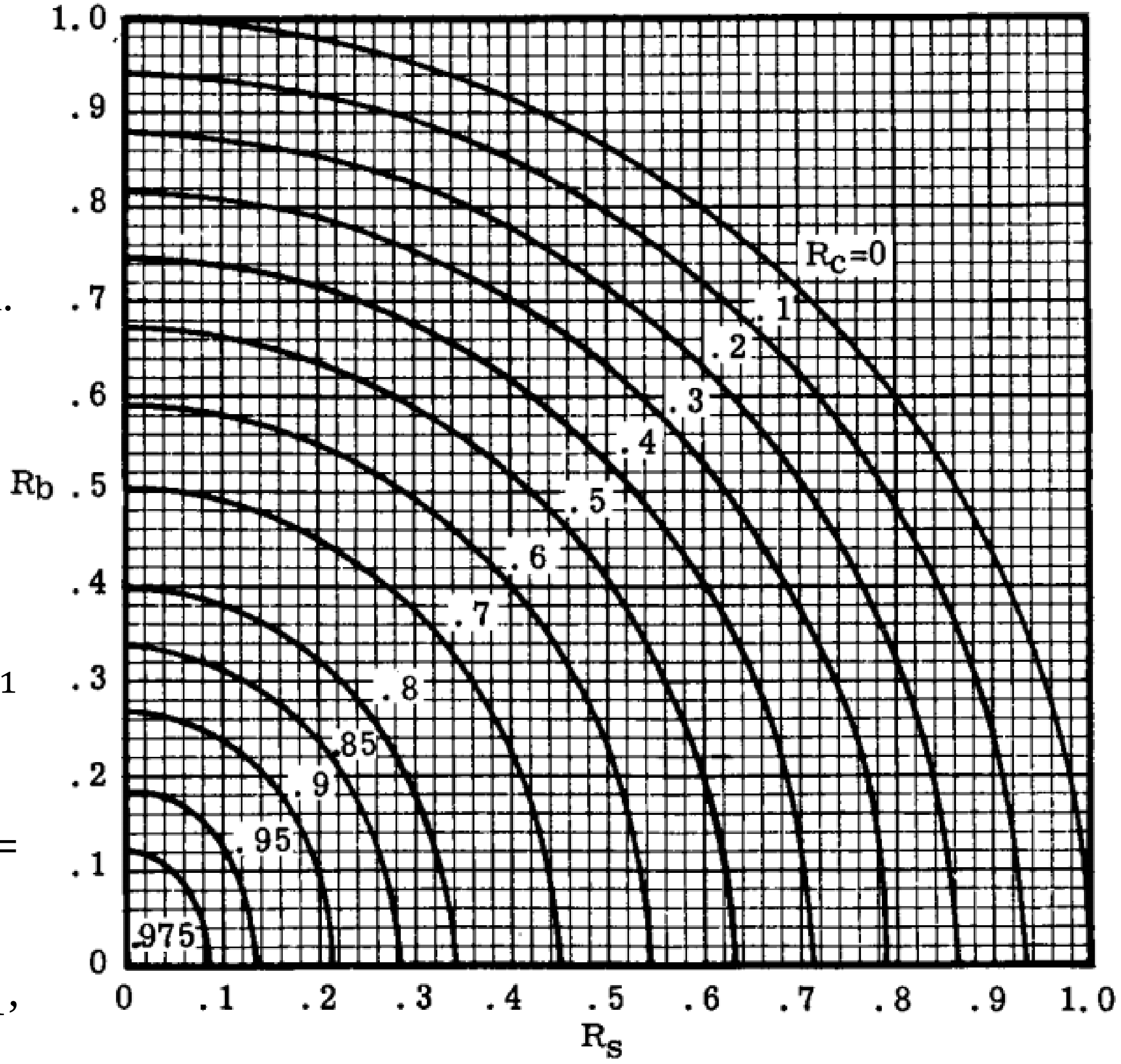


Fig. C5.18 (Ref. 5)

Example

a portion of a cantilever wing composed of sheet, stiffeners and ribs. The problem is to determine whether skin panels marked (A), (B) and (C) will buckle under the various given load cases. The sheet material is aluminum alloy 2024-T3.

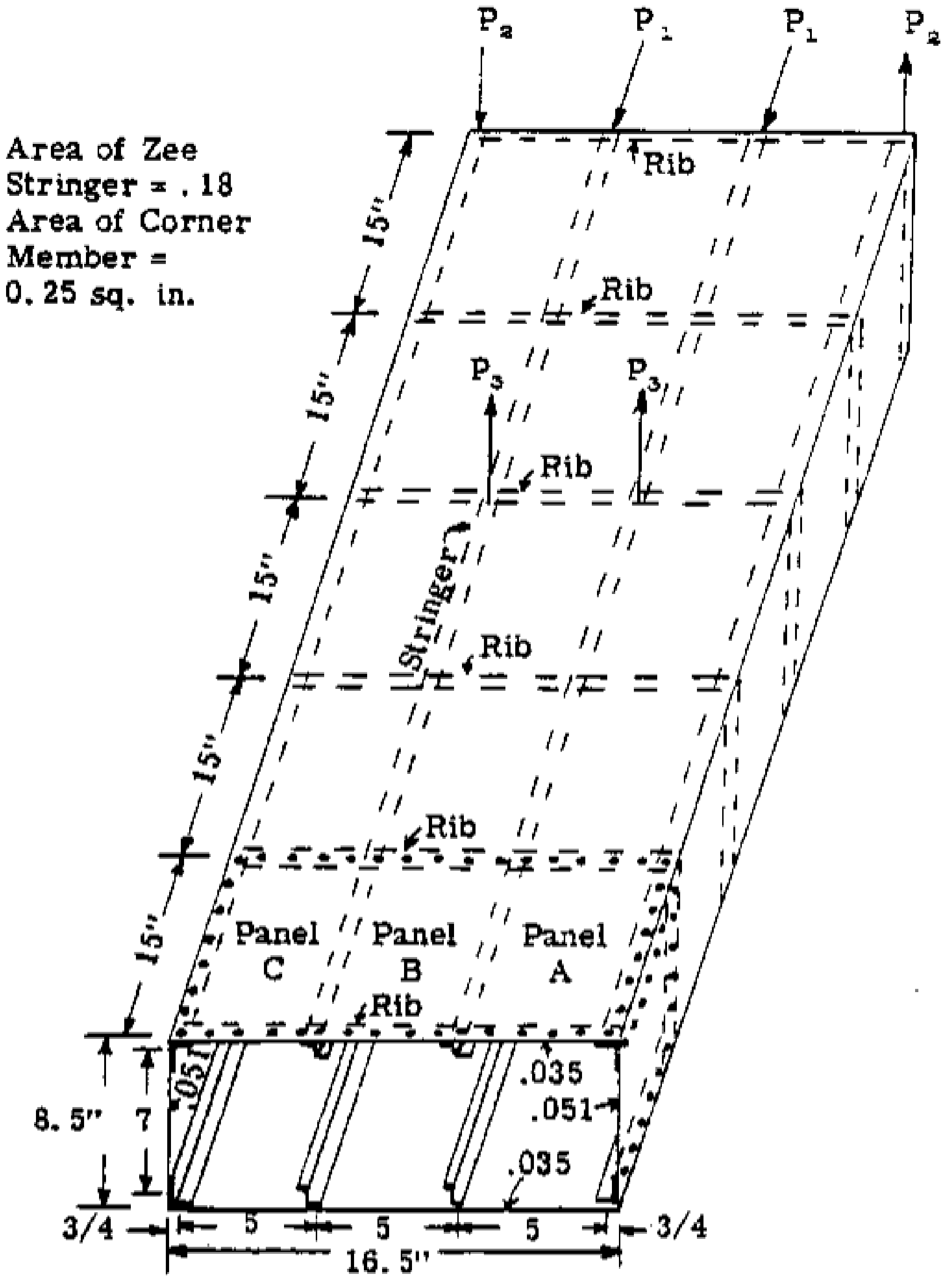


Fig. C5.20

Example

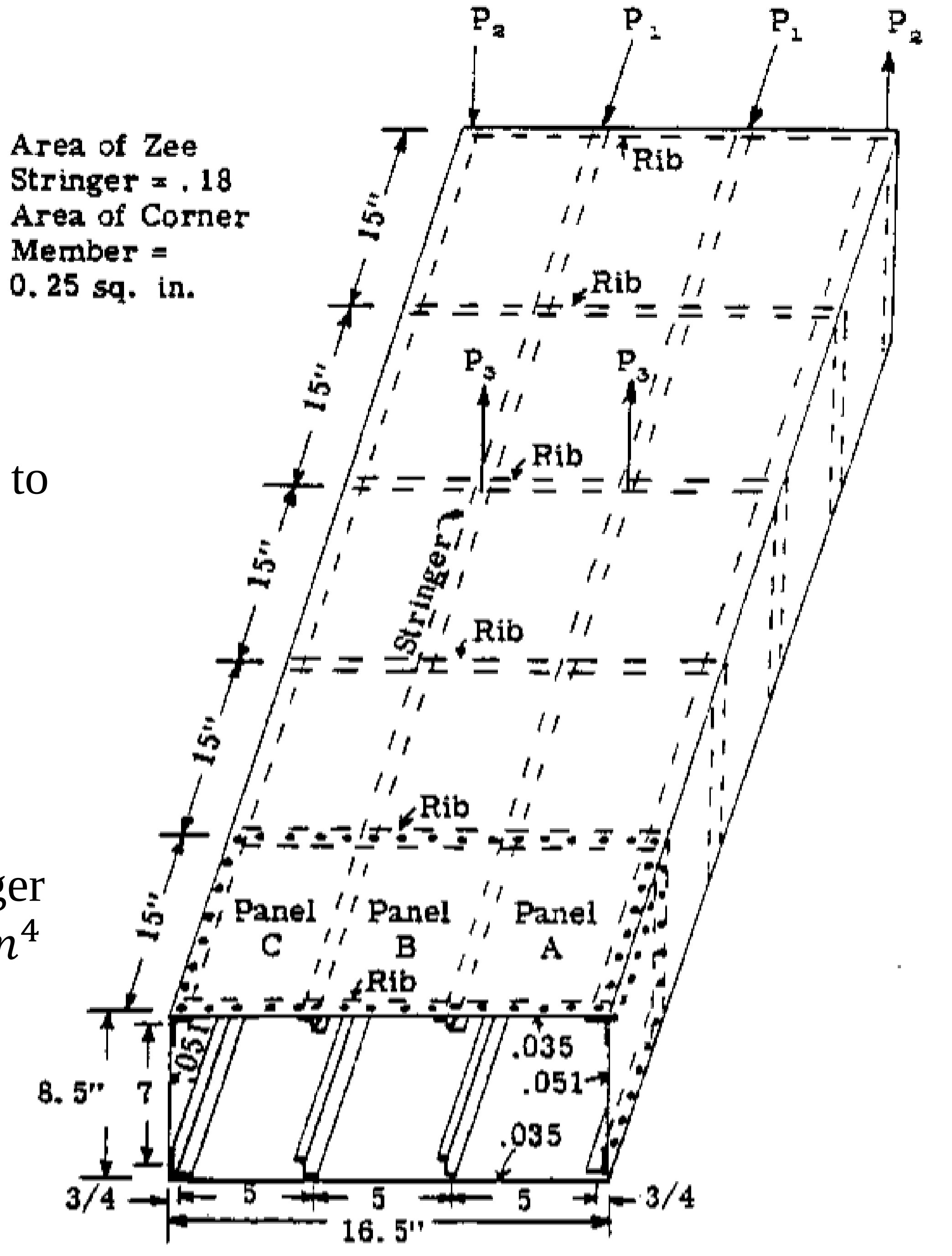


Fig. C5.20

Case 1: $P_1 = 700\ Ib., P_2 = 0, P_3 = 0$ Then, the beam is subjected to compressive load $2 * 700 = 1400\ Ib$

Thus, a bending moment is produced about x-axis $M_x = 1400 * 3.7 = \ 5170\ in.Ib,$
$where\ 3.7\ in\ is\ the\ distance\ from\ the\ load\ o\ the\ x - axis$

The total cross-section area of the section including skin and stringer = 3.73 sq.in. , and the moment of inertia about the x-axis is 4.93 in^4

Example

Since the beam section is symmetrical, the top panels A, B and C are subjected to the same stress under the P_1 load system.

Compressive stress due to transferring loads P_1 to centroid of beam cross-section is,

$f_c = 2P_1/\text{area} = 1400/3.73 = 375$ psi

Compressive stress due to constant bending moment of 5170 in. lbs. is,

$f_c = M_x Z/I_x = 5170 \times 4.233/49.30 = 444$ psi

Total $f_c = 375 + 444 = 819$ psi.

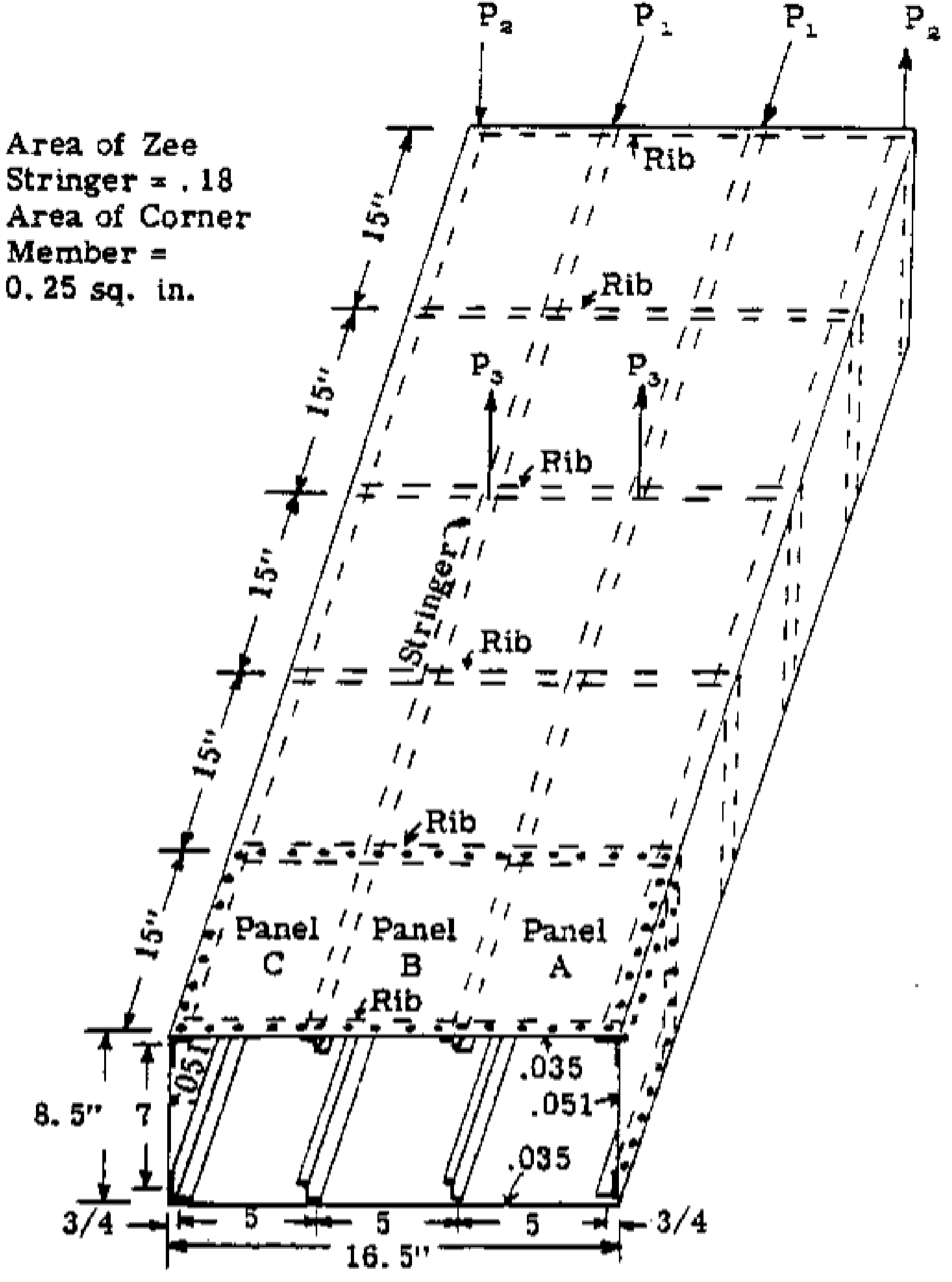


Fig. C5.20

Example

The skin panels are subjected to compression as shown in Fig. a. The boundary edge conditions given by the longitudinal sti[illegible]

and the rib flanges will be conservatively assumed as simply supported. ($F_{c_{cr}}$ is same as σ_{cr})

$$F_{c_{cr}} = \frac{\pi^2 k_c E}{12(1-\nu_e^2)} \left(\frac{t}{b}\right)^2$$

(See Eq. C5.1)

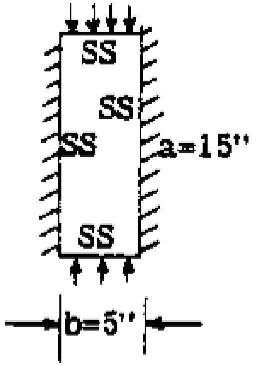


Fig. (a)

a/b of skin panel = 15/5 = 3

From Fig. C5.2 for Case C, we read k_c = 4.0

$$F_{c_{cr}} = \frac{\pi^2 \times 4.0 \times 10{,}700{,}000}{12(1-0.3^2)} \left(\frac{.035}{5}\right)^2 = 1900 \text{ psi}$$

Since $F_{c_{cr}}$ the buckling stress is less than the applied stress f_c, the panels will not buckle.

M.S. = $(F_{c_{cr}}/f_c) - 1 = (1900/819) - 1 = 1.32$

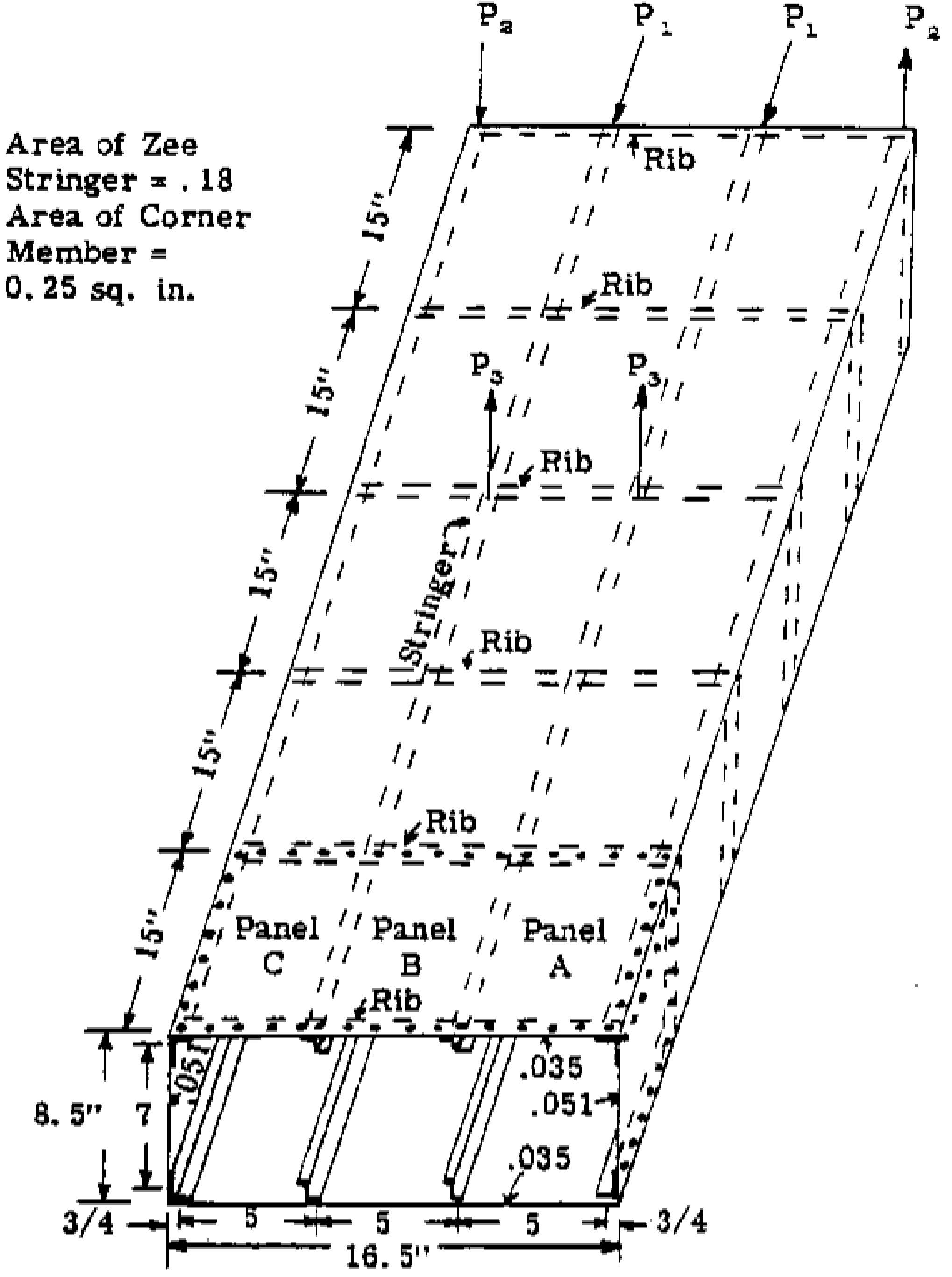


Fig. C5.20

Example

Load Case 2.

P_1 = 700 lb., P_2 = 500, P_3 = 0

The two loads P_2 acting in opposite directions produce a couple or a torsional moment of 500 x 16.5 = 8250 in. lb. on the beam structure, which means we have added a pure shear stress system to the compressive stress system of Case 1 loading.

The shear stress in the top panels A, B and C is,

f_S = T/2At = 8250/2 x 138 x .035 = 854 psi.

(Where A is the cell inclosed area)

The shear buckling stress is

$$F_{S_{cr}} = \frac{\pi^2 k_S E}{12(1-\nu_e^2)}\left(\frac{t}{b}\right)^2 \quad \text{- - - - (See Eq. C5.4)}$$

a/b = 15/5 = 3. From Fig. C5.11, for hinged or simply supported edges, we read k_S = 5.8.

$$F_{S_{cr}} = \frac{\pi^2 \times 5.8 \times 10{,}700{,}000}{12(1-.3^2)}\left(\frac{.035}{5}\right)^2 = 2760 \text{ psi.}$$

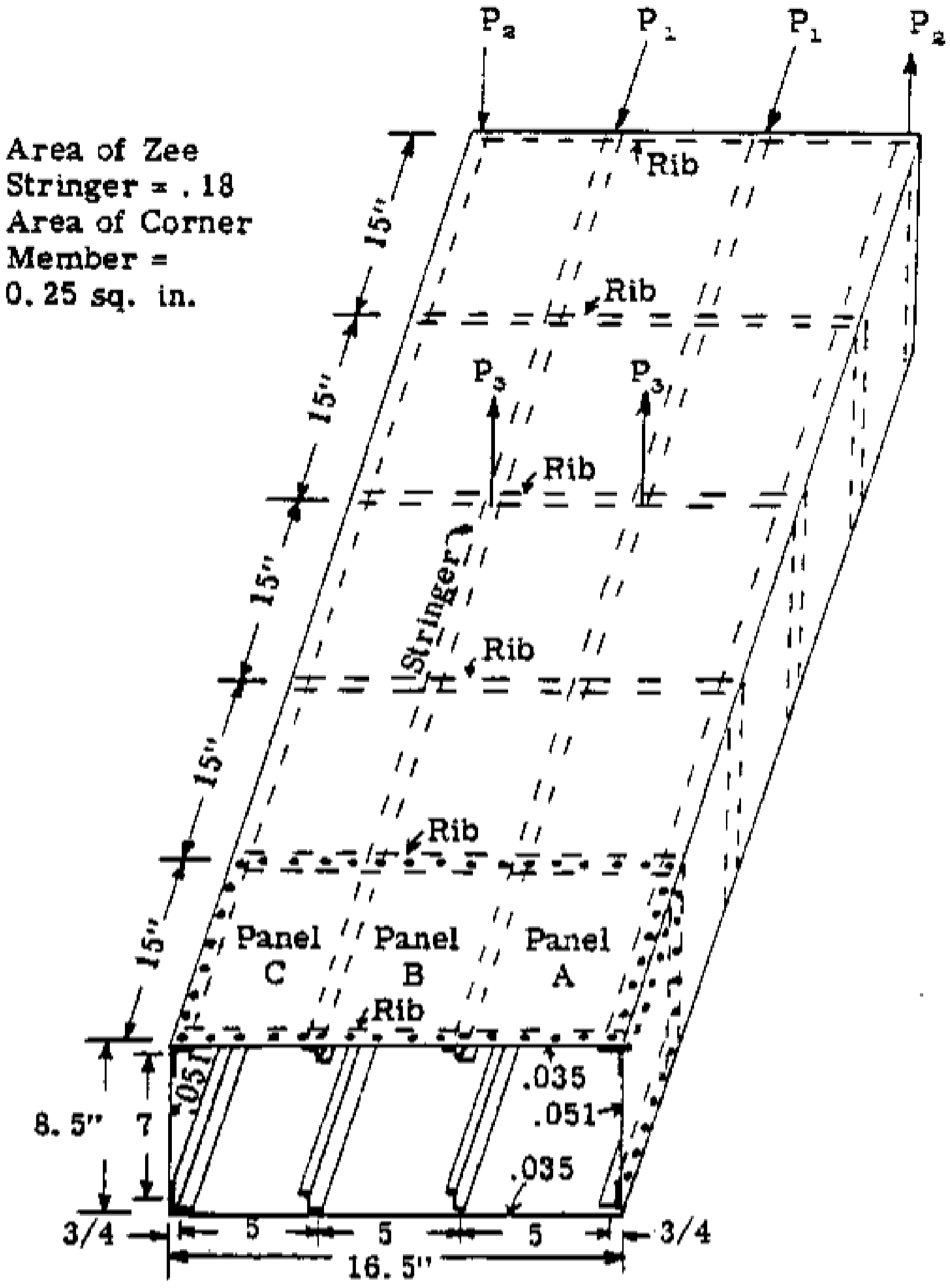


Fig. C5.20

Example

The sheet panels are now loaded in combined compression and shear so the interaction equation must be used. From Art. C5.12 the interaction equation is $R_c + R_s^2 = 1$.

$$R_c = f_c/F_{c_{cr}} = 819/1900 = .431$$

$$R_s = f_s/F_{s_{cr}} = 854/2760 = .309$$

The $R_c + R_s^2 = .431 + .309^2 = .526$. Since the result is less than 1.0, no buckling occurs.

$$\text{The M.S.} = \frac{2}{R_c + \sqrt{R_c^2 + 4R_s^2}} - 1$$

$$= \frac{2}{.431 + \sqrt{.431^2 + 4 \times .309^2}} - 1 = .69$$

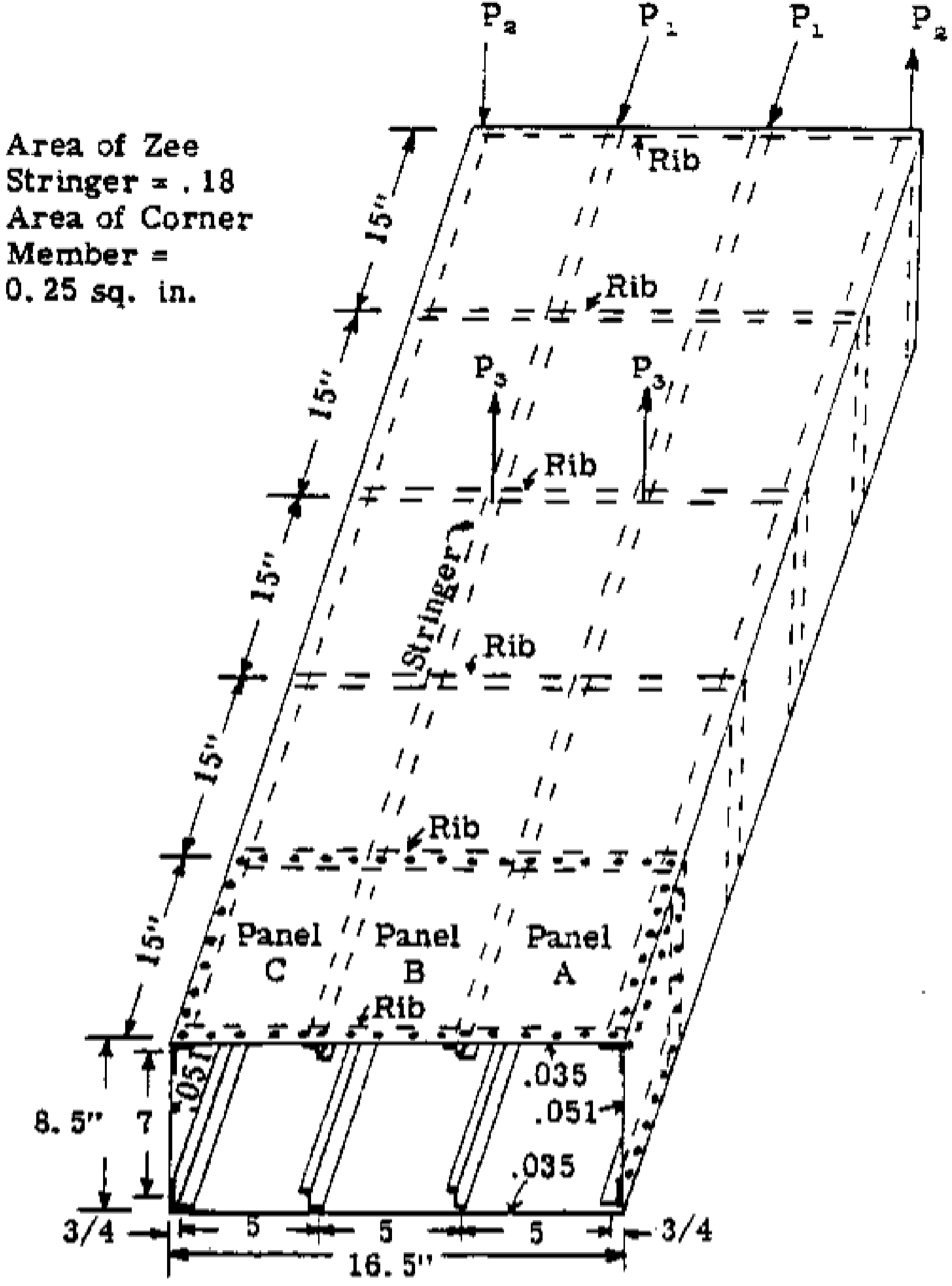


Fig. C5.20

Example

Load Case 3.

$P_1 = 700$, $P_2 = 500$, $P_3 = 100$ lb.

The two loads P_3 produce bending and flexural shear on the beam. The bending moment produces a different end compressive stress on the three sheet panels since the bending moment is not constant over the panel moment. To simplify we will take average bending moment on the panel.

Error in 52.5

$M_{x(av)} = 200 \times 52.5 = 10500$ in. lb.

f_c due to this bending $= M_x Z/I_x = 10500 \times 4.233/49.3 = 903$ psi.

Total $f_c = 903 + 819 = 1722$ psi.

$R_c = f_c/F_{c_{cr}} = 1722/1900 = .906$

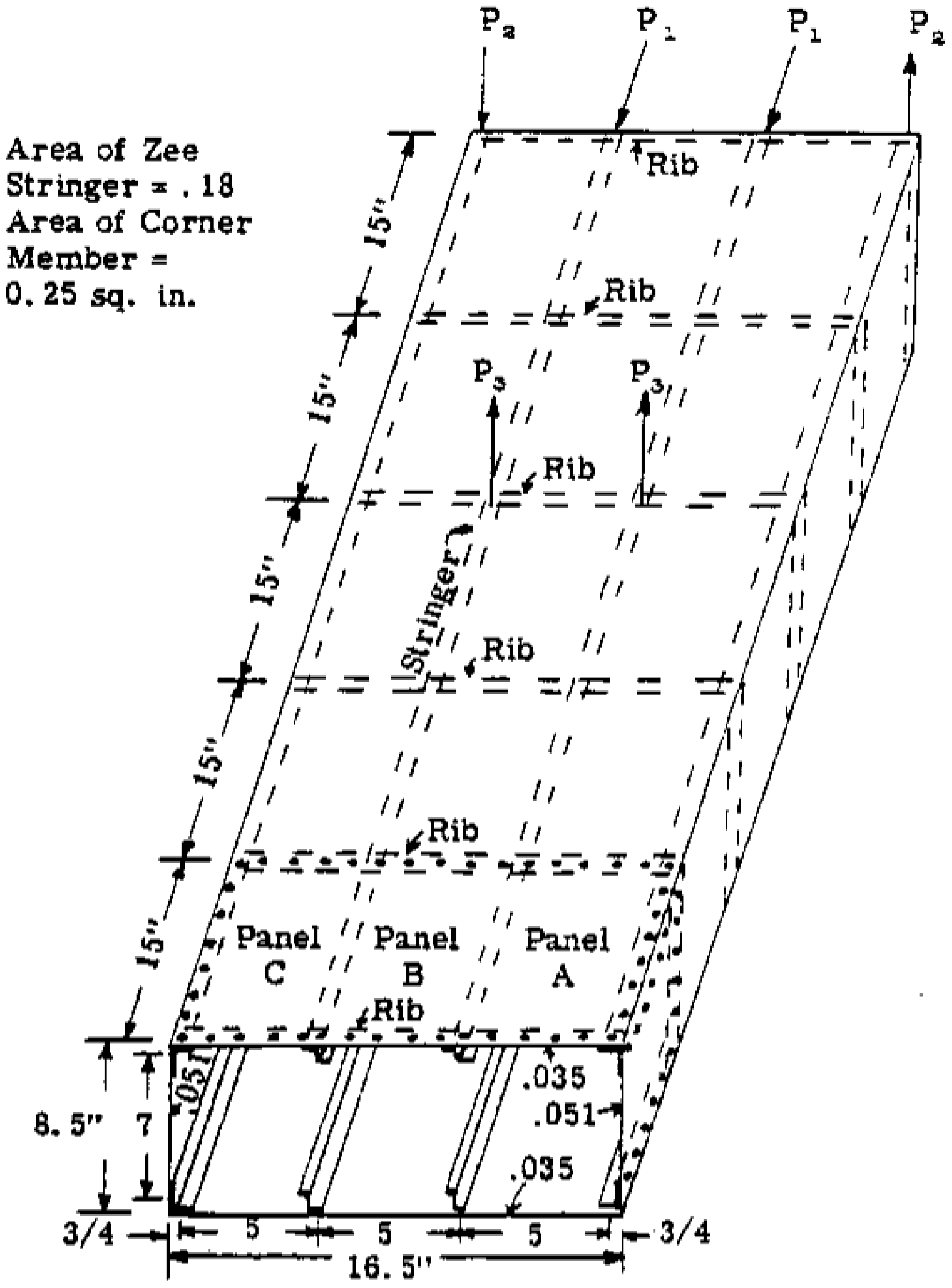


Fig. C5.20

Example

The two loads P_3 produce a traverse shear load V = 200 lb. The flexural shear stress must be added to the torsional shear stress as found in Case 2 loading.

Due to symmetry of beam section and P_3 loading the shear flow q at midpoint of sheet panel (B) is zero. We will thus start at this point and go clockwise around cell. The shear flow equation (see Chapter A15) is,

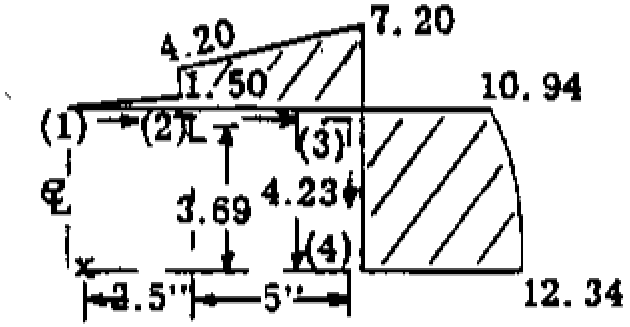


Fig. (b)

$$q = \frac{V}{I_X} \Sigma ZA$$

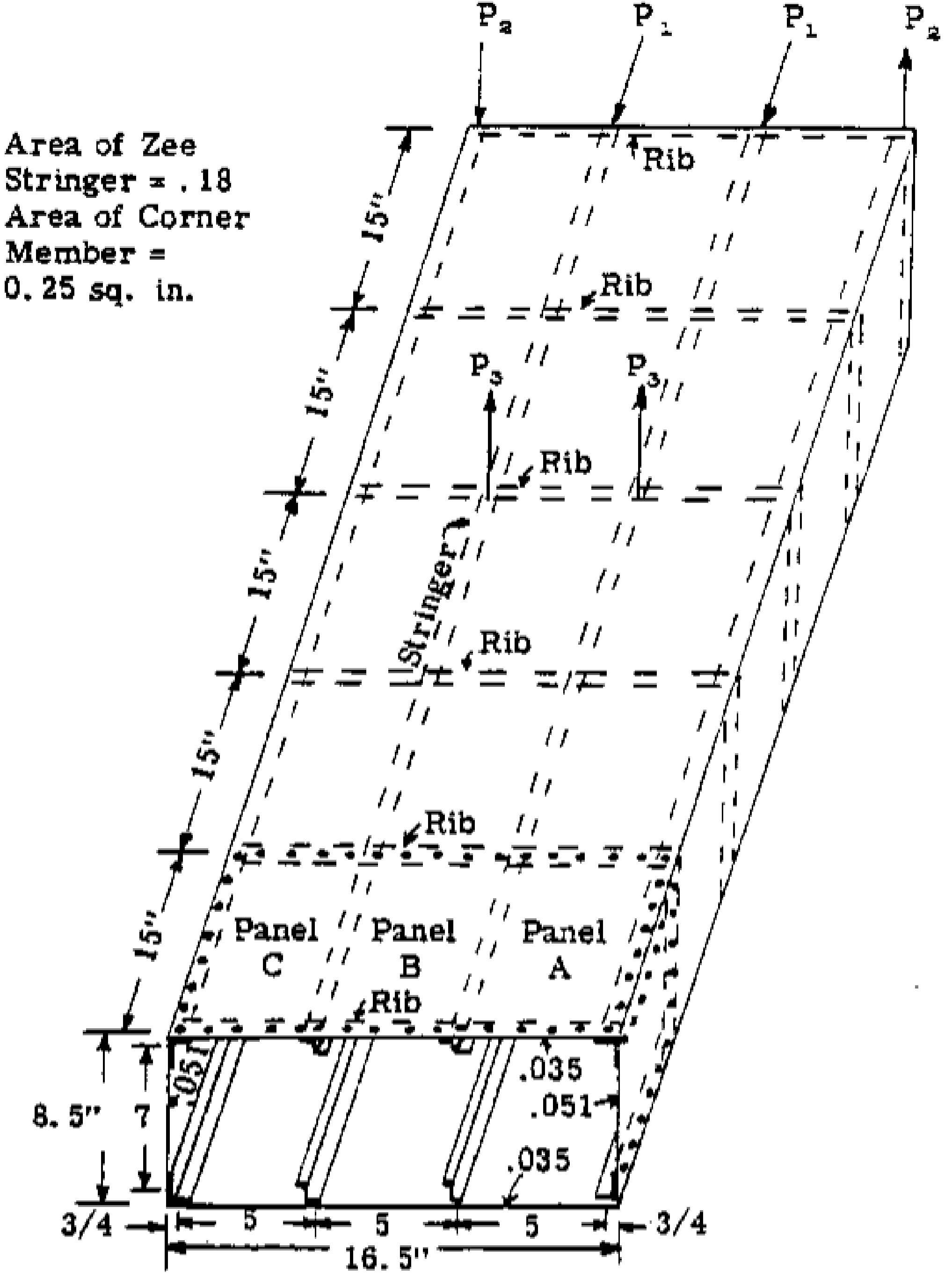


Fig. C5.20

Example

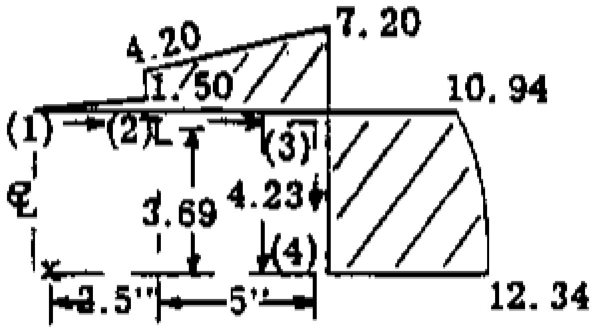


Fig. (b)

$$q = \frac{V}{I_x} \Sigma ZA$$

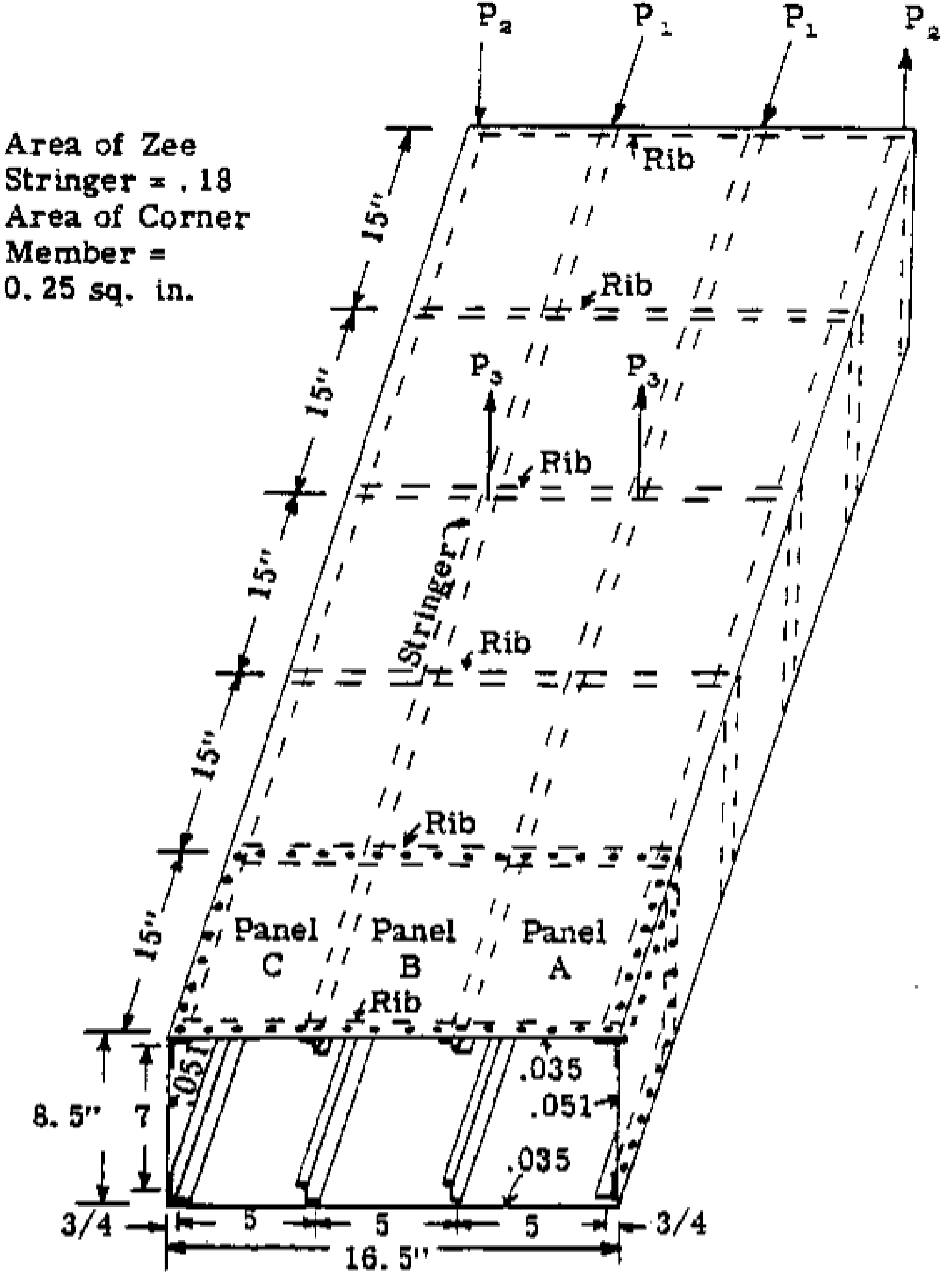


Fig. C5.20

$$q = -\frac{200}{4.93} = 4.05\ \Sigma ZA$$

$q_1 = 0$ (Refer to Fig. b)

$q_{21} = -4.05 \times 2.5 \times .035 \times 4.23 = -1.50$

$q_{23} = -1.50 - 4.05 \times .18 \times 3.69 = -4.20$

$q_{32} = -4.20 - 4.05 \times 5 \times .035 \times 4.23 = 7.20$

$q_{34} = -7.20 - 4.05 \times .25 \times 3.69 = 10.94$

$q_{43} = -10.94 - 4.05 \times .051 \times 3.69 \times 3.69/2 = -12.34$

Example

The shear flow q on panel (A) varies from 4.20 to 7.20 or the average q = (4.2 + 7.2)/2 = 5.7. Thus the average shear stress is 5.7/.035 = 163 psi. It is in the same direction as the torsional shear flow and thus is additive.

Total f_s = 163 + 854 = 917 psi

$$R_s = f_s/F_{s_{cr}} = 917/2760 = .332$$

$R_c + R_s^2 = 1$, Subt.:- $.906 + .332^2 = 1.016$, since the result is greater than 1.0, initial buckling has started. The margin of safety is slightly negative and equals,

$$M.S. = \frac{2}{.905 + \sqrt{.905^2 + 4 \times .332^2}} - 1 = -.01$$

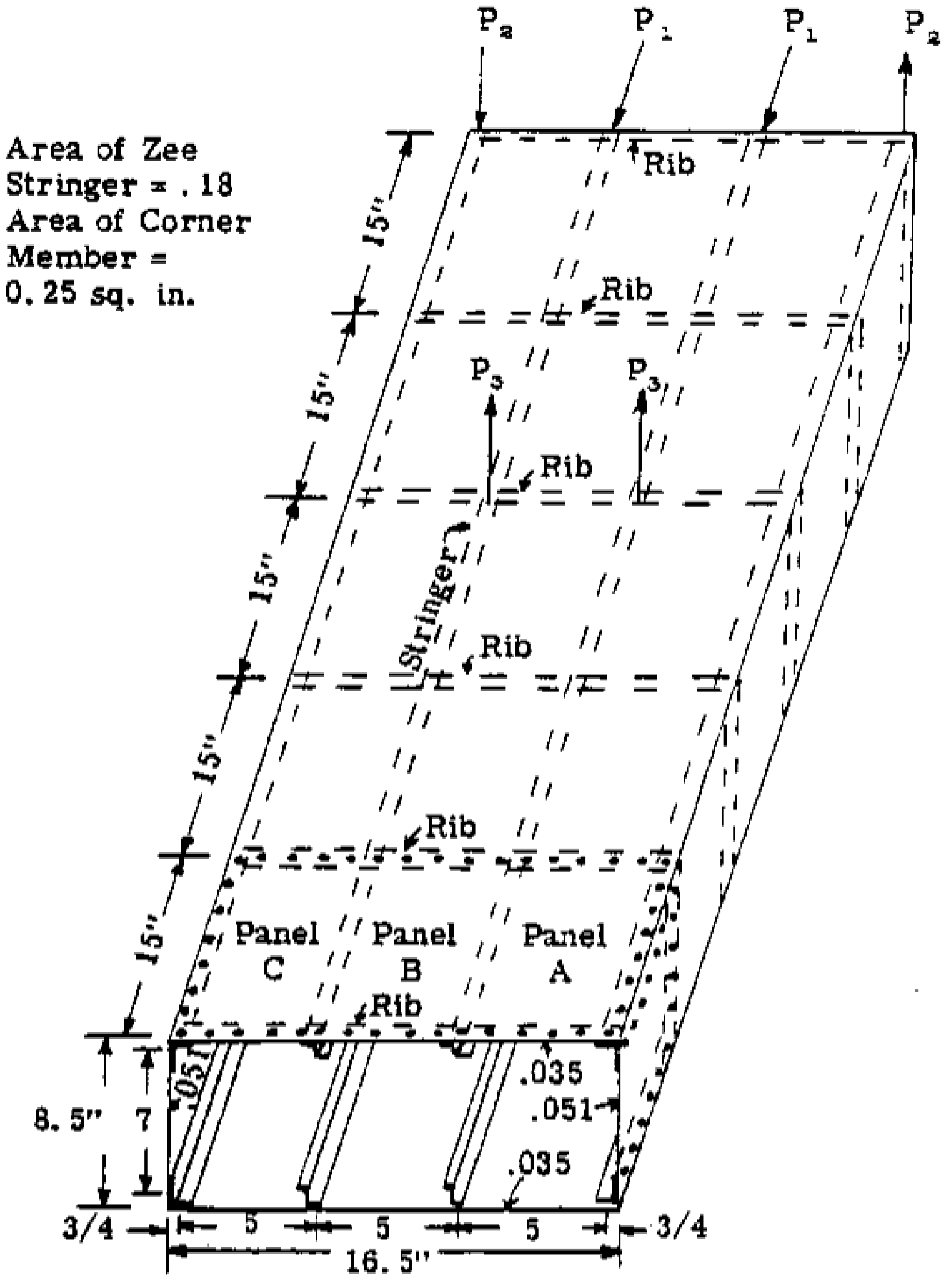


Fig. C5.20

Example

Panel (C) is less critical because the flexural shear is acting opposite to the torsional shear stress, thus f_s total = 845 - 163 = 682 psi. The R_s = 682/2760 = .246.

$R_c + R_s^2 = .905 + .246^2 = .966$. Since the result is less than 1.0, panel will not buckle.

Panel (B) carries a small shear flow, being zero at center of panel and increasing uniformly to 1.5 lb. per inch at the edges, and flowing in opposite directions from the centerline. Thus transverse shear will have negligible effect. Thus R_s = 854/2760 = .309.

$R_c + R_s^2 = .905 + .309^2 = 1.00$, or panel (B) is on the verge of buckling under the assumptions made in the solution.

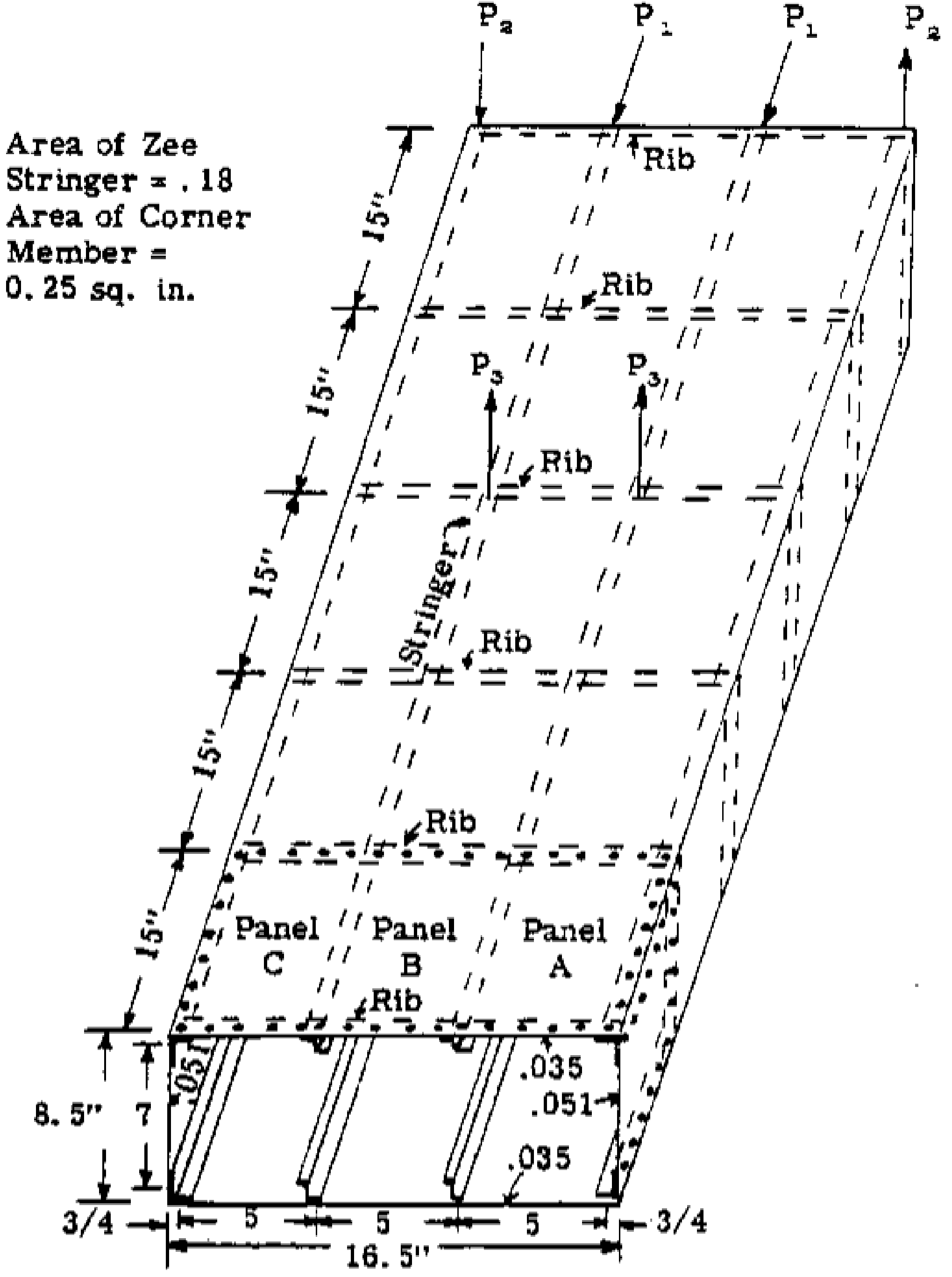


Fig. C5.20

Review sheet 2 based on Excel solution