ELC 2050: Fields & Wave Propagation

Department of Electronics and Electrical Communications Engineering

Introduced By:

Eng. Mohamed Ossama Ashour

E-mail: vert4231@gmail.com

First term 20-21

Course Information

- **Instructor:** Dr. Hanna A. Kirolous & Dr. Tamer Abuelfadl
- **TA:** Eng. Mohamed Ossama Ashour
- **Mark Distribution:** Midterm (30), 2 Quizzes (10), 2 Labs (20), Final (90)
- The tutorial slides are available on the following link:

  https://scholar.cu.edu.eg/?q=moashour/classes/elc-2050/course-material
- For any questions regarding the curriculum, Ask on this sheet:

  https://docs.google.com/spreadsheets/d/12sT3zWfz5GJQQoZ-0zJpRtb-vDTonJEOcBa9BrYqZDQ/edit?usp=sharing
- **Contact info:** vert4231@gmail.com with mail subject “FWP_F20”

Agenda

- Review of Vector Analysis
  - Orthogonal Co-ordinate Systems
  - Vector Algebra (Addition, Subtraction, and Products)
  - Problems

Orthogonal Co-ordinate Systems

- A point or vector can be represented in any curvilinear coordinate system.

- A coordinate system is orthogonal if the coordinates are mutually perpendicular.

- The three best-known orthogonal coordinate systems are the Cartesian, the cylindrical and the spherical.

Vector Analysis

Orthogonal Co-ordinate Systems

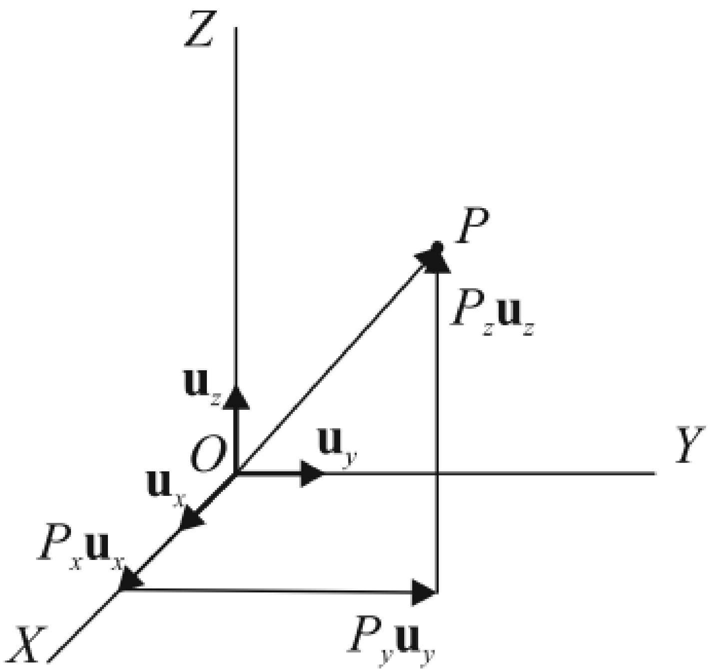
1. Cartesian Coordinates

- It specifies the position of any point ‘P’ in three-dimensional space by three Cartesian coordinates (x, y, z).

$$\underline{d\ell} = dx\,\underline{u}_x + dy\,\underline{u}_y + dz\,\underline{u}_z$$

$$\underline{dS} = dy\,dz\,\underline{u}_x + dx\,dz\,\underline{u}_y + dx\,dy\,\underline{u}_z$$

$$d\upsilon = dx\,dy\,dz$$



Orthogonal Co-ordinate Systems

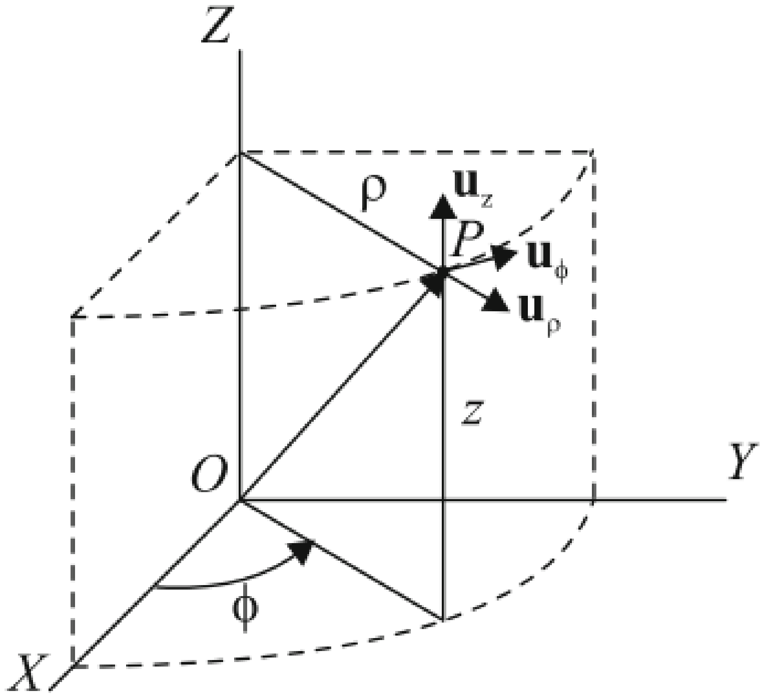
2. Cylindrical Coordinates

- It specifies the position of any point ‘P’ in three-dimensional space by the three coordinates (ρ, ϕ, z) defined as:

-- The radial distance ρ is the Euclidean distance from the Z-axis to the point P, that is, the radius of the cylinder passing through P.

-- The azimuthal angle ϕ is the angle between the X-axis and the line from the origin to the projection of P on the XY-plane.

-- The height z is the Euclidean distance from the XY-plane to the point P.



Vector Analysis

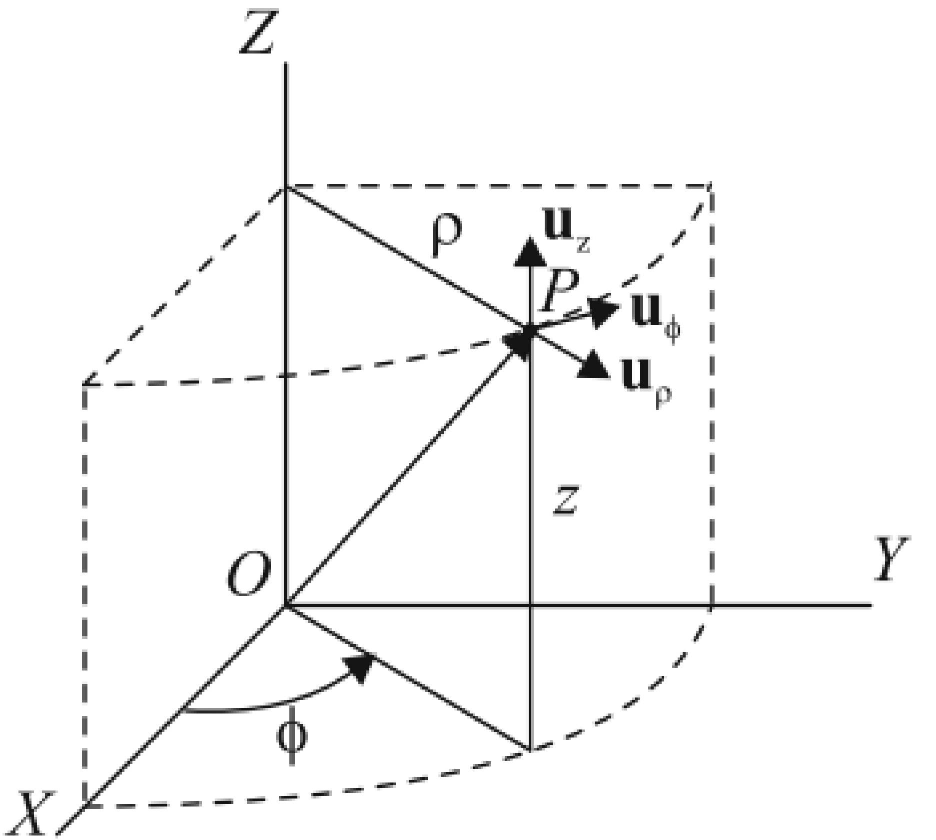
Orthogonal Co-ordinate Systems

2. Cylindrical Coordinates

$$\underline{d\ell} = d\rho\,\underline{u}_\rho + \rho\,d\varphi\,\underline{u}_\varphi + dz\,\underline{u}_z$$

$$\underline{dS} = \rho\,d\varphi\,dz\,\underline{u}_\rho + d\rho\,dz\,\underline{u}_\varphi + \rho\,d\rho\,d\varphi\,\underline{u}_z$$

$$d\upsilon = \rho\,d\rho\,d\varphi\,dz$$



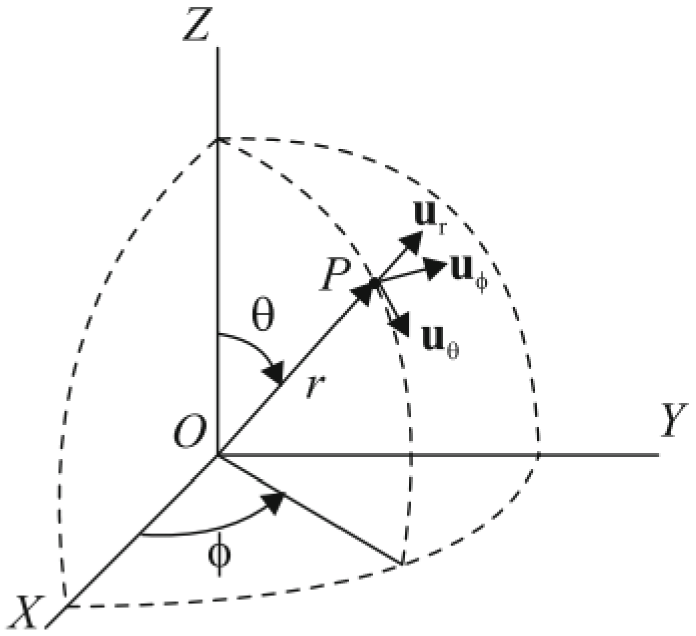
Vector Analysis

Orthogonal Co-ordinate Systems

3. Spherical Coordinates

- It specifies the position of any point ‘P’ in three-dimensional space by the three coordinates (r, θ, ϕ) defined as:

-- The radius or radial distance r is the Euclidean distance from the origin O to P, that is, the radius of a sphere centered at the origin and passing through P.
-- The inclination (or polar angle or colatitude) θ is the angle between the Z-axis and the position vector of P.
-- The azimuthal angle ϕ is the angle between the X-axis and the line from the origin to the projection of P on the XY-plane(the same one in cylindrical coordinates).



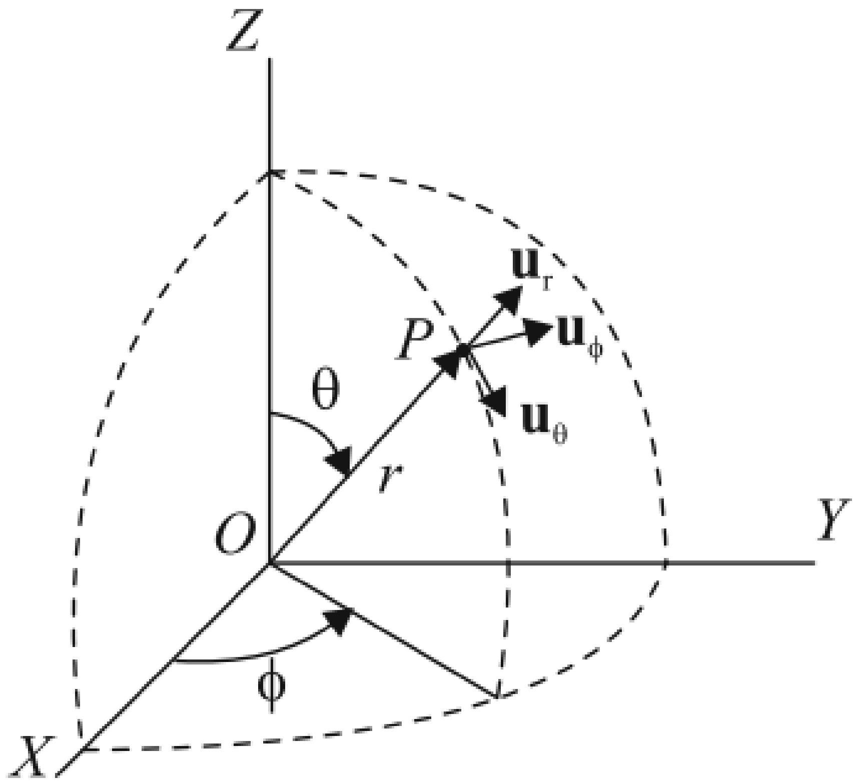
Orthogonal Co-ordinate Systems

3. Spherical Coordinates

$$\underline{d\ell} = dr\,\underline{u}_r + r\,d\theta\,\underline{u}_\theta + r\sin\theta\,d\varphi\,\underline{u}_\varphi$$

$$\underline{dS} = r^2\sin\theta\,d\theta\,d\varphi\,\underline{u}_r + r\sin\theta\,dr\,d\varphi\,\underline{u}_\theta + r\,dr\,d\theta\,\underline{u}_\varphi$$

$$d\upsilon = r^2\sin\theta\,dr\,d\theta\,d\varphi$$



Vector Analysis

Orthogonal Co-ordinate Systems

- The relationships between the variables (x, y, z) & (ρ, ϕ, z) :

$$x = \rho\cos\phi\,, \quad y = \rho\sin\phi\,, \quad z = z$$

$$\rho = \sqrt{x^2 + y^2}\,, \quad \phi = \tan^{-1}\frac{y}{x}\,, \quad z = z$$

- The relationships between the variables (x, y, z) & (r, θ, ϕ) :

$$x = r\sin\theta\cos\phi\,, \quad y = r\sin\theta\sin\phi\,, \quad z = r\cos\theta$$

$$r = \sqrt{x^2 + y^2 + z^2}\,, \quad \theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}\,, \quad \phi = \tan^{-1}\frac{y}{x}$$

Vector Analysis

Orthogonal Co-ordinate Systems

- The relationships between the rectangular & cylindrical unit vectors:

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x\cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y\cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z\cdot$	0	0	1

- The relationships between the rectangular & spherical unit vectors:

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x\cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y\cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z\cdot$	$\cos\theta$	$-\sin\theta$	0

Vector Analysis

Vector Addition and Subtraction

We know that a vector has a magnitude and a direction. A vector **A** can be written as

$$\mathbf{A} = \mathbf{a}_A A, \qquad \mathbf{A} = \mathbf{a}_{u_1} A_{u_1} + \mathbf{a}_{u_2} A_{u_2} + \mathbf{a}_{u_3} A_{u_3}$$

where A is the magnitude (and has the unit and dimension) of **A**,

$$A = |\mathbf{A}|, \qquad A = |\mathbf{A}| = (A_{u_1}^2 + A_{u_2}^2 + A_{u_3}^2)^{1/2}$$

and $\mathbf{a}_A$ is a dimensionless unit vector[+] with a unity magnitude having the direction of **A**. Thus,

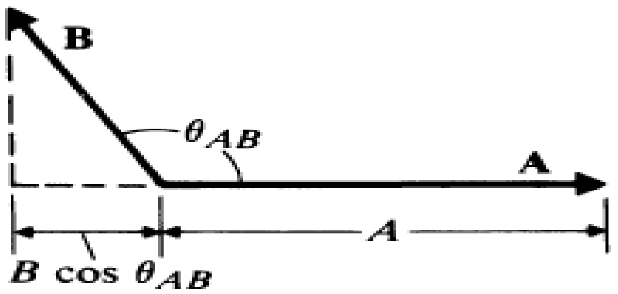
$$\mathbf{a}_A = \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{\mathbf{A}}{A}.$$

$$\mathbf{C} = \mathbf{A} + \mathbf{B} \qquad \mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$$

Products of Vectors

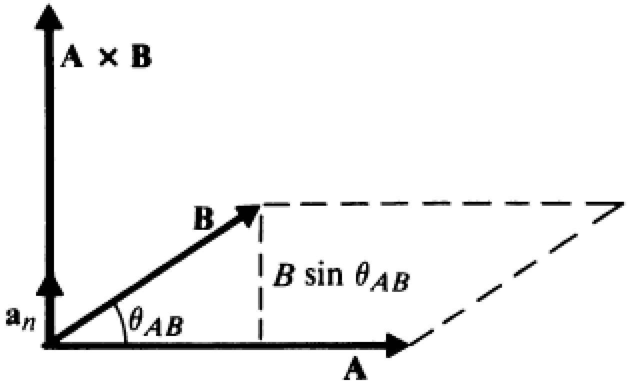
1. SCALAR DOT PRODUCT

$$\mathbf{A} \cdot \mathbf{B} \triangleq AB \cos \theta_{AB}.$$



2. VECTOR CROSS PRODUCT

$$\mathbf{A} \times \mathbf{B} \triangleq \mathbf{a}_n |AB \sin \theta_{AB}|.$$



NOTE: Division by a vector is not defined, and expressions such as B/A are meaningless.

Problems

(1) (*a*) Give the rectangular coordinates of the point $C(\rho = 4.4, \phi = -115°, z = 2)$. (*b*) Give the cylindrical coordinates of the point $D(x = -3.1, y = 2.6, z = -3)$. (*c*) Specify the distance from C to D.

Ans. $C(x = -1.860, y = -3.99, z = 2)$; $D(\rho = 4.05, \phi = 140.0°, z = -3)$; 8.36

(2) Transform to cylindrical coordinates: (*a*) $\mathbf{F} = 10\mathbf{a}_x - 8\mathbf{a}_y + 6\mathbf{a}_z$ at point $P(10, -8, 6)$; (*b*) $\mathbf{G} = (2x + y)\mathbf{a}_x - (y - 4x)\mathbf{a}_y$ at point $Q(\rho, \phi, z)$. (*c*) Give the rectangular components of the vector $\mathbf{H} = 20\mathbf{a}_\rho - 10\mathbf{a}_\phi + 3\mathbf{a}_z$ at $P(x = 5, y = 2, z = -1)$.

Ans. $12.81\mathbf{a}_\rho + 6\mathbf{a}_z$; $(2\rho\cos^2\phi - \rho\sin^2\phi + 5\rho\sin\phi\cos\phi)\mathbf{a}_\rho + (4\rho\cos^2\phi - \rho\sin^2\phi - 3\rho\sin\phi\cos\phi)\mathbf{a}_\phi$; $H_x = 22.3$, $H_y = -1.857$, $H_z = 3$

Extra watching

- **Spherical Coordinate System (With 3D Animation)**
  - https://www.youtube.com/watch?v=FDyenWWlPdU
- **Cylindrical Coordinate system**
  - https://www.youtube.com/watch?v=EthQB3325GM
    - The reference and its solution manual are available on the following links:

http://www.mediafire.com/file/2cd15byqzcqcc4e/David_K._Cheng%252C_Field_and_Wave_Electromagnetics.pdf/file

http://www.mediafire.com/file/bizl04s5c1p1v87/Field_and_Wave_Electromagnetics_Solution_Manual.pdf/file