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Different Estimators for Stochastic Parameter Panel Data Models with Serially Correlated Errors

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Abstract: This paper considers stochastic parameter panel data models when the errors are first-order serially correlated. The feasible generalized least squares (FGLS) and simple mean group (SMG) estimators for these models have been reviewed and examined. The efficiency comparisons for these estimators have been carried when the regression parameters are stochastic, non-stochastic, and mixed-stochastic. Monte Carlo simulation study and a real data application are given to evaluate the performance of FGLS and SMG estimators. The results indicate that, in small samples, SMG estimator is more reliable in most situations than FGLS estimators, especially when the model includes one or more non-stochastic parameter.

Keywords: Feasible generalized least squares estimator, First-order serial correlation, Mixed-stochastic parameter regression model, Simple mean group estimator

1 Introduction

In classical panel data models, there is an important assumption is that the individuals in our database are drawn from a population with a common regression parameter vector. In other words, the parameters of a classical panel data model must be non-stochastic. In particular, this assumption is not satisfied in most economic models, see, e.g., [1,2]. In this paper, panel data models are studied when this assumption is relaxed. In this case, the model is called stochastic parameter regression (SPR) model. This model has been examined by Swamy in several publications (Swamy [3,4,5]), and [6,7, 8,9,10,11,12]. Some statistical and econometric publications refer to this model as Swamy's model, see, e.g., Poi [13], Abonazel [14,15], and Elhorst [16].

Practically, the SPR models have been used in several fields, especially in finance and economics, e.g., Feige and Swamy [17] used this model to estimate demand equations for liquid assets, while Boness and Frankfurter [18] applied it to examine the concept of risk-classes in finance. Recently, Westerlund and Narayan [19] used the stochastic parameter approach to predict the stock returns at the New York Stock Exchange.

In classical SPR model, Swamy [3] assumed that the individuals of the dataset are drawn from a population has a common regression parameter, which is a constant component, and another stochastic component, that will allow the parameters to differ from unit to unit. This model has been developed in many papers, e.g., Anh and Chelliah [20], Murtazashvili and Wooldridge [21], and Hsiao and Pesaran [22].

The main objective of this paper is to provide the researcher with some guidelines on how to select the appropriate estimator for panel data models, in the case of small samples, when the errors are first-order serially correlated as well as with stochastic or mixed-stochastic regression parameters. To achieve this objective, we will discuss and examine the performance of different estimators in the case of small samples.

The rest of the paper is organized as follows. Section 2 provides generalized least squares (GLS) estimators for stochastic parameter model with serially correlated errors. In section 3, we present an appropriate estimator for mixed-stochastic parameter model. The feasible versions of GLS (FGLS) estimators have been suggested in section 4. While in section 5, simple mean group (SMG) estimator has been discussed. Section 6 contains the Monte Carlo simulation study.

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A real data set has been used to examine the behavior of the estimators in section 7. Finally, section 8 offers the concluding remarks.

2 Stochastic Parameter Model

Let there be observations for N cross-sectional units over T time periods. Suppose the variable y for the ith unit at time t is specified as a linear function of K strictly independent variables, x_{kit}, in the following form:

$$y_{it} = \sum_{k=1}^{K} \beta_{ki} x_{kit} + u_{it} = \mathrm{x}_{it}\beta_i + u_{it}, \quad i = 1, \ldots, N; t = 1, \ldots, T, \tag{1}$$

where u_{it} is the random error term, x_{it} is a $1 \times K$ vector of independent variables, and β_i is the $K \times 1$ vector of regression parameters. If the performance of one individual from the database is of interest, separate equation regressions can be estimated for each individual unit and then rewrite the model in (1) as:

$$y_i = X_i \beta_i + u_i; \quad i = 1, \ldots, N, \tag{2}$$

where $y_i = (y_{i1}, \ldots, y_{iT})'$, $X_i = (\mathrm{x}'_{i1}, \ldots, \mathrm{x}'_{iT})'$, $\beta_i = (\beta_{i1}, \ldots, \beta_{iK})'$, and $u_i = (u_{i1}, \ldots, u_{iT})'$.In this paper, we assume that the model in (1) or (2) under the following assumptions:

Assumption 1: The errors have zero mean, i.e., $E(u_i) = 0;\ \forall\, i = 1, \ldots, N$.

Assumption 2: The independent variables are non-stochastic (in repeated samples), and then assume independent with other variables in the model. And the value of $rank(X_i'X_i) = K;\ \forall\ i = 1, \ldots, N$, where $K < T, N$.

Assumption 3: The errors have a constant variance for each individual but they are cross-sectional heteroscedasticity as well as they are first-order serially correlated: $u_{it} = \phi_i u_{i,t-1} + \varepsilon_{it}$; $|\phi_i| < 1$, where ϕ_i for $i = 1, \ldots, N$ are first-order serial correlation coefficients and are fixed. Where $E(\varepsilon_{it}) = 0$, $E(u_{i,t-1}\varepsilon_{jt}) = 0$; $\forall\, i,\ j$, and t. And

$$E(\varepsilon_{it}\varepsilon_{js}) = \begin{cases} \sigma^2_{\varepsilon_i} & if\ t = s;\ i = j \\ 0 & otherwise \end{cases} \quad i, j = 1, \ldots, N; t, s = 1, \ldots, T,$$

it is assumed that in the initial time period the errors have the same properties as in subsequent periods. So, we assume that: $E\left(u^2_{i0}\right) = \sigma^2_{\varepsilon_i}/1 - \phi^2_i;\ \forall\, i$.

Assumption 4: The vector of regression parameters is specified as: $\beta_i = \bar{\beta} + \theta_i$, where $\bar{\beta} = \left(\bar{\beta}_1, \ldots, \bar{\beta}_K\right)'$ is a vector of non-stochastic parameter and $\theta_i = (\theta_{i1}, \ldots, \theta_{iK})'$ is a vector of random variables with:

$$E\left(\theta_i\theta_j'\right) = \begin{cases} \gamma^* & if\ i = j \\ 0 & if\ i \neq j \end{cases} \quad i, j = 1, \ldots, N;\ k = 1, .., K,$$

where γ^* is a K diagonal matrix, also assume that $E(\theta_i u_{jt}) = 0\ \forall\, i$ and j.

Using assumption 4, the model in (2) can be rewritten as: $Y = X\bar{\beta} + Z\theta + u$; where $Y = (y_1', \ldots, y_N')'$, $X = (X_1', \ldots, X_N')'$, $u = (u_1', \ldots, u_N')'$, $\theta = (\theta_1', \ldots, \theta_N')'$, for $i = 1, \ldots, N$. Under assumptions 1 to 4, the best linear unbiased estimator (BLUE) of $\bar{\beta}$ and the variance-covariance matrix of it are:

$$\hat{\bar{\beta}}_{SPR-SC} = \left(X'\Lambda^{*-1}X\right)^{-1} X'\Lambda^{*-1}Y;\ var\left(\hat{\bar{\beta}}_{SPR-SC}\right) = \left(X'\Lambda^{*-1}X\right)^{-1}, \tag{3}$$

where $\Lambda^* = V + Z(I_N \otimes \gamma^*)Z'$, with

$$V = \begin{pmatrix} \sigma^2_{\varepsilon_1}\Omega_{11} & 0 & \cdots & 0 \\ 0 & \sigma^2_{\varepsilon_2}\Omega_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma^2_{\varepsilon_N}\Omega_{NN} \end{pmatrix},$$

and

$$\gamma^* = \left[\frac{1}{N-1}\left(\sum_{i=1}^{N}\beta_i^*\beta_i^{*\prime} - \frac{1}{N}\sum_{i=1}^{N}\beta_i^*\sum_{i=1}^{N}\beta_i^{*\prime}\right)\right] - \frac{1}{N}\sum_{i=1}^{N}\sigma_{\varepsilon_i}^2\left(X_i'\Omega_{ii}^{-1}X_i\right)^{-1},$$

where $\beta_i^* = \left(X_i'\Omega_{ii}^{-1}X_i\right)^{-1}X_i'\Omega_{ii}^{-1}y_i$, with

$$\Omega_{ii} = \frac{1}{1-\phi_i^2}\begin{pmatrix} 1 & \phi_i & \phi_i^2 & \cdots & \phi_i^{T-1} \\ \phi_i & 1 & \phi_i & \cdots & \phi_i^{T-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_i^{T-1} & \phi_i^{T-2} & \phi_i^{T-3} & \cdots & 1 \end{pmatrix}.$$

Remark 1: Non-stochastic parameter model with serial correlation
In non-stochastic parameter model, the errors are cross-sectional heteroscedasticity as well as they are first-order serially correlated. However, the individuals are drawn from a population with a common regression parameter vector β, i.e., $\beta_1 = \cdots = \beta_N = \beta$. Therefore the BLUE of β, under assumptions 1 to 3, is:

$$\hat{\hat{\beta}}_{PLS-SC} = \left(X'V^{-1}X\right)^{-1}\left(X'V^{-1}Y\right),$$

this estimator has been termed pooled least squares with serial correlation (PLS-SC) estimator.

Remark 2: Standard stochastic parameter (Swamy's) model
In standard stochastic parameter model that presented by Swamy [3], he assumed that the errors are cross-sectional heteroscedasticity and they are serially independently. As for the parameters, he assumed the same conditions in assumption 4. Therefore, the BLUE of $\bar{\beta}$, under Swamy's [3] assumptions, is:

$$\hat{\hat{\bar{\beta}}}_{SPR} = \left(X'\Lambda^{-1}X\right)^{-1}X'\Lambda^{-1}Y,$$

where $\Lambda = (\Sigma_H \otimes I_T) + Z(I_N \otimes \gamma)Z'$, with $\Sigma_H = diag\left\{\sigma_i^2\right\}$; for $i = 1,\ldots,N$, $\sigma_i^2 = var(u_i)$, and γ in this estimator is equal γ^* under $\Omega_{ii} = I_T$; for $i = 1,\ldots,N$:

$$\gamma = \left[\frac{1}{N-1}\left(\sum_{i=1}^{N}\beta_i\beta_i' - \frac{1}{N}\sum_{i=1}^{N}\beta_i\sum_{i=1}^{N}\beta_i'\right)\right] - \left[\frac{1}{N}\sum_{i=1}^{N}\sigma_i^2\left(X_i'X_i\right)^{-1}\right].$$

The efficiency gains from the use of SPR-SC estimator will be explained in the following lemma.

lemma 1.
If assumptions 1 to 4 are satisfied, and Λ, V and Λ^* are known, we get:

i. The PLS and SPR are unbiased estimators and have the following variance-covariance matrices:

$$var\left(\hat{\hat{\bar{\beta}}}_{PLS-SC}\right) = F_1\Lambda^*F_1';\;\; F_1 = \left(X'V^{-1}X\right)^{-1}X'V^{-1}, \tag{4}$$

$$var\left(\hat{\hat{\bar{\beta}}}_{SPR}\right) = F_2\Lambda^*F_2';\;\; F_2 = \left(X'\Lambda^{-1}X\right)^{-1}X'\Lambda^{-1}. \tag{5}$$

ii. The efficiency gains from the use of SPR-SC estimator (let $F_0 = \left(X'\Lambda^{*-1}X\right)^{-1}X'\Lambda^{*-1}$):

$$EG_{PLS-SC} = var\left(\hat{\hat{\bar{\beta}}}_{PLS-SC}\right) - var\left(\hat{\hat{\bar{\beta}}}_{SPR-SC}\right) = (F_1 - F_0)\Lambda^*(F_1 - F_0)',$$

$$EG_{SPR} = var\left(\hat{\hat{\bar{\beta}}}_{SPR}\right) - var\left(\hat{\hat{\bar{\beta}}}_{SPR-SC}\right) = (F_2 - F_0)\Lambda^*(F_2 - F_0)'.$$

From Lemma 1, we can conclude that the SPR-SC estimator is more efficient than PLS-SC and SPR estimators because EG_{PLS-SC} and EG_{SPR} matrices are positive semi-definite matrices. And these efficiency gains given in Lemma 1 are increasing when $|\phi_i|$ and/or the variances values of the parameters are increasing. However, these efficiency gains may be not achieved in practice because the estimated matrices of Λ and Λ^* are not consistently positive definite matrices, especially in small samples, as explained below in Remark 3.

3 Mixed-Stochastic Parameter Model

In this section, we will present the GLS estimator for the model when the parameters are mixed; some of them are stochastic and the other is non-stochastic. In this case, the mixed model can be written as:

$$y_i = X_{1i}\beta_{1i} + X_{2i}\beta_2 + u_i = G_i\alpha_i + u_i, \qquad (6)$$

where y_i and u_i are defined in (2), $G_i = (X_{1i}, X_{2i})$ where X_{1i} and X_{2i} are $T \times K_1$ and $T \times K_2$ matrices of observations on K_1 and K_2 independent variables, respectively, and $\alpha_i = (\beta'_{1i},\ \beta'_2)'$, where β_{1i} is a $K_1 \times 1$ vector of parameters assumed to be stochastic with mean $\bar{\beta}_1$ and variance-covariance matrix $\gamma_{\beta 1}$, but β_2 is a $K_2 \times 1$ vector of parameters assumed to be non-stochastic, where $K_1 + K_2 = K$.

The model in (6) applies to each of N cross-sections. Under suppose that $\beta_{1i} = \bar{\beta}_1 + \theta_{\beta 1}$, these N individual equations can be combined as:

$$Y = G\bar{\alpha} + \tau,$$

where $G = (G'_1,\ \ldots,\ G'_N)'$, $\bar{\alpha} = (\bar{\beta}'_1,\ \beta'_2)'$, and $\tau = (\tau'_1,\ \ldots,\ \tau'_N)'$; $\tau_i = X_{1i}\theta_{\beta 1} + u_i$.

Under Swamy's [3] assumptions, this model has been examined by Swamy [23] and Rosenberg [24]. However, in this paper, we examine this model under our assumptions (1 to 4), therefore the variance-covariance matrix of τ is:

$$E\left(\tau\,\tau'\right) = V + Z_{\beta_1}\left(I_N \otimes \gamma_{\beta_1}\right) Z'_{\beta_1} = \Pi,$$

where $Z_{\beta_1} = diag\left\{X_{1i}\right\}$. The GLS estimator of $\bar{\alpha}$ is:

$$\hat{\bar{\alpha}}_{MSPR-SC} = \left(G'\Pi^{-1}G\right)^{-1} G'\Pi^{-1}Y = \begin{pmatrix} X'_1\Pi^{-1}X_1 & X'_1\Pi^{-1}X_2 \\ X'_2\Pi^{-1}X_1 & X'_2\Pi^{-1}X_2 \end{pmatrix}^{-1} \begin{pmatrix} X'_1\Pi^{-1}Y \\ X'_2\Pi^{-1}Y \end{pmatrix},$$

where $X_1 = (X'_{11},\ \ldots,\ X'_{1N})'$ and $X_2 = (X'_{21},\ \ldots,\ X'_{2N})'$.

It is worth noting that, the mixed model is a special case of the stochastic model when the variances of certain parameters are assumed to be equal to zero.

4 FGLS Estimators and Negative Variance Problem

Since SPR-SC, SPR, PLS-SC, and MSPR-SC estimators still involve the unknown parameters (variance-covariance matrices), therefore it needs to estimate the elements of these matrices to make these estimators feasible. For SPR-SC estimator, we suggest using the following consistent estimators for ϕ_i and $\sigma^2_{\varepsilon_i}$:

$$\hat{\phi}_i = \frac{\sum_{t=2}^{T} \hat{u}_{it}\hat{u}_{i,t-1}}{\sum_{t=2}^{T} \hat{u}^2_{i,t-1}};\ \hat{\sigma}^2_{\varepsilon_i} = \frac{\hat{\varepsilon}'_i\hat{\varepsilon}_i}{T-K}, \qquad (7)$$

where $\hat{u}_i = (\hat{u}_{i1},\ \ldots, \hat{u}_{iT})' = y_i - X_i\hat{\beta}_i$; $\hat{\beta}_i = (X'_iX_i)^{-1}X'_iy_i$, while $\hat{\varepsilon}_i = (\hat{\varepsilon}_{i1}, \hat{\varepsilon}_{i2}, \ldots,\ \hat{\varepsilon}_{iT})'$; $\hat{\varepsilon}_{i1} = \hat{u}_{i1}\sqrt{1-\hat{\phi}^2_i}$ and $\hat{\varepsilon}_{it} = \hat{u}_{it} - \hat{\phi}_i\hat{u}_{i,t-1}$ for $t = 2, \ldots, T$.[1]

By replacing ϕ_i by $\hat{\phi}_i$ in Ω_{ii} matrix, we get consistent estimators of Ω_{ii}, say $\hat{\Omega}_{ii}$. And we will use $\hat{\sigma}^2_{\varepsilon_i}$ and $\hat{\Omega}_{ii}$ to get consistent estimators of V and γ^*, say $\hat{V}$and $\hat{\gamma}^*$. By using consistent estimators ($\hat{\sigma}^2_{\varepsilon_i}, \hat{\Omega}_{ii}$, and $\hat{\gamma}^*$), we have a consistent estimator of Λ^*, say $\hat{\Lambda}^*$. And then use $\hat{\Lambda}^*$ to get a feasible estimator of SPR-SC. Summarily, using $\hat{V}$ that defined above lead to get feasible PLS-SC estimator.

For SPR estimator, Swamy [23] used the following unbiased and consistent estimator for σ^2_i: $\hat{\sigma}^2_i = \hat{u}'_i\hat{u}_i/T-K$; where $\hat{u}_i$ is defined in (7). While for MSPR-SC estimator, we suggest use of the consistent estimator of Π (say $\hat{\Pi}$) that proposed by Abonazel [25] to get the feasible estimator for it.

Remark 3: Negative variance estimates problem

Just as in the error-components model, the estimate values of γ^* and γ are not necessarily non-negative definite. So, we expect to get the negative values of the estimated variances of SPRSC and SPR estimators. To avoid this problem, it can use one of the following proposed estimators:[2]

[1] The estimator of ϕ_i in (7) is consistent, but it is not unbiased. See Srivastava and Giles [26] for other suitable consistent estimators of it that are often used in practice.

[2] These suggestions ware been proposed to correct the negative variance estimation in stochastic parameter (Swamy's) model, but we generalized these suggestions for stochastic parameter model under assumption 3.

i. The first proposed estimator (Swamy [3]):[3]

$$\hat{\gamma}^{*+}_{(1)} = \frac{1}{N-1}\left(\sum_{i=1}^{N}\hat{\beta}_i^{*}\hat{\beta}_i^{*\prime} - \frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_i^{*}\sum_{i=1}^{N}\hat{\beta}_i^{*\prime}\right).$$

ii. The second proposed estimator (Havenner and Swamy [28]):

$$\hat{\gamma}^{*+}_{(2)} = \begin{cases} \hat{\gamma}^{*} & if\ \hat{\gamma}^{*}\ is\ positive\ definite, \\ \hat{\gamma}^{*} + \left(-\hat{\lambda}_{min} + \nu\right) \mathrm{I}_K & otherwise, \end{cases}$$

where $\hat{\lambda}_{min}$ is the smallest eigenvalue of $\hat{\gamma}^{*}$ and $\nu > 0$ is a small constant number.

Abonazel [14] and Mousa et al. [12] showed that the first proposed estimator by Swamy [3] may be suitable in case of moderate or large samples $(T \geq 20)$, but it is not suitable for small samples $(T < 20)$. Therefore, in this paper, we select the second proposed estimator as a corrected estimator for γ^{*}.

5 Mean Group Estimator

Abonazel [15,25] proposed use the SMG estimator as an alternative estimator for stochastic regression models, in general, it is defined as:

$$\hat{\bar{\beta}}_{SMG} = \frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_i.$$

Note that this estimator is the simple average of ordinary least squares estimators $(\hat{\beta}_i)$. The SMG estimator is also used by Pesaran and Smith [29] for estimation of dynamic panel data (DPD) models with stochastic parameters.[4]

It is easy to verify that SMG estimator is consistent of β when both $N,\ T \to \infty$. Moreover, Abonazel [25] showed the statistical properties of SMG estimator that will be displayed in the following lemma.

lemma 2.
If assumptions 1 to 4 are satisfied and $\underset{T\to\infty}{plim}\, T^{-1}X_i^{\prime}X_i,\ \underset{T\to\infty}{plim}\, T^{-1}X_i^{\prime}\hat{\Omega}_{ii}^{-1}X_i$ are finite and positive definite for all i, we get:

i. The SMG is unbiased estimator of $\bar{\beta}$ and the consistent estimator of the variance-covariance matrix of SMG is:

$$\widehat{var}\left(\hat{\bar{\beta}}_{SMG}\right) = \frac{1}{N}\hat{\gamma}^{*} + \frac{1}{N^2}\sum_{i=1}^{N}\hat{\sigma}^2_{\varepsilon_i}\left(X_i^{\prime}X_i\right)^{-1}X_i^{\prime}\hat{\Omega}_{ii}X_i\left(X_i^{\prime}X_i\right)^{-1}. \tag{8}$$

ii. The estimated asymptotic variance-covariance matrices of SPR-SC, SPR, and SMG estimators are:

$$\underset{T\to\infty}{plim}\,\widehat{var}\left(\hat{\bar{\beta}}_{SPR-SC}\right) = \underset{T\to\infty}{plim}\,\widehat{var}\left(\hat{\bar{\beta}}_{SPR}\right) = \underset{T\to\infty}{plim}\,\widehat{var}\left(\hat{\bar{\beta}}_{SMG}\right) = \frac{1}{N}\gamma^{+}.$$

From lemma 2, we can conclude that the means and the variance-covariance matrices of the limiting distributions of SPR-SC, SPR, and SMG estimators are the same and are equal to $\bar{\beta}$ and $\frac{1}{N}\gamma^{+}$ respectively even if the errors are correlated as in assumption 3. Therefore, it is not expected to increase the asymptotic efficiency of SPR-SC about SPR and SMG. But in small samples, the efficiency of these estimators will be examined by the following Monte Carlo simulation study.

[3] Judge et al. [27] and Abonazel [15] showed that the resulted estimator using this suggestion is consistent when $T \to \infty$. Moreover, this suggestion used by Stata software for the estimation of stochastic parameter (Swamy's) model, specifically in *xtrchh* and *xtrchh2* Statas commands. See Poi [13].

[4] For more information about the estimation methods for DPD models, see, e.g., Baltagi [30], Hsiao [31], Abonazel [32], Youssef et al. [33,34], and Youssef and Abonazel [35].

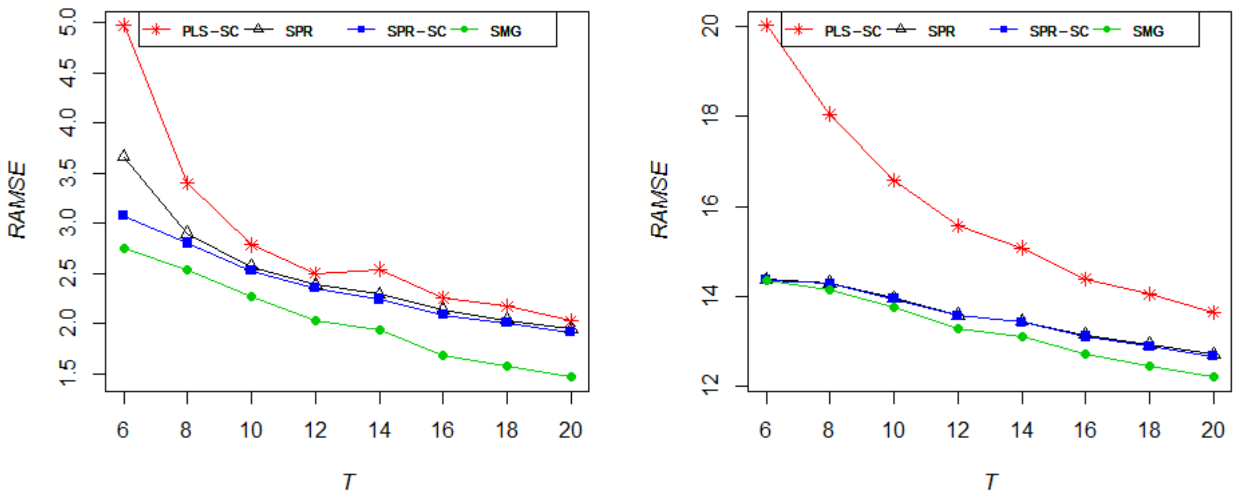

Fig. 1: Square root of AMSE for different estimators when $N = 10, \sigma_\varepsilon = 15$, and the parameters distributed student-t.

Table 1: Simulation factors.

No.	Factor	Levels
1.	Cross-sectional units (N)	$N = 5$ or 10
2.	Time periods (T)	$T = 6, 8, , 20$
3.	Standard deviation of errors (σ_ε)	$\sigma_\varepsilon = 1$ or 15
4.	First-order serial correlation coefficient ($\phi_i = \phi$)	$\phi = .45$ or $.95$
5.	Stochastic component of regression parameters $\theta_i = (\theta_{0i}, \theta_{1i})'$	I. Non-stochastic model: $\theta_{0i} = \theta_{1i} = \dot{\theta} = 0$ II. Stochastic model: $\theta_{0i} = \theta_{1i} = \dot{\theta}$; $\dot{\theta} \sim N(0,30)$, $\dot{\theta} \sim t(10)$, or $\dot{\theta} \sim U(-10,10)$ III. Mixed-stochastic model: type I: $\theta_{0i} \sim N(0,30)$, $\theta_{1i} = 0$ type II: $\theta_{0i} = 0$, $\theta_{1i} \sim N(0,30)$

6 Monte Carlo Simulation Study

In this section, we will make Monte Carlo simulation study to examine the performance of pooled least squares (PLS-SC), simple mean group (SMG), and stochastic parameter (SPR, SPR-SC, and MSPR-SC) estimators in small samples. The programs to set up the Monte Carlo simulation study, written in R language, are available upon request.[5] Monte Carlo experiments were carried out based on the following data generating process:

$$y_{it} = \beta_{0i} + \beta_{1i} x_{1it} + u_{it} = \mathrm{x}_{it} \bar{\beta} + \mathrm{x}_{it} \theta_i + u_{it}, \quad i = 1, \ldots, N;\ t = 1, \ldots, T, \tag{9}$$

where $\mathrm{x}_{it} = (1, x_{1it})$, $\bar{\beta} = (\bar{\beta}_0, \bar{\beta}_1)'$, and $\theta_i = (\theta_{0i}, \theta_{1i})'$. In this study, the values of the independent variable, x_{1it}, in (9) were generated as independent normally distributed random variable with mean 1 and standard deviation 5. The values of x_{1it} were allowed to differ for each cross-sectional unit. However, once generated for all N cross-sectional units the values were held fixed over all Monte Carlo experiments. For all experiments, we ran $L = 3000$ replications and all the results of all separate experiments are obtained by precisely the same series of random numbers. The parameters, β_{0i} and β_{1i}, were generated as in assumption 4: $\beta_i = (\beta_{0i}, \beta_{1i})' = \bar{\beta} + \theta_i$, where the vector of $\bar{\beta} = (10, 10)'$, and θ_i were generated from three different distributions (normal, student-t, and uniform).

To compare small samples performance for the different estimators, the three different types of regression parameters (non-stochastic, stochastic, and mixed-stochastic) have been designed. And the effective simulation factors and their values, are summarized in Table 1.

[5] For information about how to create Monte Carlo simulation studies in econometric models using R, see Abonazel [36].

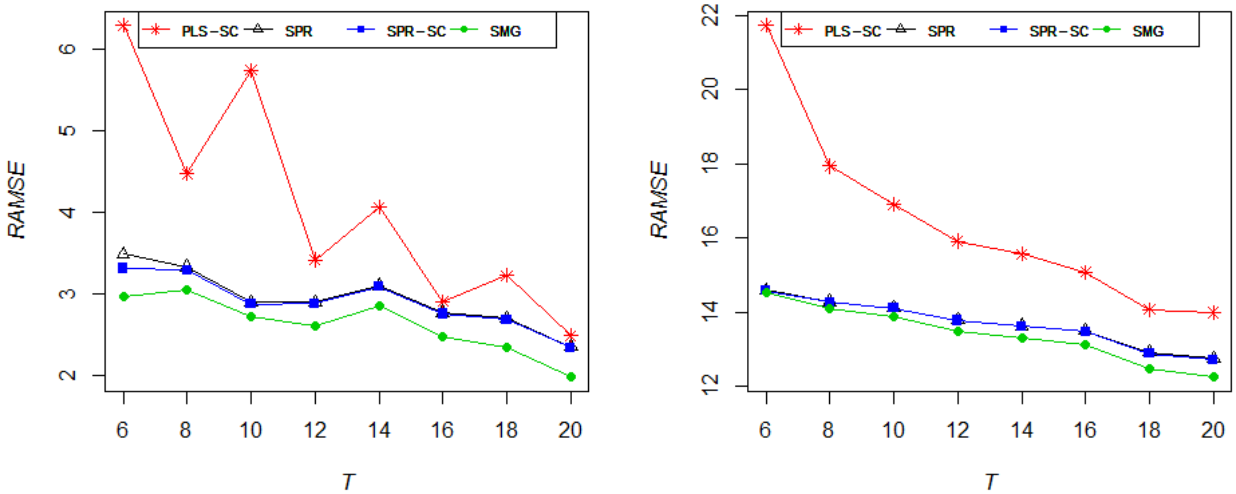

Fig. 2: Square root of AMSE for different estimators when $N = 10, \sigma_\varepsilon = 15$, and the parameters distributed uniform.

Table 2: The formulas of variances that used in the simulation study.

Model type	Results presentation	Appropriate estimator (theoretically)	Other estimators	The formula of variance
Stochastic	Table 3 & Figures 1, 2	SPR-SC		Equation (3)
			PLS-SC	Equation (4)
			SPR	Equation (5)
			SMG	Equation (8)
Non-stochastic	Table 4	PLS-SC		$\left(X'\hat{V}^{-1}X\right)^{-1}$
			SPR	$\left(X'\hat{\Lambda}^{-1}X\right)^{-1}X'\hat{\Lambda}^{-1}\hat{V}\hat{\Lambda}^{-1}X\left(X'\hat{\Lambda}^{-1}X\right)^{-1}$
			SPR-SC	$\left(X'\hat{\Lambda}^{*-1}X\right)^{-1}X'\hat{\Lambda}^{*-1}\hat{V}\hat{\Lambda}^{*-1}X\left(X'\hat{\Lambda}^{*-1}X\right)^{-1}$
			SMG	$\frac{1}{N^2}\sum_{i=1}^{N}\hat{\sigma}_{\varepsilon_i}^2\left(X_i'X_i\right)^{-1}X_i'\hat{\Omega}_{ii}X_i\left(X_i'X_i\right)^{-1}$
Mixed-stochastic	Tables 5, 6	MSPR-SC		$\left(G'\hat{\Pi}^{-1}G\right)^{-1}$
			PLS-SC	$\left(X'\hat{V}^{-1}X\right)^{-1}X'\hat{V}^{-1}\hat{\Pi}\,\hat{V}^{-1}X\left(X'\hat{V}^{-1}X\right)^{-1}$
			SPR	$\left(X'\hat{\Lambda}^{-1}X\right)^{-1}X'\hat{\Lambda}^{-1}\hat{\Pi}\,\hat{\Lambda}^{-1}X\left(X'\hat{\Lambda}^{-1}X\right)^{-1}$
			SPR-SC	$\left(X'\hat{\Lambda}^{*-1}X\right)^{-1}X'\hat{\Lambda}^{*-1}\hat{\Pi}\,\hat{\Lambda}^{*-1}X\left(X'\hat{\Lambda}^{*-1}X\right)^{-1}$
			SMG	$\frac{1}{N}\hat{\gamma}_{\beta 1}+\frac{1}{N^2}\sum_{i=1}^{N}\hat{\sigma}_{\varepsilon_i}^2\left(X_i'X_i\right)^{-1}X_i'\hat{\Omega}_{ii}X_i\left(X_i'X_i\right)^{-1}$

We calculate the average of mean square error (AMSE) for each estimator to compare between these estimators.[6] The AMSE of any estimator is calculated by:[7]

$$\text{AMSE}\,(e) = \frac{1}{2}\sum_{k=0}^{1} M.\widehat{var}\left(\tilde{\beta}_{k(e)}\right);\; M.\widehat{var}\left(\tilde{\beta}_{k(e)}\right) = \frac{1}{L}\sum_{l=1}^{L}\widehat{var}\left(\tilde{\beta}_{k(e)}\right)_l,$$

where the subscript e indicates the estimator that it calculated, i.e., $e = PLS-SC, SPR, SPR-SC, MSPR-SC$, or SMG. The different formulas of variances of estimators that used in our study are summarized in Table 2.

The simulation results are presented in Tables 3 to 6 and Figures 1 and 2. Specifically, Table 3 presents AMSE values of PLS-SC, SPR, and SMG estimates when the all regression parameters are stochastic. While the results in case

[6] In our simulation study, the AMSE for the estimator has been used as an efficiency criterion because the true parameters for intercept (β_{0i}) and slope (β_{1i}) are equal in all experiments as above.

[7] Since all estimators used in the simulation study are unbiased, the values of variance and mean square error of an estimator are equal.

Table 3: AMSE values for different estimators of stochastic parameter models.

T	6	8	10	12	14	16	18	20
Estimator				$\sigma_\varepsilon = 1$ & $\phi = .45$				
PLS-SC	1161.7078	286.5830	1590.0133	286.7348	145.5133	192.8387	129.5876	78.8564
SPR	151.0063	108.6775	400.2833	226.5832	68.1937	96.4581	90.7558	50.0262
SPR-SC	151.0052	108.6768	400.2828	226.5829	68.1935	96.4579	90.7556	50.0260
SMG	150.9839	108.6624	400.2723	226.5738	68.1847	96.4494	90.7475	50.0178
				$\sigma_\varepsilon = 1$ & $\phi = .95$				
PLS-SC	296.0779	601.3393	600.0485	415.5658	258.8698	206.5223	378.0578	376.2315
SPR	119.3964	278.9932	199.3290	245.5471	75.8107	76.9921	186.5398	236.9335
SPR-SC	119.3955	278.9920	199.3263	245.5431	75.8055	76.9883	186.5355	236.9297
SMG	119.3783	278.9643	199.2930	245.5100	75.7595	76.9442	186.4797	236.8750
				$\sigma_\varepsilon = 15$ & $\phi = .45$				
PLS-SC	1171.4790	294.2605	1596.4111	292.4377	149.9735	196.7339	133.3269	82.3236
SPR	162.2297	117.1815	407.5070	232.6752	72.8079	100.5687	94.6470	53.7125
SPR-SC	161.7574	116.9123	407.3713	232.6022	72.7359	100.5100	94.6061	53.6556
SMG	157.9103	114.0909	405.1224	230.5740	70.8367	98.6555	92.7897	51.8352
				$\sigma_\varepsilon = 15$ & $\phi = .95$				
PLS-SC	622.9230	879.2027	851.1865	648.8132	470.0074	411.1071	570.0349	560.8580
SPR	327.3646	478.9316	392.3680	434.3315	256.1690	251.0837	356.1431	399.1720
SPR-SC	327.1277	478.5953	391.6656	433.4229	254.8246	250.0546	355.0235	398.1656
SMG	323.7681	472.6823	384.6074	426.1067	244.9034	240.5681	342.8339	386.1583

of the all regression parameters are non-stochastic are presented in Table 4. This table displays AMSE values of SPR, SPR-SC, and SMG estimates. Finally, Tables 5 and 6 present AMSE values of PLS-SC, SPR, SPR-SC, MSPR-SC, and SMG estimates when the vector of regression parameters contains both stochastic and non-stochastic parameter (mixed-stochastic parameter model). Specifically, Table 5 displays the results when the intercept parameter is stochastic and the slope parameter is non-stochastic, we refer to this model as mixed-stochastic type-I model. Table 6 displays the inverse case; when the intercept parameter is non-stochastic and the slope parameter is stochastic, also we refer to this model as mixed-stochastic type-II model.[8] In Tables 3, 5, and 6, the all stochastic regression parameters were generated from a normal distribution. However, in Figures 1 and 2 the parameters were generated from student-t with degree of freedom 10 and uniform from -10 to 10 distributions, respectively.

Table 3 indicates that AMSE values for SPR, SPR-SC and SMG are very closely in all simulation situations (for every value of σ_ε and ϕ), this means that the efficiency of SPR and SMG is close to the efficiency of SPR-SC estimator even if $\sigma_\varepsilon = 15$ and $\phi = .95$, then SPR and SMG are good alternatives estimators for SPR-SC in stochastic parameter models. But PLS-SC is inefficient estimator (has highest AMSE) for this model even if $\sigma_\varepsilon = 1$ and $\phi = .45$. While the results of non-stochastic parameter models as in Table 4 indicate that SMG estimator is more efficient (has less AMSE) than SPR and SPR-SC estimators, and then it is a good alternative estimator for PLS-SC in non-stochastic parameter models.

Tables 5 and 6 indicate that PLS-SC is inefficient estimator (has highest AMSE) for these models (type-I and type-II) for every value of σ_ε and ϕ. Also, SPR and SPR-SC estimators are greater in AMSE than SMG in most situations, especially when the errors have large standard deviation ($\sigma_\varepsilon = 15$). Therefore, SMG estimator is more efficient than SPR and SPR-SC estimators and it is a good alternative estimator for MSPR-SC in mixed-stochastic parameter models.

Figures 1 and 2 confirm that PLS-SC is inefficient estimator (has highest RAMSE) for the stochastic parameter models in general, whether the parameters are distributed normal or another distribution (student-t or uniform). While the RAMSE values of SPR and SPR-SC are very closely even if $\phi = .95$, and SMG estimator has minimum AMSE. Therefore, we can conclude that the relative efficiency of SMG estimator is increasing when the regression parameters are distributed non-normal distributions (such as student-t or uniform).

[8] Note that the mixed-stochastic type-I model is equivalent to the random-effects panel data model that well-known in econometric literature such as Baltagi [30] and Hsiao [31] and others.

Table 4: AMSE values for different estimators of non-stochastic parameter models.

T	6	8	10	12	14	16	18	20
Estimator				$\sigma_\varepsilon = 1$ & $\phi = .45$				
PLS-SC	0.0226	0.0180	0.0149	0.0133	0.0117	0.0111	0.0103	0.0105
SPR	0.0281	0.0208	0.0168	0.0145	0.0126	0.0118	0.0109	0.0111
SPR-SC	0.0255	0.0194	0.0161	0.0138	0.0121	0.0114	0.0105	0.0107
SMG	0.0175	0.0116	0.0100	0.0071	0.0051	0.0044	0.0035	0.0035
				$\sigma_\varepsilon = 1$ & $\phi = .95$				
PLS-SC	0.0193	0.0272	0.0303	0.0310	0.0422	0.0389	0.0490	0.0470
SPR	0.0316	0.0389	0.0417	0.0412	0.0540	0.0497	0.0603	0.0588
SPR-SC	0.0304	0.0365	0.0385	0.0376	0.0489	0.0451	0.0546	0.0531
SMG	0.0209	0.0128	0.0092	0.0080	0.0077	0.0040	0.0016	0.0015
				$\sigma_\varepsilon = 15$ & $\phi = .45$				
PLS-SC	5.0817	4.0455	3.3524	2.9949	2.6331	2.4961	2.3092	2.3698
SPR	6.3190	4.6805	3.7806	3.2538	2.8351	2.6624	2.4434	2.5097
SPR-SC	5.7319	4.3729	3.6192	3.1125	2.7275	2.5696	2.3641	2.4159
SMG	3.9281	2.6162	2.2395	1.6071	1.1454	0.9796	0.7880	0.7910
				$\sigma_\varepsilon = 15$ & $\phi = .95$				
PLS-SC	4.3513	6.1296	6.8180	6.9697	9.4963	8.7486	11.0184	10.5803
SPR	7.1163	8.7486	9.3926	9.2803	12.1286	11.1962	13.5678	13.2238
SPR-SC	6.8458	8.2178	8.6557	8.4619	11.0079	10.1370	12.2884	11.9450
SMG	4.6985	2.8901	2.0727	1.7965	1.7241	0.9107	0.3582	0.3361

Table 5: AMSE values for different estimators of mixed-stochastic parameter type-I models.

T	6	8	10	12	14	16	18	20
Estimator				$\sigma_\varepsilon = 1$ & $\phi = .45$				
PLS-SC	42.5935	16.7176	158.1552	20.7387	76.3997	27.7728	25.5579	31.2017
SPR	28.1440	12.5011	126.8526	17.3081	65.4339	24.4185	22.8696	28.3037
SPR-SC	28.1430	12.5003	126.8521	17.3078	65.4337	24.4183	22.8695	28.3035
MSPR-SC	28.1380	12.4981	126.8499	17.3073	65.4332	24.4180	22.8692	28.3032
SMG	28.1215	12.4860	126.8416	17.2987	65.4248	24.4098	22.8613	28.2952
				$\sigma_\varepsilon = 1$ & $\phi = .95$				
PLS-SC	31.1223	113.7420	72.4561	36.4672	8.7816	2.0711	71.8286	85.2755
SPR	19.2128	79.8667	54.3945	28.8520	7.2631	1.7506	62.0244	73.0288
SPR-SC	19.2120	79.8654	54.3918	28.8481	7.2579	1.7467	62.0202	73.0249
MSPR-SC	19.2064	79.8638	54.3902	28.8462	7.2555	1.7458	62.0199	73.0247
SMG	19.1947	79.8377	54.3585	28.8149	7.2119	1.7030	61.9643	72.9703
				$\sigma_\varepsilon = 15$ & $\phi = .45$				
PLS-SC	52.5343	24.5567	164.0455	26.2783	81.2154	31.8164	29.1236	34.7906
SPR	40.0848	20.8322	133.3901	23.1781	72.3780	28.6569	26.7044	32.0908
SPR-SC	40.6557	20.5998	133.2785	23.1847	73.2956	28.6100	26.7498	32.0390
MSPR-SC	38.0632	20.1050	132.8147	22.8804	70.3133	28.5346	26.5452	31.9901
SMG	35.0916	17.8817	130.9597	21.0680	68.4397	26.7367	24.7971	30.2067
				$\sigma_\varepsilon = 15$ & $\phi = .95$				
PLS-SC	357.4704	387.1405	325.2358	269.4800	222.3834	205.1322	260.1526	266.6328
SPR	226.3993	276.3997	249.1998	218.2033	188.8947	175.6246	229.0426	232.7010
SPR-SC	226.6159	276.1689	248.7864	217.3773	187.8297	174.9711	228.1817	231.9148
MSPR-SC	224.5230	275.5007	247.9577	216.4067	186.7705	174.4181	227.8646	231.6045
SMG	222.1396	269.8575	241.0743	209.5643	177.2645	164.9754	215.6170	219.6503

Table 6: AMSE values for different estimators of mixed-stochastic parameter type-II models.

T	6	8	10	12	14	16	18	20
Estimator				$\sigma_\varepsilon = 1$ & $\phi = .45$				
PLS-SC	268.3050	55.3754	546.7841	33.2591	175.0454	54.8584	49.7008	54.8090
SPR	28.1098	12.4915	126.8922	17.2964	65.4255	24.4224	22.8722	28.2791
SPR-SC	28.1087	12.4908	126.8917	17.2969	65.4253	24.4221	22.8720	28.2789
MSPR-SC	28.0953	12.4841	126.8877	17.2923	65.4234	24.4207	22.8708	28.2778
SMG	28.0874	12.4764	126.8812	17.2858	65.4165	24.4137	22.8639	28.2707
				$\sigma_\varepsilon = 1$ & $\phi = .95$				
PLS-SC	76.8349	238.4523	216.9282	124.2924	45.3886	3.7416	168.0867	176.9515
SPR	18.3929	79.1032	53.5576	27.9590	6.5245	1.0289	61.4536	72.7310
SPR-SC	18.3920	79.1021	53.5550	27.9551	6.5192	1.0247	61.4498	72.7272
MSPR-SC	18.3762	79.0894	53.5437	27.9463	6.5096	1.0168	61.4412	72.7182
SMG	18.3750	79.0741	53.5217	27.9219	6.4734	0.9815	61.3936	72.6725
				$\sigma_\varepsilon = 15$ & $\phi = .45$				
PLS-SC	287.3080	60.5266	550.0523	37.4486	177.6554	57.4803	52.2882	57.0881
SPR	37.7539	20.3777	131.3394	22.4409	68.5303	27.3492	25.5501	30.8830
SPR-SC	37.4135	20.9536	131.1374	22.3493	68.4455	27.2948	25.4719	30.8116
MSPR-SC	35.2518	17.0909	130.6431	20.9898	68.1025	27.0299	25.3454	30.6368
SMG	33.4848	15.3430	129.1714	19.5234	66.5490	25.4568	23.7824	29.0194
				$\sigma_\varepsilon = 15$ & $\phi = .95$				
PLS-SC	80.8612	244.5464	224.0472	131.3575	56.3350	12.6885	179.9614	188.0358
SPR	27.1524	88.9928	64.1088	38.3350	20.0432	12.6620	75.4609	86.5948
SPR-SC	26.9147	89.0226	63.7921	37.3767	18.5995	11.6541	74.5337	85.7907
MSPR-SC	23.2522	85.3927	60.6137	35.2174	16.4916	9.8758	72.7077	83.4960
SMG	23.0028	81.9484	55.6523	29.7454	8.3503	1.9382	61.9973	73.2101

7 Real Data Application

In this section, PLS-SC, SPR, SPR-SC, and SMG estimates are computed for Grunfeld [37] investment data set. This data is a classic data set that has been used for decades to develop and demonstrate estimators for panel data models.[9] The used data of our application consists of time series of 10 yearly observations (1935-1944) on 5 large US manufacturing firms for the following three variables: gross investment (y), market value of the firm at the end of the previous year (X_1), and value of the stock of plant and equipment at the end of the previous year (X_2).

First, the randomness of the parameters has been examined by using Swamy's [3] test.[10] The value of the test statistic $\chi^2_{(12)}$ = 245.72 with p-value = 0.0001, so the appropriate model for this data is the stochastic parameter model. Estimation results have been presented in Table 7. These results indicate that SMG estimator have smallest standard errors and highs t-values. Moreover, SMG estimator has the smallest values of all goodness-of-fit measures as in Table 8.

8 Conclusion

In this paper, we examined FGLS (PLS-SC, SPR, SPR-SC, and MSPR-SC) and SMG estimators of panel data models when the errors are first-order serially correlated and the regression parameters are stochastic, non-stochastic, or mixed-stochastic. Moreover, we carried out Monte Carlo simulation study to investigate small samples performance for these estimators. Simulation results indicate that SPR and SMG are efficient alternatives estimators for SPR-SC in stochastic parameter models. But in non-stochastic parameter models, the SMG estimator is more efficient than SPR and SPR-SC estimators and then it is an efficient alternative estimator for PLS-SC, practically. While in mixed-stochastic parameter models, the SMG estimator only is an efficient alternative estimator for MSPR-SC. Also, the results of real data application indicate that SMG estimator has the smallest values of all goodness-of-fit measures. Consequently, we conclude that the SMG estimator is more efficient than FGLS estimators of panel data models, especially in small samples and the model includes one or more non-stochastic parameter.

[9] This data set, even though dated, is of manageable size for classroom use and has been used by Zellner [38] and Taylor [39]. For more details about this data set, see Kleiber and Zeileis [40]. It available at https://www.wiley.com/legacy/wileychi/baltagi/supp/Grunfeld.fil

[10] The null hypothesis of this test: $H_0 : \beta_1 = \cdots = \beta_N = \bar{\beta}$. The power of this test examined by Abonazel [25].

Table 7: Estimation results of different estimators for Grunfeld investment data set.

Estimator	Variable	Coefficient	Standard Error	t-value	p-value
PLS-SC					
	intercept	6.4948	34.1773	0.1900	0.4251
	X_1	0.0629	0.0405	1.5511	0.0638
	X_2	0.0096	0.2197	0.0439	0.4826
SPR					
	intercept	12.8415	20.4223	0.6288	0.2663
	X_1	0.0736	0.0308	2.3869	0.0105
	X_2	0.1237	0.1079	1.1463	0.1287
SPR-SC					
	intercept	11.1976	8.1809	1.3687	0.0888
	X_1	0.0721	0.0300	2.4020	0.0102
	X_2	0.0931	0.1045	0.8903	0.1889
SMG					
	intercept	10.2926	4.0986	2.5113	0.0078
	X_1	0.0772	0.0299	2.5871	0.0064
	X_2	0.1291	0.1005	1.2841	0.1027

Table 8: Goodness-of-fit measures.

Measure	PLS-SC	SPR	SPR-SC	SMG
MAE: Mean absolute error	93.4543	81.8989	82.4011	80.2713
MSE: Mean square error	17195.4489	11636.9245	12357.5340	11130.9532
RMSE: Root of mean square error	135.2511	111.2638	114.6564	108.8181
AIC: Akaike's information criterion	686.1274	666.6043	669.6084	664.3816
BIC: Bayesian information criterion	691.8635	672.3404	675.3445	670.1177

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