A Monte Carlo Study for Swamy's Estimate of Random Coefficient Panel Data Model

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A particularly useful approach for analyzing pooled cross sectional and time series data is Swamy's random coefficient panel data (RCPD) model. This paper examines the performance of Swamy's estimators and tests associated with this model by using Monte Carlo simulation. The Monte Carlo study shed some light into how well the Swamy's estimate perform in small, medium, and large samples, in cases when the regression coefficients are fixed, random, and mixed. The Monte Carlo simulation results suggest that the Swamy's estimate perform well in small samples if the coefficients are random and but it does not when regression coefficients are fixed or mixed. But if the samples sizes are medium or large, the Swamy's estimate performs well when the regression coefficients are fixed, random, or mixed.

Key words: Random Coefficient Panel Data Model, Mixed RCPD Model, Panel Data, Monte Carlo Simulation, Pooling Cross Section and Time Series Data.

1. Introduction

Econometrics commonly uses "Time Series Data" describing a single entity. Another type of data called "Panel Data" which means any data base describing number of individuals across a sequence of time periods. To realize the potential value of the information contained in a panel data see Carlson (1978), Hsiao (1985, 2003), and Baltagi (2008). `

When the performance of one individual form the panel data is interest, separate regression can be estimated for each individual unit. Each relationship, on our model studied, is written as follows:

$$y_{it} = \beta_{0i} + \beta_{1i} x_{it} + \varepsilon_{it}, \qquad i = 1,2,3,\ldots,N \qquad t = 1,2,3,\ldots,T, \tag{1}$$

where i denotes cross-sections and t denotes time-periods. The ordinary least squares (OLS) estimators of β_{0i} and β_{1i} will be best linear unbiased estimators (BLUE) under the following assumptions:

A1: $E(\varepsilon_i) = 0$

A2: $E(\varepsilon_i \varepsilon_i') = \sigma_i^2 I_T$

A3: $E(\varepsilon_i \varepsilon_j') = 0, \quad for\, all\, i \neq j$.

These conditions are sufficient but not necessary for the optimality of the OLS estimator, see Rao and Mitra (1971). If assumption 2 is violated and disturbances are either serially correlated or heteroskedastic, generalize least squares (GLS) will provide relatively more efficient estimator than OLS, see Gendreau and Humphrey (1980). If assumption 3 is violated and contemporaneous correlation is present, we have what Zellner (1962) termed seemingly unrelated regression (SUR) equations. There is gain in efficiency by using SUR estimator rather than OLS, equation by equation estimator, see Zellner (1962, 1963).

Suppose that each regression coefficient in equation (1) is viewed as a random variable, that is, the coefficients β_{0i} and β_{1i} are viewed as invariant over time and varying from one unit to anther.

So, we are assuming that the individuals in our panel data are drawn from a population with a common regression parameter,($\overline{\beta}_j,\ j = 0,1$), which is fixed component, and a random component v_i which will allow the coefficients to differ from unit to unit, i.e.

A4: $\beta_{ji} = \overline{\beta}_j + v_{ji}$, *f*or *i*= 1,2,….,*N*, *j*=0,1.

Model (1) can be rewritten, under assumptions (1) to (4), as:

$$y_{it} = \overline{\beta}_{0i} + \vec{\beta}_{1i}\, x_{it} + e_{it}, \tag{2}$$

where

$$e_{it} = v_{0i} + x_{it}\, v_{1i} + \varepsilon_{it}, \quad i=1,2,\ldots,N,\ t=1,2,\ldots T,$$

model (2) is called "Random Coefficient Panel Data" model examined by Swamy (1970, 1971, 1973, 1974), Kelejian and Stephan (1983), Hsiao and Pesaran (2004), and Murtazashvili and Wooldridge (2008).

Equation (2) can be written in matrix form as

$$Y = X\overline{\beta} + e, \tag{3}$$

where

$$Y' = [Y_1 Y_2 \cdots Y_N], \qquad Y_i' = [y_{1i}\, y_{2i} \cdots y_{Ti}], \qquad X' = [X_1 X_2 \cdots X_N],$$

$$X_i = \begin{bmatrix} 1 & x_{i1} \\ 1 & x_{i2} \\ \vdots & \vdots \\ 1 & x_{iT} \end{bmatrix}, \qquad \overline{\beta} = \begin{bmatrix} \overline{\beta}_0 \\ \overline{\beta}_1 \end{bmatrix}, \qquad e = DV + \varepsilon,$$

$$D=\begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_N \end{bmatrix}, \qquad V=\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix}, \qquad v_i=\begin{bmatrix} v_{0i} \\ v_{1i} \end{bmatrix}.$$

The following assumptions are added to the previous assumptions:

A5: The vector V_i are independently and identically distributed with $E(v_i)=0,$ and $E(v_i v_i')=\psi,$ i=1,2,…,N.

A6: The ε_{it} and v_i are independent for every i and j, so the variance-covariance matrix of e is

$$E(ee')=\begin{bmatrix} X_1\psi X_1'+\sigma_1^2 I_T & 0 & \cdots & 0 \\ 0 & X_2\psi X_2'+\sigma_2^2 I_T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & X_N\psi X_N'+\sigma_N^2 I_T \end{bmatrix}=\Omega,$$

where zeros are $T\times T$ null matrices and ψ is the variance-covariance matrix of β_i as given in assumption (5). If assumptions (1) till (6) hold, then the GLS estimator of $\bar{\beta}$ is given by

$$\hat{\bar{\beta}}=(X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y\,. \tag{4}$$

Swamy (1970) showed that

$$\hat{\bar{\beta}}=\left\{\sum_{i=1}^{N}\left[\psi+\sigma_i^2(X_i'X_i)^{-1}\right]^{-1}\right\}^{-1}\sum_{i=1}^{N}\left[\psi+\sigma_i^2(X_i'X_i)^{-1}\right]^{-1}\hat{\beta}_i \tag{5}$$

where $\hat{\beta}_i$ is the OLS estimator of β_i. The GLS estimator cannot be used in practice, since ψ and σ_i^2 are unknowns. Swamy (1971) suggested the following unbiased and consistent estimators

$$\hat{\sigma}_i^2=\frac{1}{T-K}\hat{\varepsilon}_i'\hat{\varepsilon}_i\,, \tag{6}$$

and

$$\hat{\psi}=\frac{1}{N-1}S_{\hat{\beta}}-\frac{1}{N}\sum_{i=1}^{N}\hat{\sigma}_i^2(X_i'X_i)^{-1}, \tag{7}$$

where

$$S_{\hat{\beta}}=\sum_{i=1}^{N}\hat{\beta}_i\hat{\beta}_i'-\frac{1}{N}\sum_{i=1}^{N}\hat{\beta}_i\sum_{i=1}^{N}\hat{\beta}_i'. \tag{8}$$

Note that $\hat{\sigma}_i^2$ is the mean square error from the OLS regression of Y_i on X_i, and $S_\beta/(N-1)$ is the sample variance-covariance matrix of β_i. Substitute (6), (7), and (8) in (5), we get the feasible generalized lest square (FGLS) estimator of $\hat{\hat{\beta}}$ as follows:

$$\hat{\hat{\beta}} = \left\{ \sum_{i=1}^{N} \left[\hat{\psi} + \hat{\sigma}_i^2 (X_i' X_i)^{-1} \right]^{-1} \right\}^{-1} \sum_{i=1}^{N} \left[\hat{\psi} + \hat{\sigma}_i^2 (X_i' X_i)^{-1} \right]^{-1} \hat{\beta}_i, \tag{9}$$

and the estimated variance-covariance matrix for the RCPD model is

$$Var\left[\hat{\hat{\beta}}\right] = (X' \Omega^{-1} X)^{-1},$$

$$= \left\{ \sum_{i=1}^{N} \left[\hat{\psi} + \hat{\sigma}_i^2 (X_i' X_i)^{-1} \right]^{-1} \right\}^{-1}, \tag{10}$$

Swamy (1973, 1974) showed that the estimator $\hat{\hat{\beta}}_i$ is consistent as both *N* and $T \to \infty$ and is asymptotically efficient as $T \to \infty$.

Because v_i is fixed for given *i*, we can test for random variation indirectly by testing whether or not the fixed coefficient vectors β_i are all equal. That is, we form the null hypothesis

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_N = \overline{\beta}.$$

If different cross-sectional units have the same variance, $\sigma_i^2 = \sigma^2$, *i*=1,...,*N*, the conventional analysis of covariance test for homogeneity. If σ_i^2 are assumed different, as postulated by Swamy (1970, 1971), we can apply the modified test statistic

$$F = \sum_{i=1}^{N} \frac{(\hat{\beta}_i - \hat{\hat{\beta}}^*)' X_i' X_i (\hat{\beta}_i - \hat{\hat{\beta}}^*)}{\hat{\sigma}_i^2}, \tag{11}$$

where

$$\hat{\hat{\beta}}^* = \left[\sum_{i=1}^{N} \frac{1}{\hat{\sigma}_i^2} X_i' X_i \right]^{-1} \left[\sum_{i=1}^{N} \frac{1}{\hat{\sigma}_i^2} X_i' y_i \right].$$

Under H_0, (11) is asymptotically chi-square distributed, with *K* (*N* - 1) degrees of freedom, as *T* tends to infinity and *N* is fixed.

If the regression coefficients in model (3) contain both random and fixed coefficients, the model will be called "Mixed RCPD" model. The Mixed RCPD model is simply a special case of the RCPD model where the variance of certain coefficients, which will be considered as fixed coefficients, are assumed to be equal to zero. Thus equation (9) still applies to estimation after certain elements of the ψ matrix are constrained to equal zero.

2. Simulation Design

A Monte Carlo simulation was conducted to study the behavior of certain estimators and tests in small, medium and large samples. The simulation was designed primarily to investigate estimation and hypothesis tests for the RCPD model discussed before. The settings of the model and results of the simulation study are discussed below.

The values of the independent variable x_{it}, were generated as independent normally distributed random variates with mean μ_X and standard deviation σ_X. The values of x_{it} were allowed to differ for each cross-sectional unit: However, once generated for all *N* cross-sectional units the values were held fixed over all Monte Carlo trials. The value of μ_X was set equal to zero and the value of σ_X was set equal to one. The disturbances, ε_{it}, were generated as independent normally distributed random variates, independent of the x_{it} values, with mean zero and standard deviation σ_ε. The disturbances were allowed to differ for each cross-sectional unit on a given Monte Carlo trial and were allowed to differ between trials. The standard deviation of the disturbances was set equal to either 1, 3, or 5 and held fixed for each cross-sectional unit. The values of *N* and *T* were chosen to be 10, 25, and 100 to represent small, medium and large samples for the number of individuals and the time dimension.

The parameters, β_{0i} and β_{1i}, were set at several different values to allow study of the estimators under conditions where the model was both properly and improperly specified. Also, test of hypothesis for randomness was examined to determine the observed level of significance and to obtain an idea of the power of the test. Five different combinations of β_{0i} and β_{1i} are used as given in Table (1). Note that a variance of zero simply means that the coefficient is fixed and equal over all cross-sectional units. These models will be estimated using Swamy's estimators in order to study the behavior of the coefficient mean estimator under misspecification of the model and to study the behavior of the tests for randomness of coefficients.

Table (1) Values of Coefficient Means and Variances Used in the Simulation

Model	$\bar{\beta}_0$	$Var(\beta_0)$	$\bar{\beta}_1$	$Var(\beta_1)$
1	5	5	5	5
2	5	25	5	25
3	5	0	5	0
4	5	0	5	5
5	5	5	5	0

There are 45 experimental settings for the simulation, and 10,000 Monte Carlo trials were used for each settings. The results were recorded in Tables (2) through (6), with each table consisting of three panels, numbered I through III, for the different samples sizes (10, 25, and 100). And each panel from this panels corresponding to three settings of the disturbance standard deviation (1, 3, and 5). Each of the tables provides the results for a particular scheme of generation of the regression coefficients.

3. Monte Carlo Results

Tables (2) through (6) are set up to show the following information:

The coefficient mean estimators (or the estimators of the fixed coefficients), $\hat{\bar{\beta}}_0$ and $\hat{\bar{\beta}}_1$, that are computed as in equation (9). The values shown in the first row of each panel of each table are the averages over all 10,000 Monte Carlo trials at a particular setting.

Table (2) Results of RCPD Estimation When $\beta_0 \sim N\,(5, 5)$ and $\beta_1 \sim N\,(5, 5)$

N=T	σ_ε	1		3		5	
		β_0	β_1	β_0	β_1	β_0	β_1
I. 10	$\hat{\bar{\beta}}$	4.999	5.012	5.000	4.981	5.835	6.215
	$\hat{\psi}$	4.966	4.992	4.999	5.014	5.023	4.989
	Bias of $\hat{\bar{\beta}}$	0.001	-0.012	0.000	0.019	-0.835	-1.215
	MSE of $\hat{\bar{\beta}}$	0.508	0.515	0.592	0.627	1.404	2.209
	% Negative Variance Estimates	0.0	0.0	0.1	0.2	1.0	1.9
	%Rejections $H_0: \sigma_\beta^2 = 0$	100.0	100.0	99.0	97.3	86.1	80.8
II. 25	$\hat{\bar{\beta}}$	5.000	4.999	4.998	5.001	4.992	5.002
	$\hat{\psi}$	5.020	5.003	4.996	5.010	4.969	4.970
	Bias of $\hat{\bar{\beta}}$	0.000	0.001	0.002	-0.001	0.008	-0.002
	MSE of $\hat{\bar{\beta}}$	0.202	0.202	0.214	0.216	0.239	0.240
	% Negative Variance Estimates	0.0	0.0	0.0	0.0	0.0	0.0
	%Rejections $H_0: \sigma_\beta^2 = 0$	100.0	100.0	100.0	100.0	100.0	100.0
III. 100	$\hat{\bar{\beta}}$	4.999	4.998	4.999	5.000	5.002	5.000
	$\hat{\psi}$	5.000	4.996	4.999	4.999	5.002	5.007
	Bias of $\hat{\bar{\beta}}$	0.001	0.002	0.001	0.000	-0.002	0.000
	MSE of $\hat{\bar{\beta}}$	0.050	0.050	0.051	0.051	0.053	0.053
	% Negative Variance Estimates	0.0	0.0	0.0	0.0	0.0	0.0
	%Rejections $H_0: \sigma_\beta^2 = 0$	100.0	100.0	100.0	100.0	100.0	100.0

Table (3) Results of RCPD Estimation When $\beta_0 \sim$ N (5, 25) and $\beta_1 \sim$ N (5, 25)

N=T	σ_ε	1		3		5	
		β_0	β_1	β_0	β_1	β_0	β_1
I. 10	$\hat{\bar{\beta}}$	4.997	5.026	5.008	4.998	5.015	4.998
	$\hat{\psi}$	24.816	24.943	24.972	25.089	25.062	24.986
	Bias of $\hat{\bar{\beta}}$	0.003	-0.026	-0.008	0.002	-0.015	0.002
	MSE of $\hat{\bar{\beta}}$	2.493	2.511	2.597	2.649	2.775	2.865
	% Negative Variance Estimates	0.0	0.0	0.0	0.0	0.0	0.0
	%Rejections $H_0: \sigma^2_\beta = 0$	100.0	100.0	100.0	100.0	99.8	99.7
II. 25	$\hat{\bar{\beta}}$	5.000	4.998	4.998	5.001	4.981	5.001
	$\hat{\psi}$	25.101	25.011	24.978	25.071	24.866	24.886
	Bias of $\hat{\bar{\beta}}$	0.000	0.002	0.002	-0.001	0.019	-0.001
	MSE of $\hat{\bar{\beta}}$	1.006	1.002	1.014	1.018	1.036	1.038
	% Negative Variance Estimates	0.0	0.0	0.0	0.0	0.0	0.0
	%Rejections $H_0: \sigma^2_\beta = 0$	100.0	100.0	100.0	100.0	100.0	100.0
III. 100	$\hat{\bar{\beta}}$	4.999	4.995	4.997	5.000	5.003	4.999
	$\hat{\psi}$	25.001	24.982	24.992	24.997	25.010	25.036
	Bias of $\hat{\bar{\beta}}$	0.001	0.005	0.003	0.000	-0.003	0.001
	MSE of $\hat{\bar{\beta}}$	0.250	0.250	0.251	0.251	0.253	0.253
	% Negative Variance Estimates	0.0	0.0	0.0	0.0	0.0	0.0
	%Rejections $H_0: \sigma^2_\beta = 0$	100.0	100.0	100.0	100.0	100.0	100.0

The estimated variance of each coefficient, $\hat{Var}(\beta_k) = \hat{\psi}$, averaged over 10,000 trials, is shown in the second row. The estimates are computed as the diagonal elements in equation (7).

The bias value of the coefficient mean estimators, $\hat{\bar{\beta}}_0$ and $\hat{\bar{\beta}}_1$, are computed as $bias(\hat{\bar{\beta}}) = \bar{\beta} - \hat{\bar{\beta}}$ where $\hat{\bar{\beta}}$ is a vector of coefficients mean estimators and $\bar{\beta}$ is a true vector of coefficients mean. The bias values shown in the row three of each panel.

Table (4) Results of RCPD Estimation When $\beta_0 = 5$ and $\beta_1 = 5$

N=T	σ_ε	1		3		5	
		β_0	β_1	β_0	β_1	β_0	β_1
I. 10	$\hat{\bar{\beta}}$	4.939	5.054	5.389	4.579	4.761	5.026
	$\hat{\psi}$	0.000	0.000	0.006	0.004	0.011	-0.017
	Bias of $\hat{\bar{\beta}}$	0.061	-0.054	-0.389	0.421	0.239	-0.026
	MSE of $\hat{\bar{\beta}}$	0.013	0.015	0.217	0.248	0.201	0.198
	% Negative Variance Estimates	12.5	15.2	12.4	15.3	12.5	15.3
	%Rejections $H_0:\sigma^2_\beta=0$	25.0	25.0	25.7	24.9	25.4	25.4
II. 25	$\hat{\bar{\beta}}$	5.006	5.008	4.991	4.994	5.033	5.017
	$\hat{\psi}$	0.000	0.000	-0.001	-0.002	-0.004	-0.002
	Bias of $\hat{\bar{\beta}}$	-0.006	-0.008	0.009	0.006	-0.033	-0.017
	MSE of $\hat{\bar{\beta}}$	0.001	0.001	0.013	0.012	0.036	0.035
	% Negative Variance Estimates	1.4	4.4	1.4	4.5	1.1	4.5
	%Rejections $H_0:\sigma^2_\beta=0$	12.3	12.4	12.1	11.7	12.5	12.3
III. 100	$\hat{\bar{\beta}}$	5.000	5.000	5.000	5.000	5.000	5.000
	$\hat{\psi}$	0.000	0.000	0.000	0.000	0.000	0.000
	Bias of $\hat{\bar{\beta}}$	0.000	0.000	0.000	0.000	0.000	0.000
	MSE of $\hat{\bar{\beta}}$	0.000	0.000	0.001	0.001	0.002	0.002
	% Negative Variance Estimates	0.0	0.0	0.0	0.0	0.0	0.0
	%Rejections $H_0:\sigma^2_\beta=0$	8.4	8.2	8.0	7.6	8.0	8.1

The Mean Square Error (MSE) of coefficient mean estimators that are computed as $MSE(\hat{\bar{\beta}}_k) = \hat{Var}(\hat{\bar{\beta}}_k) + [bias(\hat{\bar{\beta}}_k)]^2$ where $\hat{Var}(\hat{\bar{\beta}}_k)$ is the estimated variance of the coefficient mean estimator, and is computed as the kth diagonal element of the variance-covariance matrix given in equation (10). The MSE values shown in the row four of each panel.

Table (5) Results of RCPD Estimation When β_0 = 5 and β_1 ~ N (5, 5)

N=T	σ_ε	1		3		5	
		β_0	β_1	β_0	β_1	β_0	β_1
I. 10	$\hat{\bar{\beta}}$	5.058	4.966	4.298	4.593	4.893	4.970
	$\hat{\psi}$	0.000	5.019	0.002	4.973	0.000	5.046
	Bias of $\hat{\bar{\beta}}$	-0.058	0.034	0.702	0.407	0.107	0.030
	MSE of $\hat{\bar{\beta}}$	0.014	0.517	0.339	0.700	0.211	0.799
	% Negative Variance Estimates	10.5	0.0	11.1	0.6	11.0	2.6
	%Rejections $H_0:\sigma_\beta^2=0$	45.4	100.0	27.9	97.1	25.8	79.9
II. 25	$\hat{\bar{\beta}}$	5.002	5.002	4.997	5.010	4.988	5.003
	$\hat{\psi}$	0.000	4.993	0.000	5.013	0.003	5.019
	Bias of $\hat{\bar{\beta}}$	-0.002	-0.002	0.003	-0.010	0.012	-0.003
	MSE of $\hat{\bar{\beta}}$	0.001	0.201	0.013	0.216	0.037	0.241
	% Negative Variance Estimates	0.8	0.0	0.8	0.0	0.9	0.0
	%Rejections $H_0:\sigma_\beta^2=0$	21.1	100.0	12.8	100.0	12.3	100.0
III. 100	$\hat{\bar{\beta}}$	5.000	5.001	5.000	5.000	5.000	5.001
	$\hat{\psi}$	0.000	5.000	0.000	4.994	0.000	5.005
	Bias of $\hat{\bar{\beta}}$	0.000	-0.001	0.000	0.000	0.000	-0.001
	MSE of $\hat{\bar{\beta}}$	0.000	0.050	0.001	0.051	0.002	0.053
	% Negative Variance Estimates	0.0	0.0	0.0	0.0	0.0	0.0
	%Rejections $H_0:\sigma_\beta^2=0$	14.7	100.0	8.3	100.0	7.8	100.0

It is possible to obtain negative estimates of the coefficient variances, $\sigma^2_{\beta_K}$, when equation (7) is used to compute the variance-covariance matrix. The percentages of negative variance estimates shown in the row five of each panel.

The sixth row of each panel records the percentage of rejections of the null hypothesis $H_0:\sigma^2_{\beta_k} = 0$, for $k = 0$ and 1, at a nominal 5% level of significance. The chi-squared statistic in equation (11) is used to perform the test.

Table (6) Results of RCPD Estimation When β_0 ~ N (5, 5) and β_1 = 5

N=T	σ_ε	1		3		5	
		β_0	β_1	β_0	β_1	β_0	β_1
I. 10	$\hat{\bar{\beta}}$	4.987	4.829	4.845	4.953	3.388	4.690
	$\hat{\psi}$	5.018	0.000	5.000	-0.013	5.039	0.012
	Bias of $\hat{\bar{\beta}}$	0.013	0.171	0.155	0.047	1.612	0.310
	MSE of $\hat{\bar{\beta}}$	0.511	0.034	0.613	0.135	3.078	0.359
	% Negative Variance Estimates	0.0	14.4	0.3	14.1	1.4	13.9
	%Rejections $H_0:\sigma_\beta^2=0$	100.0	48.6	98.8	27.6	86.2	26.7
II. 25	$\hat{\bar{\beta}}$	5.001	4.976	5.005	4.993	4.989	4.989
	$\hat{\psi}$	4.993	0.000	5.016	0.002	5.019	0.003
	Bias of $\hat{\bar{\beta}}$	-0.001	0.024	-0.005	0.007	0.011	0.011
	MSE of $\hat{\bar{\beta}}$	0.201	0.002	0.215	0.013	0.240	0.032
	% Negative Variance Estimates	0.0	4.1	0.0	4.3	0.0	4.4
	%Rejections $H_0:\sigma_\beta^2=0$	100.0	20.6	100.0	13.2	100.0	12.8
III. 100	$\hat{\bar{\beta}}$	5.001	5.000	5.000	5.000	5.001	5.000
	$\hat{\psi}$	5.001	0.000	4.997	0.000	5.005	-0.001
	Bias of $\hat{\bar{\beta}}$	-0.001	0.000	0.000	0.000	-0.001	0.000
	MSE of $\hat{\bar{\beta}}$	0.050	0.000	0.051	0.001	0.053	0.002
	% Negative Variance Estimates	0.0	0.0	0.0	0.0	0.0	0.0
	%Rejections $H_0:\sigma_\beta^2=0$	100.0	14.3	100.0	8.4	100.0	8.0

As a guide to interpreting the tables, let us consider Table (2) as an example. When $\sigma\varepsilon = 1$ and N=T=10 (small samples), the averages mean and variance for β_0 over all 10,000 Mote Carlo trials are 4.999 and 4.966 respectively. Note that the true coefficients values for mean and variance are 5 and 5, and the values of bias and MSE for $\hat{\bar{\beta}}_0$ are 0.001 and 0.508. And the averages mean and variance for β_1 are 5.012 and 4.992 respectively. While the true coefficients values for mean and variance are 5 and 5. While the percentage of negative variance estimates for $\hat{\bar{\beta}}_0$ and $\hat{\bar{\beta}}_1$ is zero. Note that this percentage should be zero. And the percentage of rejections of the null hypothesis $H_0:\sigma^2_{\beta_k} = 0$ for β_0 and β_1 is 100. This means that the randomness test is performing as designed even in small samples. As the variation in the disturbances increase, from $\sigma\varepsilon = 1$ to $\sigma\varepsilon = 3$, the estimators get worst. Increasing both the number of individuals and the time series data will make the estimators better.

4. Concluding Remarks

From Tables (2) till (6), several observations concerning the RCPD estimators and the test statistics for the randomness test can be made:

1- The Swamy's estimators performs well when the coefficients are random, even though the samples are small (*T*=10). The biases, (true coefficient – estimated coefficient), of the Swamy's estimators of β_i and ψ decrease when the time series observations and the number individual units getting large. From Tables (2) and (3), the bias and MSE are doing better in small and large variation of the parameters. In general, the Swamy's estimate perform best when both coefficients are random.

2- When the coefficients are random, a small number of negative variance estimates occurs for the small sample size, the negative variance estimates does not appear in medium and large samples.

3- When both coefficients are fixed, Table (4), and the sample size is small, the RCPD model is inappropriate and a large number of negative variance estimates occurs as suggested in Dielman (1980). Thus, the appearance of negative variance estimates would suggest the possibility that the coefficient be treated as fixed.

4- When both coefficients are fixed and the samples sizes are medium or large, the RCPD model is appropriate and the negative variance estimates will not appear.

5- When one of the coefficients is fixed and the sample size is small, the Swamy's estimators will not perform as well as might be expected. The appearance of negative variance estimates, in Tables (5) and (6), would suggest misspecification occurrence in the assumptions. But if the samples sizes are medium or large, the Swamy's estimators perform well.

6- The test for randomness performs well overall. The best produces a high percentage of rejections of the hypothesis $H_0 : \sigma^2_{\beta_k} = 0$ is when the coefficients are random and a low percentage when the null hypothesis is true.

7- As the variation in the disturbances increases (relative to the variation due to the explanatory variable), the performance of the Swamy's estimators deteriorates. This is also true for the power of the test for significance of the coefficient means.

8- The behavior of Swamy's estimators is not affected by the changes in the parameter mean but it is affected by the changes in the parameter variance. This conclusion applies to the three models (RCPD, Fixed, and Mixed RCPD models).

The Monte Carlo simulation results suggest that the Swamy's estimators perform well in small samples if the coefficients are random and but it does not in fixed or Mixed RCPD models. But if the samples sizes are medium or large, the Swamy's estimators performs well for the three models. Finally, some caution must be taken before using the Swamy's estimators, and pretesting procedures of the randomness of the coefficients must be made. This simulation has been limited in scope, as all simulations must be. Hopefully it will shed some light on performance of Swamy's estimators in panel data.

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