Airframe Design and Construction

Section properties - revision

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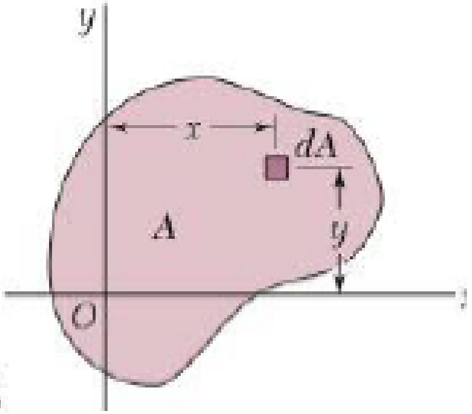
First Moment of area

Consider an area A located in the xy plane (Fig. A.1). Denoting by x and y the coordinates of an element of area dA, we define the *first moment of the area A with respect to the x axis* as the integral

$$Q_x = \int_A y\,dA$$



Similarly, the *first moment of the area A with respect to the y axis* is defined as the integral

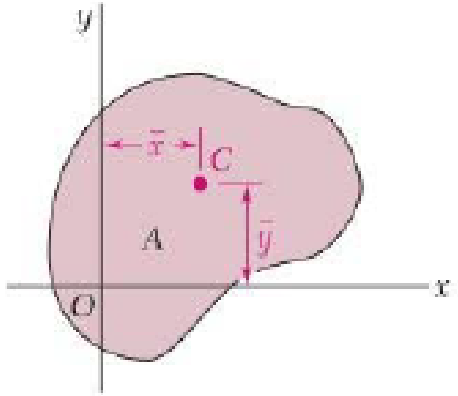
$$Q_y = \int_A x\,dA$$

First moment of area units is $[m^3]$ in SI system and $[in^3\ or ft^3]$ in U.S system.

Centroid of an area

The *centroid of the area A* is defined as the point C of coordinates $\bar{x}$ and $\bar{y}$ (Fig. A.2), which satisfy the relations

$$\int_A x\,dA = A\bar{x} \qquad \int_A y\,dA = A\bar{y}$$



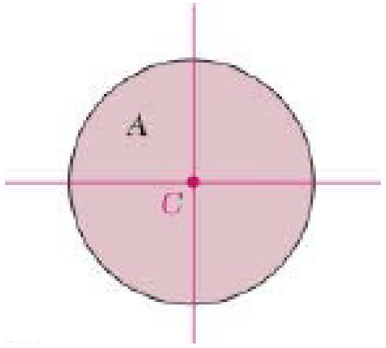
Then the centroid position can be calculated from the relation

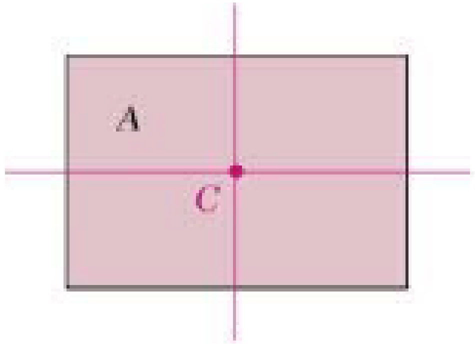
$$\bar{x} = \frac{\int x\,dA}{A} = \frac{\sum_{i=1}^{N} x_i A_i}{\sum_{i=1}^{N} A_i} \qquad \bar{y} = \frac{\int y\,dA}{A} = \frac{\sum_{i=1}^{N} y_i A_i}{\sum_{i=1}^{N} A_i}$$

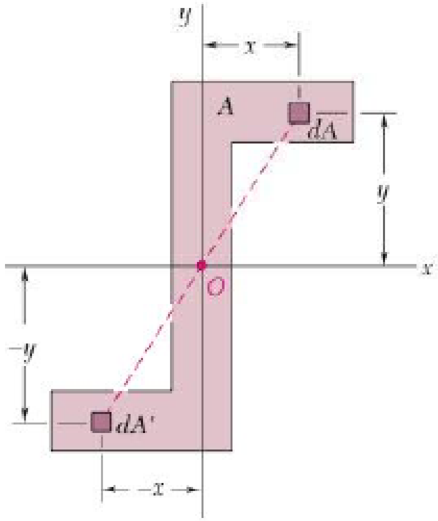
Symmetric sections

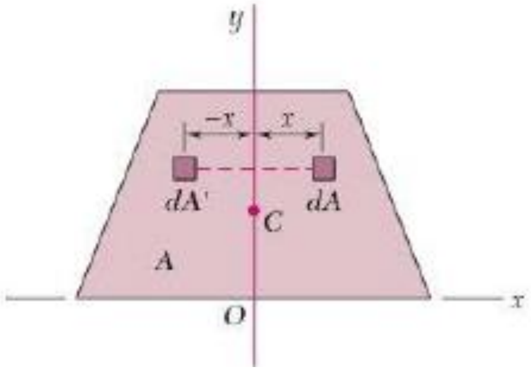
If an area A Possesses an axis of symmetry, its centroid C is located on that axis. Because the first moment of area will vanish (i.e. $Q_x = 0, or Q_y = 0$).

And if an area possesses a center of symmetry O, the first moment of area about any axis through O is zero.







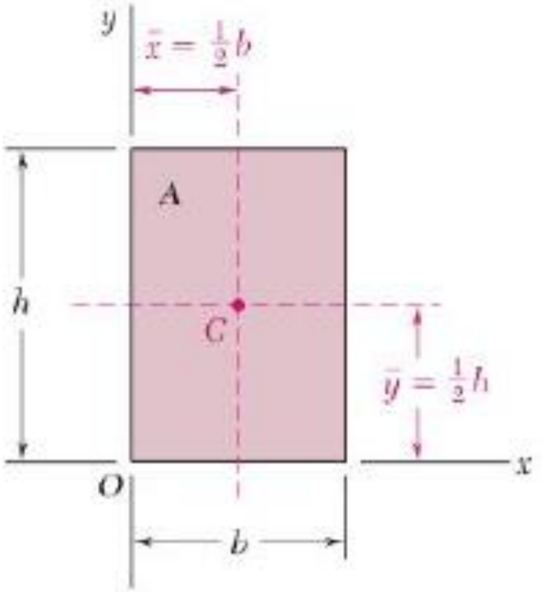


Symmetric sections

If the centroid is located by symmetry, then the first moment of area with respect to any axis can directly be obtained using the relations.

$$Q_x = A\bar{y} = (bh)(\tfrac{1}{2}h) = \tfrac{1}{2}bh^2$$

$$Q_y = A\bar{x} = (bh)(\tfrac{1}{2}b) = \tfrac{1}{2}b^2h$$



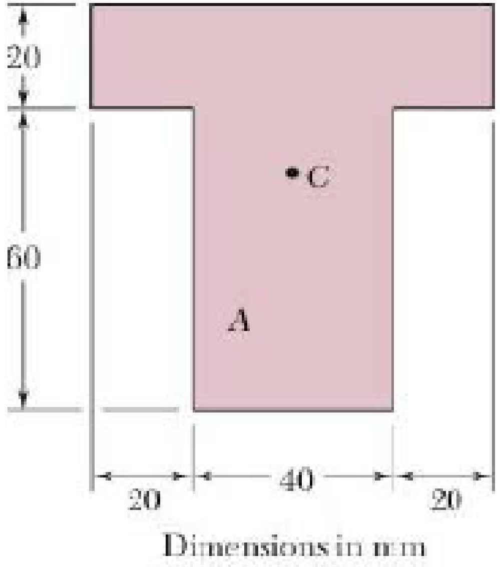
Example 1 – T section

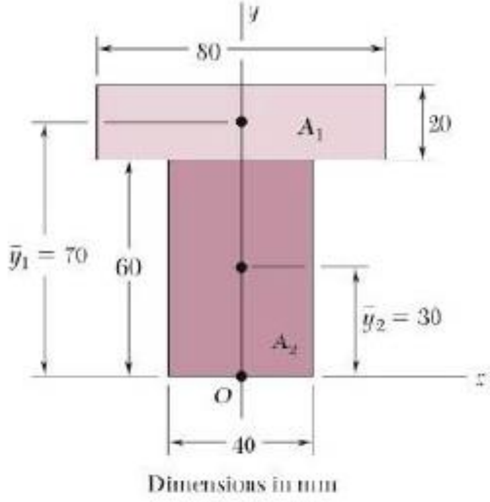
Selecting the coordinate axes shown in Fig. A.11, we note that the centroid C must be located on the y axis, since this axis is an axis of symmetry; thus, $\overline{X} = 0$.

Dividing A into its component parts A_1 and A_2, we use the second of Eqs. (A.6) to determine the ordinate $\overline{Y}$ of the centroid. The actual computation is best carried out in tabular form.

	Area, mm²	$\bar{y}_i$, mm	$A_i\bar{y}_i$, mm³
A_1	$(20)(80) = 1600$	70	112×10^3
A_2	$(40)(60) = 2400$	30	72×10^3
	$\sum_i A_i = 4000$		$\sum_i A_i\bar{y}_i = 184 \times 10^3$

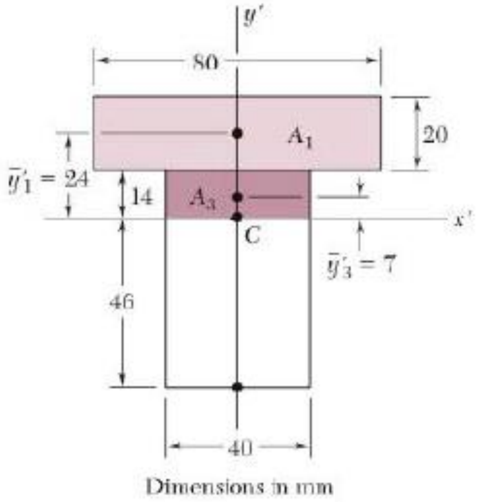
$$\overline{Y} = \frac{\sum_i A_i\bar{y}_i}{\sum_i A_i} = \frac{184 \times 10^3 \text{ mm}^3}{4 \times 10^3 \text{ mm}^2} = 46 \text{ mm}$$



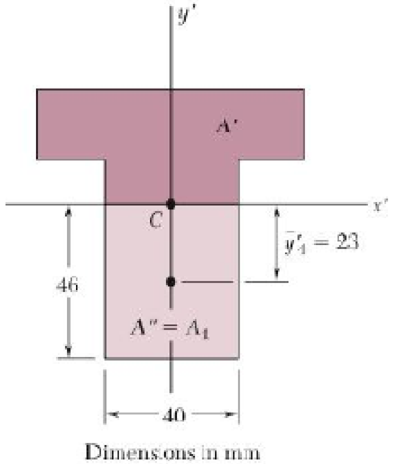


Example 1 – T section

The first moment of area can be calculated w.r.t. the axis *x'*

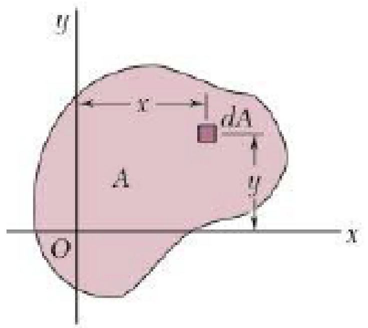


$$Q'_{x'} = A_1\bar{y}'_1 + A_3\bar{y}'_3$$
$$= (20 \times 80)(24) + (14 \times 40)(7) = 42.3 \times 10^3 \text{ mm}^3$$



Second moment of area

The second moments of area (moment of inertia) of A with respect to x-axis and y-axis are



$$I_x = \int_A y^2\, dA \qquad I_y = \int_A x^2\, dA$$

Example 2 – rectangular section

Determine the moment of inertia around the x-axis for the given rectangular section?

$$dI_x = y^2\,dA = y^2(b\,dy)$$

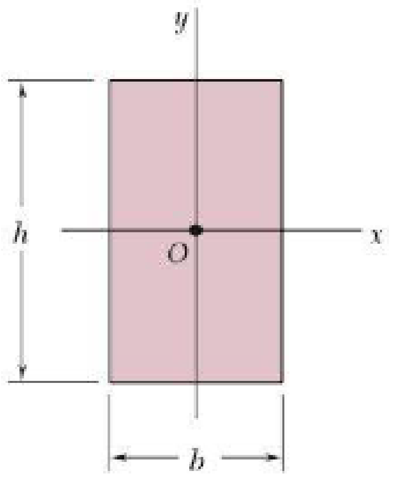
Integrating from $y = -h/2$ to $y = +h/2$, we write

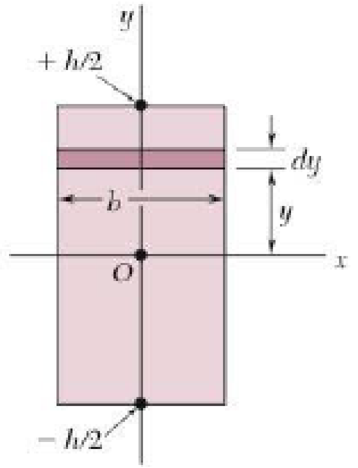
$$I_x = \int_A y^2\,dA = \int_{-h/2}^{+h/2} y^2(b\,dy) = \tfrac{1}{3}b[y^3]_{-h/2}^{+h/2}$$

$$= \tfrac{1}{3}b\left(\frac{h^3}{8} + \frac{h^3}{8}\right)$$

or

$$I_x = \tfrac{1}{12}bh^3$$





Parallel-axis theorem

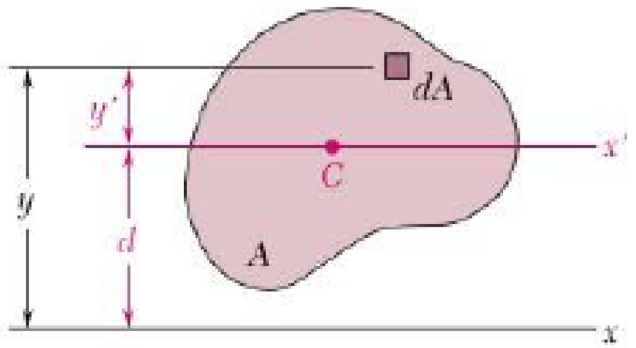
Consider the moment of inertia I_x of an area A with respect to an arbitrary x axis (Fig. A.20). Denoting by y the distance from an element of area dA to that axis, we recall from Sec. A.3 that

$$I_x = \int_A y^2 \, dA$$

Let us now draw the *centroidal x′ axis*, i.e., the axis parallel to the x axis which passes through the centroid C of the area. Denoting by y' the distance from the element dA to that axis, we write $y = y' + d$, where d is the distance between the two axes. Substituting for y in the integral representing I_x, we write

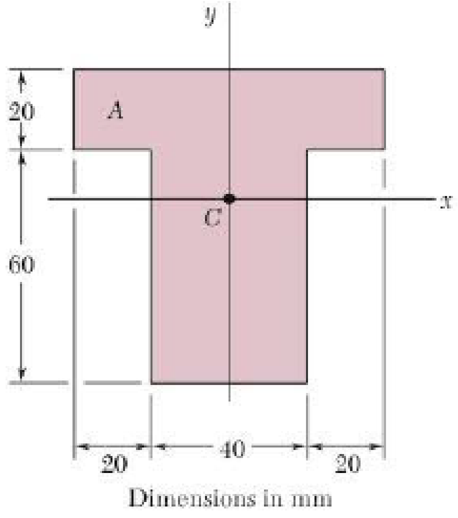
$$I_x = \int_A y^2 \, dA = \int_A (y' + d)^2 dA$$

$$I_x = \int_A y'^2 \, dA + 2d \int_A y' \, dA + d^2 \int_A dA$$



$$I_x = \bar{I}_{x'} + Ad^2$$

Example 3 – T section



Rectangular Area A_1.

$$(\bar{I}_{x'})_1 = \tfrac{1}{12}bh^3 = \tfrac{1}{12}(80\text{ mm})(20\text{ mm})^3 = 53.3 \times 10^3\text{ mm}^4$$

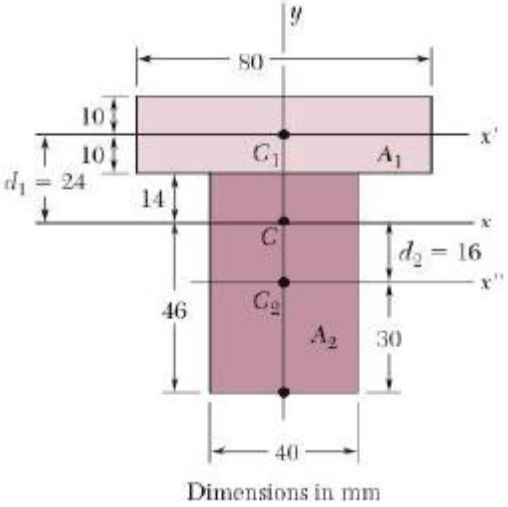
$$(I_x)_1 = (\bar{I}_{x'})_1 + A_1 d_1^2 = 53.3 \times 10^3 + (80 \times 20)(24)^2$$
$$= 975 \times 10^3\text{ mm}^4$$

Rectangular Area A_2.

$$(\bar{I}_{x''})_2 = \tfrac{1}{12}bh^3 = \tfrac{1}{12}(40)(60)^3 = 720 \times 10^3\text{ mm}^4$$
$$(I_x)_2 = (\bar{I}_{x''})_2 + A_2 d_2^2 = 720 \times 10^3 + (40 \times 60)(16)^2$$
$$= 1334 \times 10^3\text{ mm}^4$$

Entire Area A.

$$\bar{I}_x = (I_x)_1 + (I_x)_2 = 975 \times 10^3 + 1334 \times 10^3$$
$$\bar{I}_x = 2.31 \times 10^6\text{ mm}^4$$



Wing section properties

1	2	3	4	5	6	7	8
Stringer Number	Stringer Area Plus Effective Skin	Stringer Effectiveness Factor	Effective Area (A)	z'	Az'	z=z' - $\bar{z}$	Az²
1	0.37	1.0	0.370	4.50	1.664	5.47	11.05
2	0.185	0.808	0.149	4.60	0.685	5.57	4.62
3	0.185	0.808	0.149	4.60	0.685	5.57	4.62
4	0.185	0.808	0.149	4.60	0.685	5.57	4.62
5	0.370	1.0	0.370	4.50	1.664	5.47	11.05
6	0.417	1.0	0.417	-4.60	-1.920	-3.63	5.50
7	0.320	1.0	0.320	-4.63	-1.480	-3.66	4.28
8	0.320	1.0	0.320	-4.63	-1.480	-3.66	4.28
9	0.320	1.0	0.320	-4.63	-1.480	-3.66	4.28
10	0.417	1.0	0.417	-4.63	-1.920	-3.63	5.50
		Σ	2.981		-2.897	I_x =	59.80

z' = distance from ℄ x axis to centroid of stringer area

$\bar{z} = \Sigma Az'/\Sigma A = -2.897/2.981 = -.97$ in.